Cosmological density perturbations from conformal field: global anisotropy and non-Gaussianity

M.V. Libanov and V.A. Rubakov

work in progress

Institute for Nuclear Research, Moscow

Properties of scalar cosmological perturbations

- Primordial perturbations (energy density, gravitational potential): Gaussian (or nearly Gaussian) random field $\delta(\mathbf{x})$
  - $\delta(\mathbf{x})$ obeys Wick theorem
  - This suggests the origin: enhanced vacuum fluctuations of some free (linear) quantum field
- Flat or nearly flat power spectrum

$$\langle \delta(\mathbf{k})\delta^*(\mathbf{k}')\rangle = k^3 \mathscr{P}(k)\delta(\mathbf{k}-\mathbf{k}')$$

with

$\mathscr{P}(k)$ (almost) independent of k $\Longrightarrow$

$$\langle \delta^2(\mathbf{x})\rangle = \int_0^\infty \frac{d^3k}{k^3}\mathscr{P}(k) \quad \text{(almost) scale invariant}$$

  - There must be some symmetry behind this property

Candidate theory for origin: inflation

(Almost) exponential expansion of the Universe,

$$ds^2 = dt^2 - e^{2Ht} d\mathbf{x}^2 \, , \quad H \approx \mathrm{const}$$

- Efficient enhancement of vacuum fluctuations of inflaton (or curvaton) field
- Symmetry: spatial dilatations supplemented by time translations

$$\mathbf{x} \to \lambda \mathbf{x} \, , \quad t \to t - \frac{1}{2H} \log \lambda$$

Flat spectrum of field fluctuations for constant $H \Longrightarrow$ flat spectrum of density perturbations (and also gravity waves)

Conformal plus global symmetry instead of de Sitter symmetry

- Simple mechanism for producing flat primordial spectrum
- Requires long evolution before the hot stage

- But otherwise insensitive to regime

of cosmological expansion:
works at inflation, contracting (ekpyrotic) phase, "starting the Universe" scenario, etc.

Model:

Conformal complex scalar field ϕ with negative quartic potential.

Conformal symmetry broken at large fields. To be discussed later on.

$$S = \int \sqrt{-g} \left[g^{\mu\nu} \partial_\mu \phi^* \partial_\nu \phi + \frac{R}{6} |\phi|^2 - (-h^2 |\phi|^4) \right]$$

Conformal symmetry in 4 dimensions. Global symmetry $U(1)$.

Homogeneous and isotropic Universe,

$$ds^2 = a^2(\eta)[d\eta^2 - d\mathbf{x}^2]$$

In terms of the field $\chi(\eta, \mathbf{x}) = a(\eta)\phi(\eta, \mathbf{x}) = \chi_1 + i\chi_2$, evolution is Minkowskian,

