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Asymptotic safety of gravity and the Higgs boson mass

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Outline

- Asymptotic safety versus renormalizability
- Asymptotically safe gravity
- Standard Model, observations, and asymptotic safety
- Higgs boson mass
- Conclusions

Asymptotic safety versus renormalizability

Generic quantum field theory

- Take some field theory and write the most general Lagrangian.
- Compute all amplitudes in all orders of perturbation theory.
- Require that the theory is unitary, Lorentz - invariant, causal, etc - infinite number of conditions for infinite number of processes.
- Solve these consistency equations. Hopefully, the theory will be characterised by a finite number of essential parameters - coupling constants, making the predictions possible.

RG approach

Introduce dimensionless coupling constants g_i constants for all terms in the action:

$$g_i = \mu^D G_i, \quad G_i \text{ are dimensionfull in general}$$

D is the dimension of coupling constant.

RG equations: from requirement that physical amplitudes are μ-independent,

$$\mu \frac{\partial g_i}{\partial \mu} = \beta_i(g)$$

Different possibilities

- Renormalizable asymptotically free theories – Gaussian UV fixed point: essential couplings $g_i \to 0$ at $\mu \to \infty$. The number of these couplings is finite - only operators with dimension ≤ 4 are allowed.
- Asymptotically safe theories – non-Gaussian UV fixed point $g^* \neq 0$: $\beta_i(g^*) = 0$. If the dimensionality of the critical surface in the space of coupling constants (which points are attracted to g^*) is finite, the theory is predictable.

Known solutions

- Asymptotically free theories:
  - QCD
  - Certain GUTS
  - Renormalizable theories in 2d and 3d
- Asymptotically safe, but non-renormalizable theories
  - Scalar field theory in 3d at Wilson-Fischer fixed point (critical surface is 2-dimensional)
  - Non-linear σ model in 3d
  - Complete theory of pions and nucleons in 4d

The Standard Model is neither asymptotically free nor asymptotically safe!

Methods to study asymptotic safety

- ϵ - expansion
- Lattice simulations
- Functional renormalisation group:

$$\mu\,\partial_\mu S_\mu = \frac{1}{2}\,\mathrm{STr}\left[\left(S_\mu^{(2)} + \mathcal{R}_\mu\right)^{-1}\mu\,\partial_\mu \mathcal{R}_\mu\right]\;.$$

where

$S_\mu \longrightarrow S_{class}$ for $\mu \longrightarrow \infty$,

$S_\mu \longrightarrow S_{eff}$ for $\mu \longrightarrow 0$,

$\mathcal{R}_\mu$ is the “window" function.

Gravity

Conjecture Weinberg '79: Gravity may be asymptotically safe.

ϵ-expansion argument -

$$S_G = -\frac{1}{16\pi G_0}\int d^D x\sqrt{g}R$$

$$G(\mu) = \mu^{D-2}G_0, \quad \mu\frac{d}{d\mu}G(\mu) = (D-2)G(\mu) - bG^2(\mu)\ .$$

