Standard Model in adS slice with UV-localized Higgs field

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Introduction

- We discuss the 5D Standard Model on the background of the metric

$$ds^2 = e^{-2k|y|}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2,$$

  k is the adS curvature, y denotes the coordinate of the fifth warped dimension. The fifth dimension is the S_1/Z_2 orbifold of size R.
- Two branes are placed at $y = 0$ and $y = \pi R$. These are ultraviolet (UV-brane, $y = 0$) and infrared branes (IR-brane, $y = \pi R$), respectively.
- We assume that the fermions and the gauge fields live in the 5D bulk.
- The Higgs field is localized towards or on the UV-brane.
- We treat the warp factor $e^{k\pi R}$ as the large parameter in what follows:

$$e^{k\pi R} \gg 1,$$

  while $\frac{m_n}{k} \sim e^{-k\pi R}$ is considered to be the small parameter: $\frac{m_n}{k} \ll 1$.

Outline

- First, we consider different scenarios of spontaneous symmetry breaking in the bulk.
- We show that the realistic pattern of fermion masses and mixings can be obtained without introducing the hierarchy in the parameters of the initial 5D theory.
- We discuss the possibility of the neutrino mass generation via the interaction with the excited modes of the Higgs field.
- Finally, we put constraints on the masses of Kaluza–Klein excited modes. We do it for the scenarios of the bulk Higgs field and the brane-localized Higgs field.

Higgs field in the 5D space-time

The action for the Higgs field:

$$S_5 = \int d^4x dy \sqrt{g}\Big(\frac{1}{2}g^{MN}\partial_M H \partial_N H - \frac{1}{2}m_H^2 H^2 - V(H)\Big) + S_b \ ,$$

where the brane term S_b is added to have zero mode:

$$S_b = \Big(1 - \frac{\alpha}{2}\Big) k \int d^4x dy \sqrt{g} (\delta(y - \pi R) - \delta(y)) H^2 \ ,$$

the constant α is tuned to $\alpha = \sqrt{4 + \frac{m_H^2}{k^2}}$. With negative sign in front of α in the brane term and with $\alpha > 1$, the Higgs field is localized near the UV-brane.

Let us switch off the potential for a moment and consider the free scalar field:

$$V(H) = 0.$$

Profiles of the KK modes of the Higgs field

The Higgs field can be expanded in the infinite sum of the Kaluza–Klein modes:

$$H(x,y)=\sum_{n=0}^{\infty} h_n(x)H_n(y)$$

Profile of the zero mode:

$$H_0(y)=\sqrt{2k(\alpha-1)}e^{(2-\alpha)ky}, \quad \widetilde{H}_0(y)=e^{-ky}H_0(y).$$

Profiles of the excited KK modes of the Higgs field:

$$H_n(y)=\frac{m_n e^{2ky}}{\sqrt{k\int_0^{\beta_n} sJ_\alpha^2(s)ds}}\left[J_\alpha\left(\frac{m_n}{k}e^{ky}\right)+\frac{J_{\alpha-1}\left(\frac{m_n}{k}\right)}{J_{-\alpha+1}\left(\frac{m_n}{k}\right)}J_{-\alpha}\left(\frac{m_n}{k}e^{ky}\right)\right],$$

$$\widetilde{H}_n(y)=e^{-ky}H_n(y).$$

We see that with $\alpha>1$ the zero mode is localized near the UV-brane. The excited modes of the Higgs field are always localized towards the IR-brane.

Scenarios of spontaneous symmetry breaking 1

To obtain non-trivial VEVs of the Higgs field, we turn on the potential. In the first scenario we consider the potential with the mass term living in the bulk:

$$V(H) = -\frac{\mu^2}{2} H^2 + \lambda H^4.$$

We assume that excited KK modes of the Higgs field are small quantities as compared to the zero one: $h_n \ll h_0$. Then we treat the interaction between the zero mode and excited KK modes in the linear approximation in h_n. With these assumptions we obtain the following VEVs:

$$v_0 = v_{SM},$$

$$v_n \sim \left(\frac{v_{SM}}{m_n}\right)^2 \left(\frac{m_n}{k}\right)^{\alpha+1} v_{SM}, \quad \alpha > 2,$$

$$v_n \sim \left(\frac{v_{SM}}{m_n}\right)^2 \left(\frac{m_n}{k}\right)^{3(\alpha-1)} v_{SM}, \quad \alpha < 2.$$

Scenarios of spontaneous symmetry breaking 2

So strong suppression is absent in a model with another mechanism of spontaneous symmetry breaking. Let us introduce the potential:

$$V(H) = -\frac{1}{2}M\delta(y)H^2 + \lambda H^4,$$

so that the mass term resides on the UV-brane.