$$\eta^{\mu\nu}\partial_\mu\partial_\nu\chi - 2h^2|\chi|\chi = 0$$

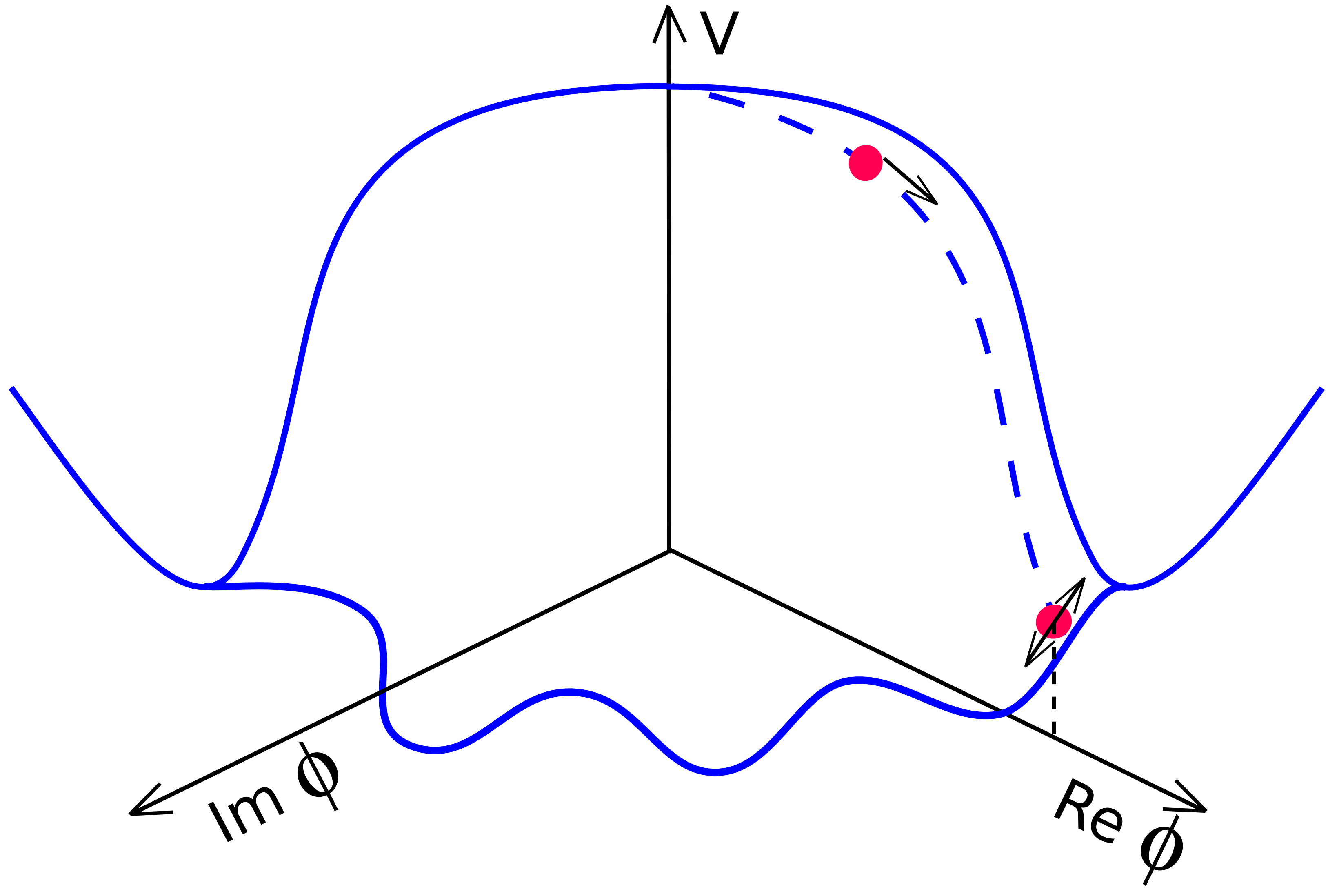
Homogeneous background solution

Attractor (real without loss of generality)

$$\chi_c(\eta) = \frac{1}{h(\eta_* - \eta)}$$

$\eta_* =$ constant of integration, end time of roll.

NB: Particular behavior $\chi_c \propto (\eta_* - \eta)^{-1}$
dictated by conformal symmetry.



V
Im φ
Re φ

Fluctuations of Im χ

automatically have flat spectrum

Linearized equation for fluctuation $\delta\chi_2 \equiv \mathrm{Im}\chi$. Mode of 3-momentum k:

$$\frac{d^2}{d\eta^2}\delta\chi_2 + k^2\delta\chi_2 - 2h^2\chi_c^2\delta\chi_2 = 0$$

[recall $h\chi_c = (\eta_* - \eta)$]

Regimes of evolution:

- Early times, $k \gg 1/(\eta_* - \eta)$, sub-"horizon" regime, χ_c negligible, free Minkowskian field

$$\delta\chi_2 = \frac{1}{(2\pi)^{3/2}\sqrt{2k}} \mathrm{e}^{-ik\eta} A_{\mathbf{k}} + \text{ h.c.}$$