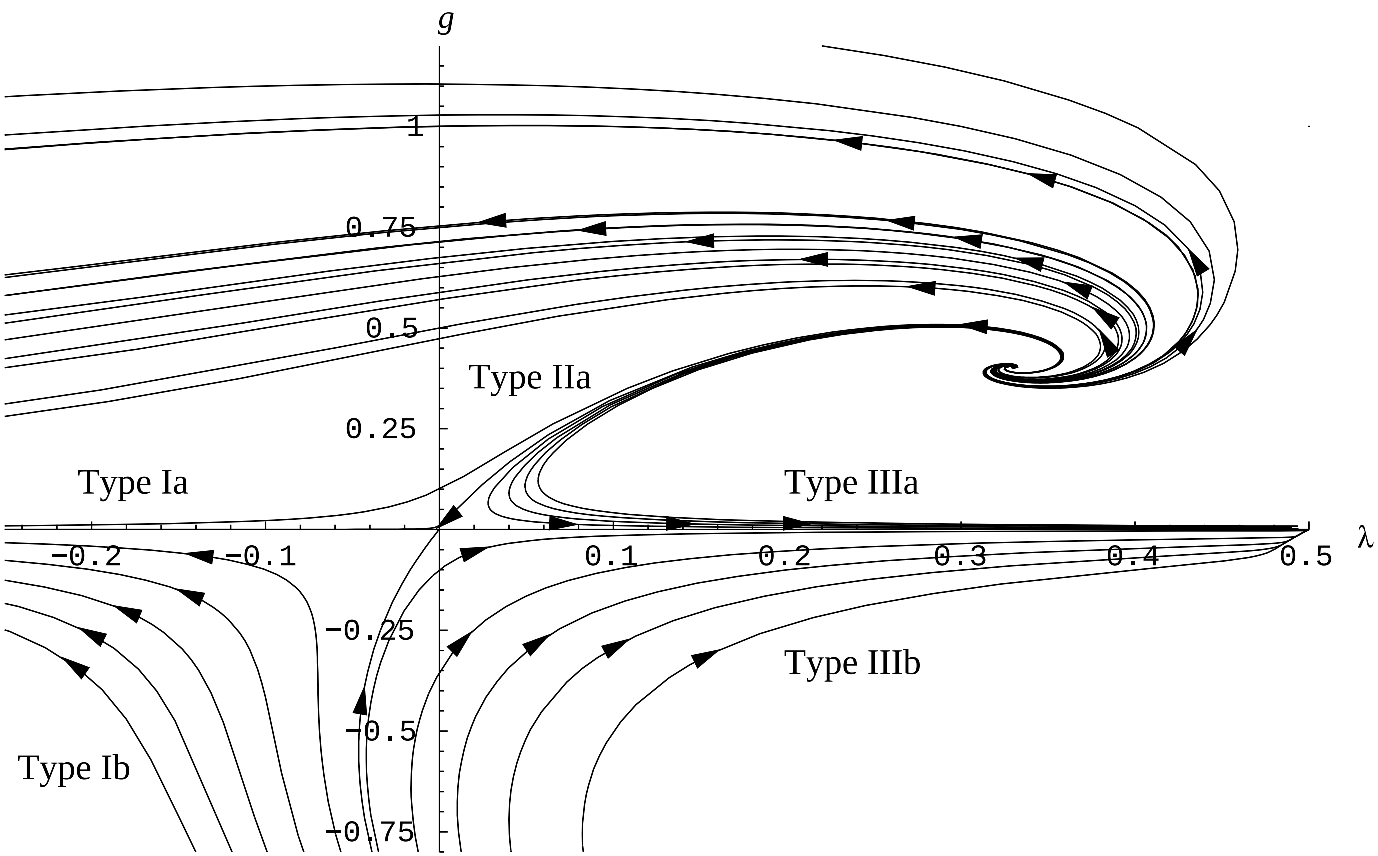
Fixed point:

$$G^* = \frac{D-2}{b}, \quad G_0^*(\mu) = \frac{G^*}{\mu^{D-2}} \to 0 \text{ if } \mu \to \infty$$

Computations give $b > 0$. Gastmans et al '77, Christensen and Duff '77, Kawaiand and Ninomiya '90, Percacci '06,...

Functional RG analysis - Reuter ’96, Percacci et al, Niedermaier ’09, ...

$$S_{\mathbf{EH}} = \frac{1}{16\pi G}\int d^4x\,\sqrt{-g}\,\{-R + 2\Lambda\}\ ,$$



Reuter and Saueressig ’02

Extra evidence

- Higher derivative gravity – Stelle ’77, Fradkin & Tseytlin ’82, Avramidi& Barvinsky ’85,...
- Large N (matter fields) expansion – Tomboulis ’77, ’80, Smolin ’82, Percacci ’06,...
- Perturbation theory – Niedermaier, ’09

What if indeed gravity is asymptotically safe?

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Any predictions for particle physics and LHC?

Possible consequence: Electroweak theory + Gravity is a final theory

Experimental evidence for physics beyond the SM

- i. Neutrino masses and oscillations
- ii. Dark matter
- iii. Baryon asymmetry of the Universe
- iv. Inflation

require only a modest extension of the SM (νMSM) by 3 singlet right-handed fermions (needed for i-iii) with masses in keV - GeV area, and non-minimal coupling of the Standard Model Higgs field to Ricci scalar (needed for iv).

