

Internal structure of Maxwell-Gauss-Bonnet black hole

K.A. Rannu

SAI MSU

QUARKS-2010

Kolomna

09.06.2010

Subject

The talk is based on the paper

S.O. Alexeyev, A. Barrau, K.A. Rannu

“Internal structure of a Maxwell-Gauss-Bonnet black hole”

Phys. Rev. **D 79** 067503 (2009)

S.O. Alexeyev, M.V. Pomazanov

Phys. Rev. D **55** 2110 (1997)

S.O. Alexeyev, M.V. Sazhin

Gen. Relativ. Grav **30** 1187 (1998)

S.O. Alexeyev, M.V. Sazhin, M.V. Pomazanov

Int. J. Mod. Phys **D 10** 225 (2001)

Action and metric

Low-energy effective string action with second order curvature corrections:

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left(-R + 2\partial_\mu \phi \partial^\mu \phi - e^{-2\phi} F_{\mu\nu} F^{\mu\nu} + \lambda e^{-2\phi} S_{GB} \right),$$
$$S_{GB} = R_{ijkl} R^{ijkl} - 4R_{ij} R^{ij} + R^2$$

Spherically-symmetric metric in GHS coordinates:

$$ds^2 = \Delta dt^2 - \frac{1}{\Delta} dr^2 - f^2 (d\theta^2 + \sin^2 \theta d\varphi^2)$$

Anzats for Maxwell tensor: $F = q \sin \theta d\theta \wedge d\varphi$

Variable change: $E = e^{-2\phi}$, $E_0 = e^{-2\phi_0}$

Lagrange-Euler (Einstein) equations

$$\begin{pmatrix} 0 & f - 4\lambda f' E' & -2\lambda(\Delta f'^2 - 1) \\ f - 4\lambda \Delta f' E' & 2\Delta(1 - 2\lambda \Delta' E') & -4\lambda \Delta \Delta' f' \\ 4\lambda E(\Delta f'^2 - 1) & 8\lambda \Delta \Delta' f' E & \frac{\Delta f'^2}{E} \end{pmatrix} \begin{pmatrix} \partial \Delta' / \partial r \\ \partial f' / \partial r \\ \partial E' / \partial r \end{pmatrix} =$$

$$= \begin{pmatrix} -\frac{f^2 E'^2}{4 E^2} \\ 2E \frac{q^2}{f^3} - 2\Delta' f' - \frac{\Delta f E'^2}{2 E^2} + 4\lambda \Delta'^2 f' E' \\ 2E \frac{q^2}{f^2} - f(\Delta' f + 2\Delta f') \frac{E'}{E} + \Delta f^2 \frac{E'^2}{E^2} - 4\lambda \Delta'^2 f'^2 E \end{pmatrix}$$

Constraint:

$$(1 + \Delta f'^2 (\phi')^2 - \Delta' f f' - \Delta (f')^2) + 4E\lambda \Delta' \phi' (1 - 3\Delta (f')^2) - E q^2 f^{-2} = 0$$

Asymptotics

- Asymptotics near the horizon $((r - r_h) \ll 1)$:

$$\Delta = d_1(r - r_h) + d_2(r - r_h)^2 + O\left((r - r_h)^2\right)$$

$$f = f_0 + f_1(r - r_h) + f_2(r - r_h)^2 + O\left((r - r_h)^2\right)$$

$$E = E_0 + \phi_1(r - r_h) + \phi_2(r - r_h)^2 + O\left((r - r_h)^2\right)$$

- Asymptotics at the infinity — GM-GHS solution:

$$ds^2 = \left(1 - \frac{2M}{r}\right) dt^2 - \left(1 - \frac{2M}{r}\right)^{-1} dr^2 - r \left(r - \frac{q^2 E_0}{M}\right) d\Omega$$