- Late times, $k \ll 1/(\eta_* - \eta)$, super-"horizon" regime, χ_c dominates,

$$\delta\chi_2 = \frac{1}{(2\pi)^{3/2}\sqrt{2k}} \cdot \frac{1}{k(\eta_* - \eta)} \cdot A_{\mathbf{k}}$$

  - Phase of the field ϕ freezes out:

$$\delta\theta = \frac{\delta\chi_2}{\chi_c} = \frac{1}{(2\pi)^{3/2}\sqrt{2k}} \cdot \frac{h}{k} \cdot A_{\mathbf{k}}$$

  - Power specrum of phase is flat:

$$\langle \delta\theta^2 \rangle \propto h^2 \int \frac{d^3k}{k^3}$$

  - This is automatic consequence of global $U(1)$

Super-"horizon" regime:

k negligible,

equation for $\delta\chi_2$ is equation for spatially homogeneous perturbation.

χ_c is solution to full field equation, $e^{i\alpha}\chi_c$ also $\Longrightarrow$

$\delta\chi = i\alpha\chi_c$ is solution to perturbation equation $\Longrightarrow$

$$\delta\chi_2 : \quad e^{-ik\eta} \quad \Longrightarrow \quad C(k)\chi_c(\eta) = \frac{1}{k(\eta_* - \eta)}$$

NB: $1/k$ on dimensional grounds.

NB: In fact, equation for $\delta\chi_2$ is precisely the same as equation for minimally coupled massless scalar field in inflating Universe

- Mechanism requires long cosmological evolution: need

$$(\eta_* - \eta) \gg 1/k$$

early times, sub-"horizon" regime,
well defined vacuum of the field $\delta\chi_2$.

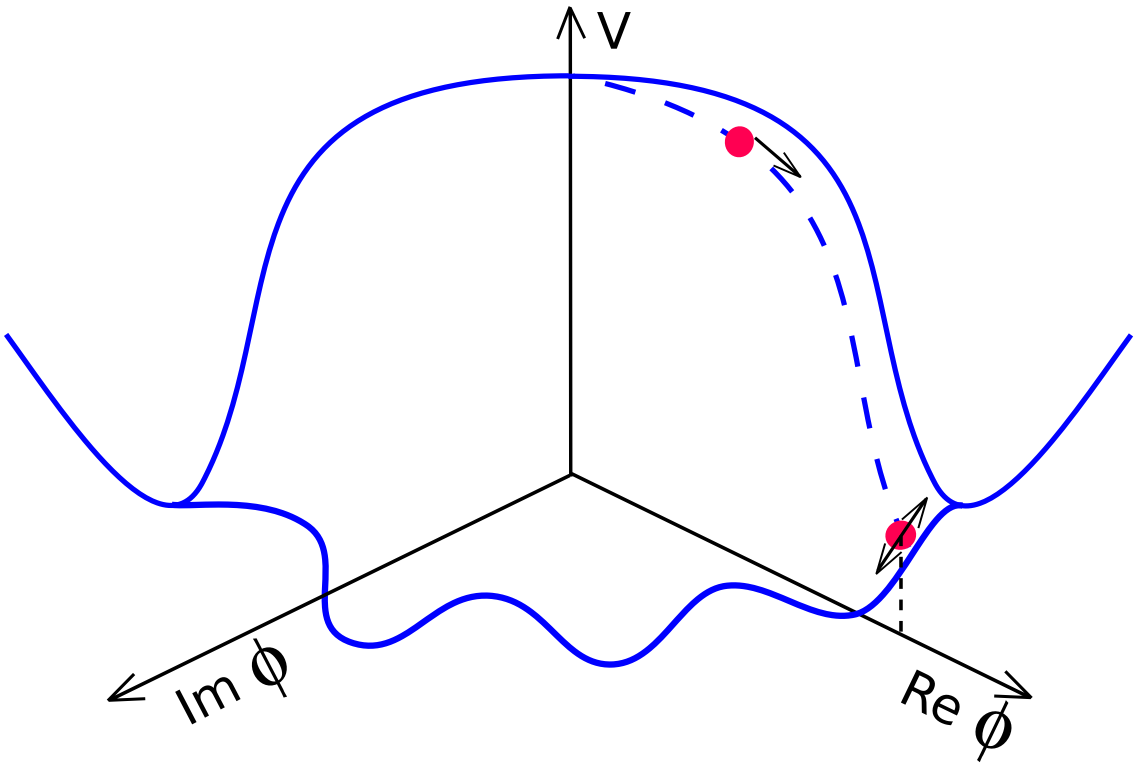
For $k \sim H_0$ this is precisely the requirement that the horizon problem is solved.