Realisation: νMSM

SM fermions

	quarks					
left	u	d	c	s	t	b
right	u	d	c	s	t	b
left	ν_e	e	ν_μ	μ	ν_τ	τ
right		e		μ		τ
	leptons					

νMSM fermions

	quarks					
left	u	d	c	s	t	b
right	u	d	c	s	t	b
left	ν_e	e	ν_μ	μ	ν_τ	τ
right	N_e	e	N_μ	μ	N_τ	τ
	leptons					

Role of N_e with mass in keV region: dark matter

Role of $N_\mu,\ N_\tau$ with mass in 100 MeV – GeV region: "give" masses to neutrinos and produce baryon asymmetry of the Universe

Role of the Higgs: give masses to quarks, leptons, Z and W and inflate the Universe.

To be true: all the couplings of the SM must be asymptotically safe or asymptotically free

Problem for:

- U(1) gauge coupling g_1, $\mu \frac{dg_1}{d\mu} = \beta_1^{\mathrm{SM}} = \frac{41}{96\pi^2} g_1^3$
- Scalar self-coupling λ, $\mu \frac{d\lambda}{d\mu} = \beta_\lambda^{\mathrm{SM}} =$

$$= \frac{1}{16\pi^2} \left[(24\lambda + 12h^2 - 9(g_2^2 + \frac{1}{3} g_1^2))\lambda - 6h^4 + \frac{9}{8} g_2^4 + \frac{3}{8} g_1^4 + \frac{3}{4} g_2^2 g_1^2 \right]$$

- Fermion Yukawa couplings, t-quark in particular h, $\mu \frac{dh}{d\mu} = \beta_h^{\mathrm{SM}} =$

$$= \frac{h}{16\pi^2} \left[\frac{9}{2} h^2 - 8 g_3^2 - \frac{9}{4} g_2^2 - \frac{17}{12} g_1^2 \right]$$

Landau pole behaviour

Gravity contribution to RG running

Let x_j is a SM coupling. Gravity contribution to RG:

$$\mu \frac{dx_j}{d\mu} = \beta_j^{\rm SM} + \beta_j^{grav} \ .$$

On dimensional grounds

$$\beta_j^{grav} = \frac{a_j}{8\pi} \frac{\mu^2}{M_P^2(\mu)} x_j \ .$$

where

$$M_P^2(\mu) = M_P^2 + 2\xi_0 \mu^2 \ ,$$

with $M_P = (8\pi G_N)^{-1/2} = 2.4 \times 10^{18}$ GeV, $\xi_0 \approx 0.024$
from a numerical solution of FRGE

Remarks

- The couplings are not in $\overline{\mathrm{M}S}$ scheme
- The couplings are not in MOM scheme
- Pretty vague definition based on physical scattering amplitudes at large momentum transfer - never actually worked out in details

Thus, computations of a_j are ambiguous and controversial.

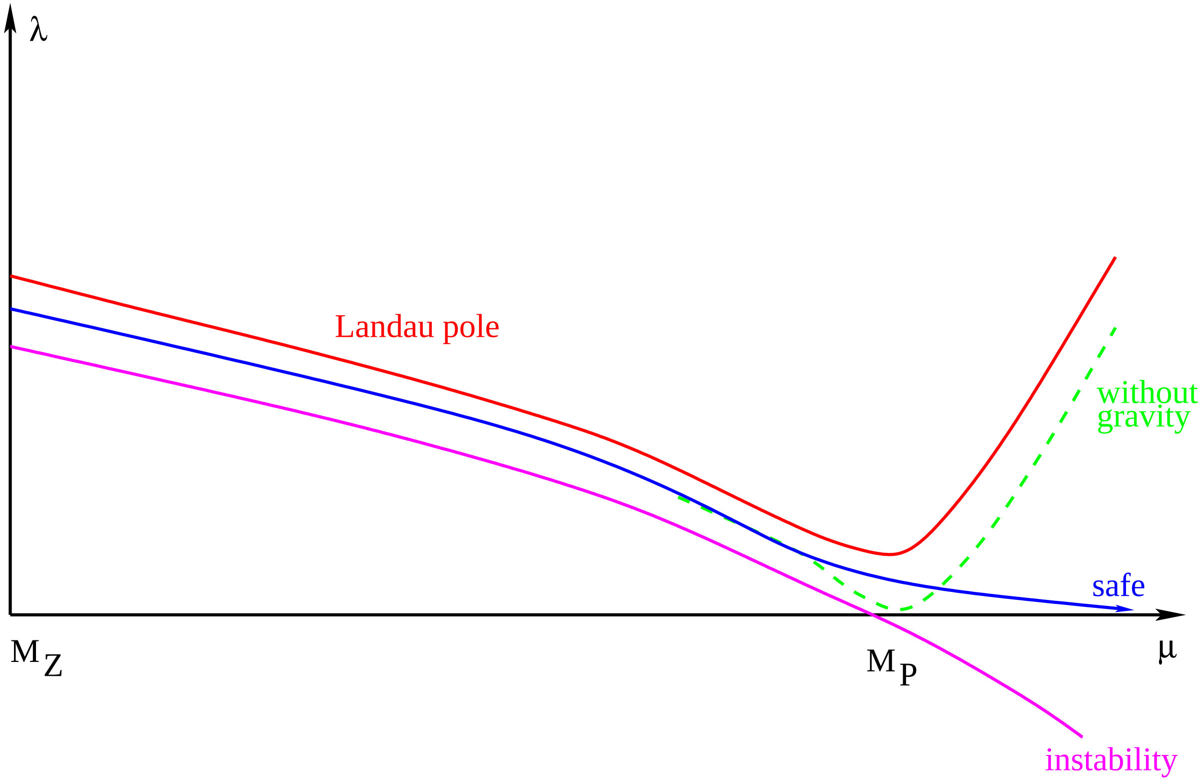
Still, even without exact knowledge of a_j a lot can be said about the Higgs mass