$$E = E_0 - \frac{q^2}{Mr}$$

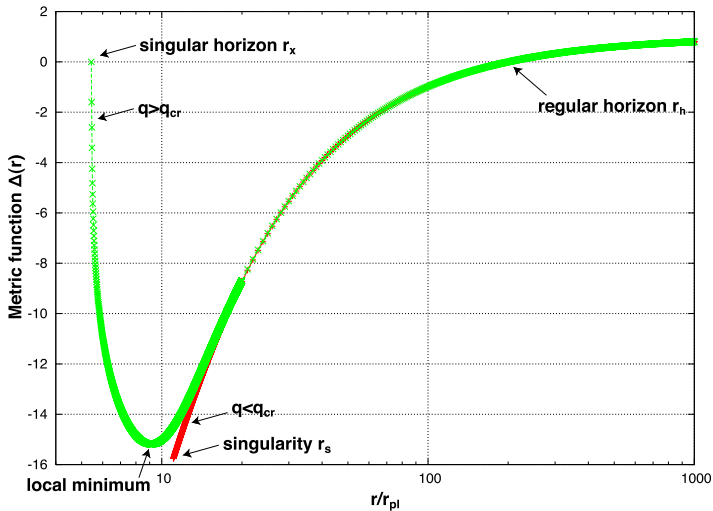
Curvature invariant in GHS coordinates

$$R_{ijkl}R^{ijkl} = \Delta'^2 + 4\Delta'^2 \frac{f'^2}{f^2} + 8\Delta^2 \frac{f''^2}{f^2} + 8\Delta\Delta' \frac{f'f''}{f^2} + \frac{4}{f^4} - 8\Delta \frac{f'^2}{f^4} + 4\Delta^2 \frac{f'^4}{f^4}$$

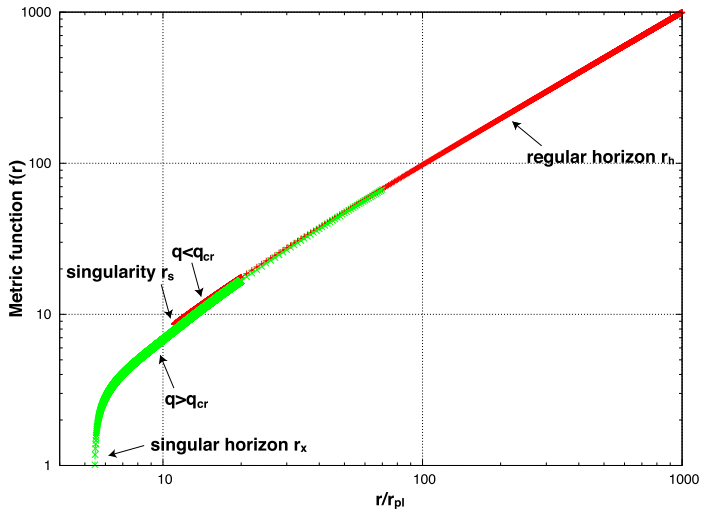
Determining the singularity type:

- curvature invariant diverges \Rightarrow real scalar singularity (Clarke, 1993)
- curvature invariant is finite \Rightarrow coordinate singularity, that can be neglected by appropriate coordinate transformation