This is probably a pre-requisite for any mechanism that generates density perturbations

Reprocessing field perturbations into density perturbations

- Assume that conformal evolution ends up at late time. Modulus of the field ϕ freezes out. Assume that energy density of ϕ is negligible at that time.

- Let the phase θ be pseudo-Goldstone field interacting with matter



- Generically, phase θ ends up at a slope of its potential
- θ serves as pseudo-Goldstone curvaton

K. Dimopoulos et.al.' 2003

If mass of θ is small enough, it does not evolve until $H \sim m_\theta$ at radiation domination

Then θ rolls down its potential, oscillates near the minimum and in the end delivers its energy to matter particles.

Perturbations in θ become adiabatic density perturbations,

$$\frac{\delta\rho}{\rho} \sim \zeta \simeq \Omega_\theta \frac{\delta\theta}{\theta_0} \qquad \text{flat power spectrum}$$

Ω_θ: relative energy density of θ at the time its oscillations decay
θ_0: distance to minimum from landing point

Without fine tuning

$$\zeta \sim \Omega_\theta \frac{h}{2\pi}$$

Work in progress: back to conformal evolution

Peculiarity: perturbations of modulus.

- Linear analysis of perturbations of $\chi_1 = \mathrm{Re}\chi$: in super-"horizon" regime, $k \ll 1/(\eta_* - \eta)$

$$\delta\chi_1 = \mathrm{const}\frac{1}{k^2\sqrt{k}(\eta_* - \eta)^2}$$

- Red spectrum:

$$\langle \delta\chi_1^2 \rangle \propto \int \frac{d^3k}{k^5}$$

- Large $\delta\chi_1$ at small $(\eta_* - \eta)$

  [Recall $\chi_c = 1/[h(\eta_* - \eta)]$]

- Again by symmetry: now translations of conformal time: $\chi_c \propto 1/(\eta_* - \eta) \Longrightarrow$ spatially homogeneous solution to perturbation equation $\delta\chi = \partial_\eta \chi_c$.
- Interpretation: shift $\eta_* \longrightarrow \eta_* + \delta\eta_*(\mathbf{x})$
  - Background for perturbations $\delta\chi_2 = \mathrm{Im}\chi$ (in other words, for phase θ) is no longer spatially homogeneous.
  - Red spectrum of $\delta\eta_*(\mathbf{x})$:

$$\langle \delta\eta_*^2 \rangle \propto \int \frac{d^3k}{k^5}$$

- Have to study perturbations of $\mathrm{Im}\chi$ in spatially inhomogeneous background, slowly varying in space,

$$\chi_c = \frac{1}{h(\eta_*(\mathbf{x}) - \eta)}$$

- Back to equation for perturbations of $\delta\chi_2 = \mathrm{Im}\chi$

$$\frac{d^2}{d\eta^2}\delta\chi_2 - \frac{\partial^2}{\partial \mathbf{x}^2}\delta\chi_2 - \frac{2}{(\eta_*(\mathbf{x}) - \eta)}\delta\chi_2 = 0$$