Robinson and Wilczek '05, Pietrykowski '06, Toms '07&'08, Ebert, Plefka and Rodigast '07, Narain and Percacci '09, Daum, Harst and Reuter '09, Zanusso et al '09, ...

- Most works get for gauge couplings a universal value $a_1 = a_2 = a_3 < 0$: U(1) gauge coupling get asymptotically free in asymptotically safe gravity
- $a_\lambda \simeq 2.6 > 0$ according to Percacci and Narain '03 for scalar theory coupled to gravity
- $a_h >< 0$?? The case $a_h > 0$ is not phenomenologically acceptable - only massless fermions are admitted

Suppose that indeed $a_1 < 0,\ a_h < 0,\ a_\lambda > 0$. Then the Higgs mass is predicted with theoretical uncertainty $\simeq \pm 2.2$ GeV

$$m_H = [126.3 + \frac{m_t - 171.2}{2.1} \times 4.1 - \frac{\alpha_s - 0.1176}{0.002} \times 1.5]\ \text{GeV}\ ,$$



Possible understanding of the amazing fact that $\lambda(M_P) = 0$ and $\beta_\lambda^{SM}(M_P) = 0$ simultaneously at the Planck scale.

To decrease uncertainty: (the LHC accuracy can be as small as $\mathbf{200}$ MeV!)

- Measure better t-quark mass (present error in m_H due to this uncertainty is $\simeq 4$ GeV)
- Measure better α_s (present error in m_H due to this uncertainty is $\simeq 1.5$ GeV)
- Compute two-loop EW corrections to pole - $\overline{\text{M}S}$ matching for the Higgs mass (has never been done)
- Compute 3-loop running of all couplings of the Standard Model (has never been done)

If done, the uncertainty will be reduced to $\sim \mathbf{0.5}$ GeV, due to unremovable non-perturbative contribution $\sim \Lambda_{QCD}$ to top quark mass.