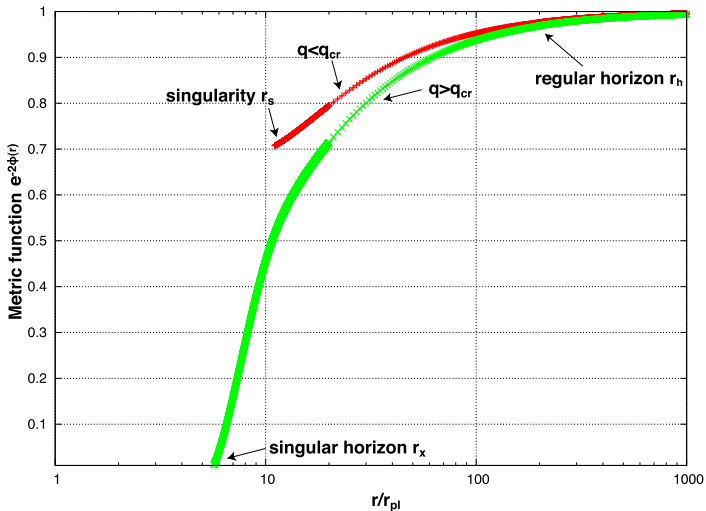
Metric function $\Delta(r)$



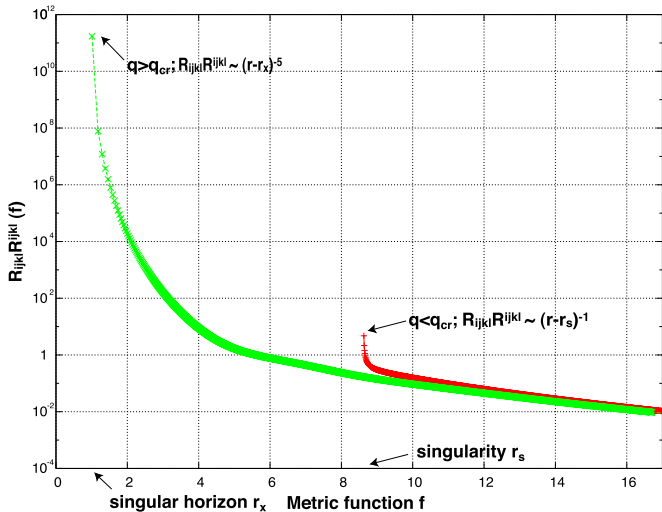
Metric function $f(r)$



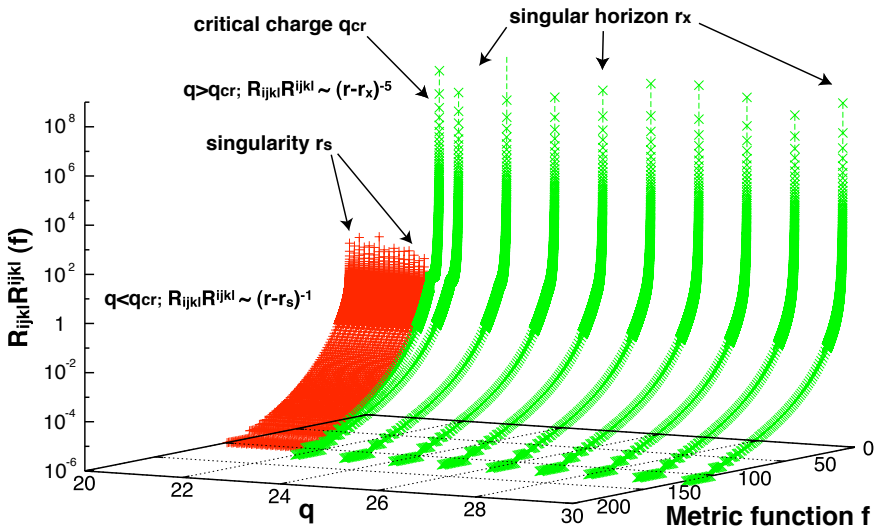
Dilatonic exponent $\exp(-2\phi(r))$



Curvature invariant $R_{ijkl}R^{ijkl}(f)$



Curvature invariant $R_{ijkl}R^{ijkl}(q,f)$



Asymptotics near the particular points

Metric function $f(r)$ should be regarded as a radial coordinate.

$$f(r \rightarrow r_s) = f_s + f_{s2}(\sqrt{r - r_s})^2 + f_{s3}(\sqrt{r - r_s})^3 + \dots$$

$$f(r \rightarrow r_x) = f_x + f_{x1}\sqrt{r - r_x} + f_{x2}(\sqrt{r - r_x})^2 + \dots$$

$$\text{for } f \rightarrow f_s \quad R_{ijkl}R^{ijkl} \sim \text{const}_1 \times (f - f_s)^{-1}$$

$$\text{for } f \rightarrow f_x \quad R_{ijkl}R^{ijkl} \sim \text{const}_2 \times (f - f_x)^{-5}$$

Results

- When the black hole charge becomes larger than the critical value the singularity r_s is replaced by a local minimum of the function $\Delta(r)$ and the solution exists till the singular horizon r_x .
- Function $f(r)$ is the radius of S^2 , so it plays the role of the radial coordinate. If $q < q_{cr}$ it decreases monotonously till $r = r_s$ like in GHS. When r_s disappears the function $f(r)$ reaches its zero in the new point r_x .
- Curvature invariant increases much more rapidly (as $(r - r_x)^{-5}$) near the singular horizon r_x than near the singularity r_s (as $(r - r_s)^{-1}$), so the singularity in r_x is much stronger than the one in r_s .
- New kind of singularity inside black hole was found. Unfortunately Maxwell-Gauss-Bonnet black hole cannot help wormholes' or multiverse theories because this singularity is very strong.

Thank you
for attention

Critical charge

Critical charge value — a meaning at which the inner singularity disappears being replaced by a local minimum for $\Delta(r)$