  - Initial condition as $\eta \to -\infty$:

$$\delta\chi_2 = \frac{1}{(2\pi)^{3/2}\sqrt{2k}} \mathrm{e}^{i\mathbf{kx} - ik\eta} A_{\mathbf{k}} + \text{ h.c.}$$

  - $\eta_*(\mathbf{x})$: long ranged field

- Solution in approximation of small wavelength k^{-1} compared to scale of spatial variation of η_*:

$$\delta\chi_2 = \frac{1}{(2\pi)^{3/2}\sqrt{2k}} \mathrm{e}^{i\mathbf{kx} - ik\eta} \left[1 - \frac{i}{q(\eta_*(\mathbf{x}) - \eta)} + \frac{k_i k_j}{k^3}\partial_i\partial_j\eta_* \cdot f(\eta_* - \eta)\right] A_{\mathbf{k}}$$

with $q = k - k_i \partial_i \eta_*$.

Interpretation: local time shift plus local Lorentz boost: background is locally homogeneous and isotropic in a reference frame other than cosmic frame.

- Modes of $\delta\eta_*$ longer than present cosmological horizon: global anisotropy, constant in space vector $\partial_i \eta_*$

Leading order effect cancels out nevertheless

Global anisotropy determined by constant in space tensor

$$\partial_i \partial_j \eta_* |_{\text{long wavelengths}}$$

Induces correlators in CMB temperature anisotropy

$$\langle a_{lm} a_{l\pm 2, m'} \rangle$$

NB: Power spectrum is blue,

$$\langle (\partial_i \partial_j \eta_*)^2 \rangle \propto h^2 \int \frac{d^3k}{k}$$

- Unclear how to disentangle long wavelengths (longer than current horizon) and shorter wavelengths
- Unclear whether global anisotropy can beat cosmic variance:

On dimensional grounds, global anisotropy effect on perturbations with present wave vector k_0 is proportional to

$$h \frac{H_0}{k_0}$$

$\Longrightarrow$ effect on CMB suppressed by $1/l$

Non-Gaussianity

- Perturbations of phase $\delta\theta$ frozen out at

$$\delta\theta(\mathbf{x}) = \frac{1}{(2\pi)^{3/2}\sqrt{2k}q(\mathbf{x})} \mathrm{e}^{i\mathbf{kx} - ik\eta_*(\mathbf{x})} A_{\mathbf{k}} + \text{ h.c.}$$

$\eta_*(\mathbf{x})$ and $q(\mathbf{x}) = k - k_i \partial_i \eta_*$ are random fields $\Longrightarrow$ Non-Gaussianity of very special kind $\Longrightarrow$ non-Gaussianity of the same kind in density perturbations.

- Density correlation functions involve

$$\langle \mathrm{e}^{-ik\eta_*(\mathbf{x})} e^{ik\eta_*(\mathbf{y})} \rangle = \mathrm{e}^{k^2[D(\mathbf{x},\mathbf{y}) - D(\mathbf{x},\mathbf{x})]}$$

where

$$D(\mathbf{x},\mathbf{y}) - D(\mathbf{x},\mathbf{x}) \propto -h^2 \int \frac{d^3k}{k^5} \left(1 - \mathrm{e}^{i\mathbf{kx}}\right)$$

NB: Infrared log. Probably cut off at the present horizon scale.

To summarize:

- Flat (or nearly flat) spectrum of density perturbations may be consequence of conformal + global symmetry, rather than de Sitter symmetry

  - A simple model of this sort: conformally coupled complex scalar field with negative quartic potential

  - Evolution of scale factor arbitrary, except that it must be long

- Peculiar property which hopefully has potentially observable consequences: strong fluctuations along roll down direction
  - Perturbations of wavelengths exceeding the present horizon: global anisotropy

$$\langle a_{lm} a_{l\pm 2, m'} \rangle \neq 0$$

  - Perturbations of wavelengths smaller than the present horizon: non-Gaussianity of a special kind.

What if the world started out conformal indeed?