Remarks

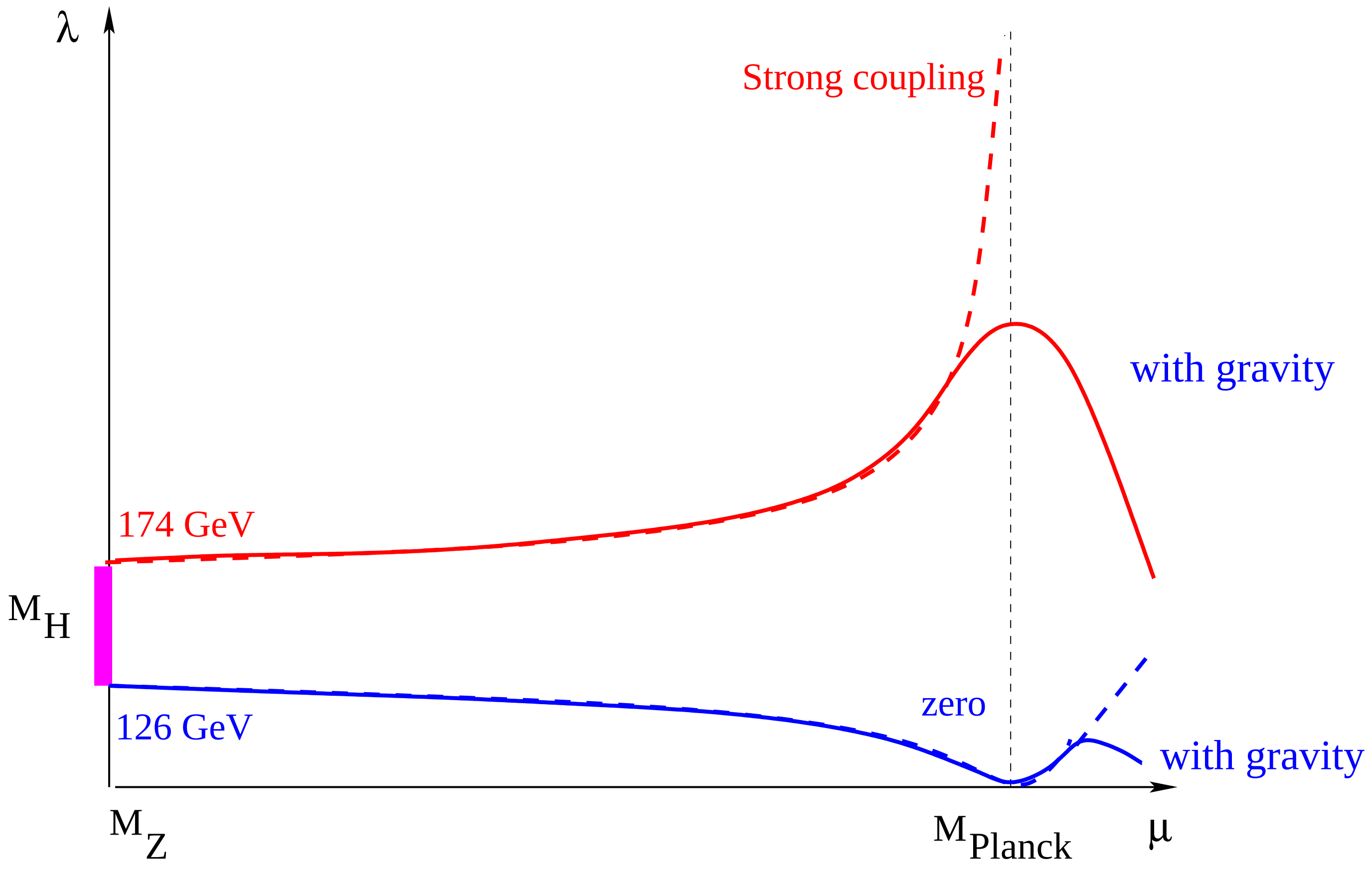
Prediction is quite model independent and may be valid for GUTs, higher dimensional theories, etc.

It stays approximately true provided:

- We have SM running of couplings up to the high energy scale
- a_λ is relatively large and positive
- If there are other light particles, the number for m_H will be different.

Suppose that $a_1 < 0,\ a_h < 0,\ a_\lambda < 0$. Then the Higgs mass is predicted with theoretical uncertainty $\simeq 50$ GeV

$$126\ \mathrm{GeV} < m_{\mathrm{H}} < 174\ \mathrm{GeV}$$



Asymptotic safety and the Fermi scale

Assumed: the Fermi scale is fixed to its experimental value. Gravity contribution to running of the mass parameter in the Higgs-potential $m^2(\mu)$:

$$\mu\frac{\partial}{\partial\mu}m^2 = A_m m^2 \ ,$$

If: $A_m > 2$ the dimensionless ratio m^2/μ^2 is attracted to zero, leading, possibly, to understanding why $G_F \gg G_N$.

Percacci '03, Narain '09: $A_m = 1.83 < 2$ for scalar-Gravity system.

SM+Gravity - not known

Conclusions

If gravity is asymptotically safe then the possible outcome of the LHC experiments is:

- Higgs and nothing else
- $m_H \simeq 126$ GeV (for central values of m_t and α_s) if, as some computations show, $a_\lambda > 0$
- $126\ \mathrm{GeV} < m_\mathrm{H} < 174\ \mathrm{GeV}$ if $a_\lambda < 0$
- Waiting time $\sim$ 6 years (?)
- Asymptotic safety may shed light to the smallness of the Fermi scale