In this case, we obtain the following estimate for the VEVs of the excited modes:

$$v_n \sim \left(\frac{v_{SM}}{m_n}\right)^2 \left(\frac{m_n}{k}\right)^{\alpha-1} v_{SM}.$$

Quarks in the 5D space-time

The interaction of quarks and the Higgs field in the 5D space-time is described by the following action

$$S_5^q = \int d^4x dy \sqrt{g} \left(\lambda_{ij}^d \bar{Q}_i H d_j + \lambda_{ij}^u \bar{Q}_i \widetilde{H} u_j + h.c. \right) .$$

Zero modes of fermions are given by

$$Q_0(y) = N_L e^{(2-c_L)ky} , \quad u_0(y) = N_R^u e^{(2-c_R^u)ky} ,$$
$$d_0(y) = N_R^d e^{(2-c_R^d)ky},$$

with the normalization constants

$$N_L = \sqrt{\frac{(1-2c_L)k}{e^{(1-2c_L)k\pi R} - 1}} , \quad N_R^{u,d} = \sqrt{\frac{(1-2c_R^{u,d})k}{e^{(1-2c_R^{u,d})k\pi R} - 1}} .$$

The constants c are related to the bulk masses of fermions

$$c_R = \frac{m_\Psi}{k} , \; c_L = -\frac{m_\Psi}{k} .$$

Quark masses and mixings

We do not introduce the hierarchy of the Yukawa couplings $\lambda_{ij}^{u,d}$ and set $\lambda_{ij}^{u,d} \sim k^{-1/2}$. From the 5D action describing the interaction of the quarks with the Higgs field, we can obtain the effective 4D mass term of quarks. Diagonalizing the latter, we express quark mixings and masses in terms of the initial 5D theory:

$$\theta_{ij} \sim \frac{N_{Li}}{N_{Lj}}, \quad m_i^{u,d} \sim N_{Li} N_{Ri}^{u,d} \frac{v_{SM}}{k}.$$

The right pattern of quark masses and mixings is obtained for

$$\frac{N_{L1}}{N_{L2}} \approx \frac{1}{10} , \quad \frac{N_{L2}}{N_{L3}} \approx \frac{1}{25} ,$$

$$\frac{N_{R1}^u}{N_{R2}^u} \approx \frac{1}{30} , \quad \frac{N_{R2}^u}{N_{R3}^u} \approx \frac{1}{5} , \quad \frac{N_{R1}^d}{N_{R2}^d} \approx \frac{1}{2} , \quad \frac{N_{R2}^d}{N_{R3}^d} \approx 1 , \quad \frac{N_{R3}^d}{N_{R3}^u} \approx \frac{1}{40} ,$$

$$N_{L3}^u = 1.3\sqrt{k} , \quad N_{R3}^u = 0.6\sqrt{k} .$$

Quark parameters in the 5D SM with warp factor $k\pi R = 10$

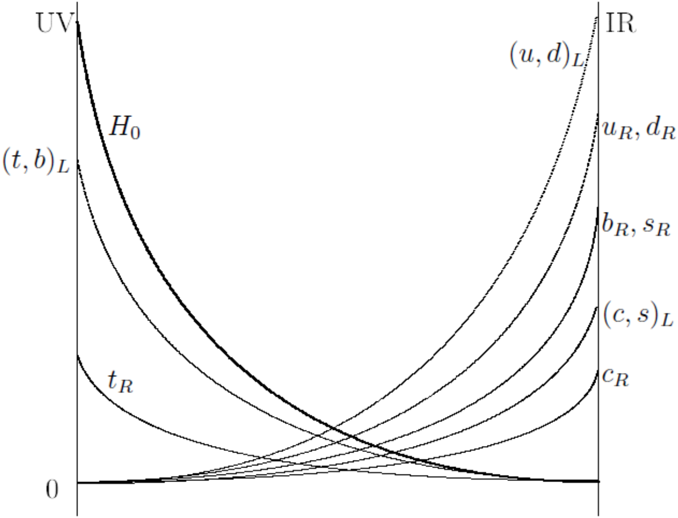
c	Q	u	d
c_{L1}	-0.1	-	-
c_{L2}	0.2	-	-
c_{L3}	1.3	-	-
c_{R1}	-	0.0	0.0
c_{R2}	-	0.4	0.1
c_{R3}	-	0.7	0.1

Table: Quark parameters in the 5D SM with warp factor $k\pi R = 10$.

Note that the effective profiles of the fermions in the curved space are given by

$$\Psi_0(y) = Ne^{(1/2-c)ky}.$$

Effective profiles of quarks in the 5D space-time



5D Yukawa couplings

Allowing 5D Yukawa couplings to vary within the interval $(1/3,\ 3)$, one can adjust masses and flavor mixings:

$$\lambda^u_{ij}\sqrt{k} = \begin{pmatrix} 1.2 & 0.4 & -1.9 \\ 1.7 & 1.1 & -0.9 \\ -0.8 & 0.6 & 1.3 \end{pmatrix}, \quad \lambda^d_{ij}\sqrt{k} = \begin{pmatrix} 0.9 & -0.4 & 1.3 \\ -1.3 & 1.3 & -0.5 \\ 1.8 & 0.3 & 1.1 \end{pmatrix},$$

With these values we obtain the following mixing matrix

$$|C| = \begin{pmatrix} 0.97 & 0.25 & 0.0040 \\ 0.25 & 0.97 & 0.040 \\ 0.0010 & 0.040 & 0.998 \end{pmatrix} \tag{1}$$

and quark masses $m_u \approx 3$ MeV, $m_d \approx 9$ MeV, $m_c \approx 1.5$ GeV, $m_s \approx 90$ MeV, $m_b \approx 4.5$ GeV, $m_t \approx 170$ GeV. Obviously, all these values are in a reasonable agreement with the experimental data.

Leptons

The case of leptons is studied in the analogous way. The main difference is that the lepton mixings do not have any essential hierarchy. We repeat the estimate for the mixing angles:

$$\theta_{ij} \sim \frac{N_{Li}}{N_{Lj}}.$$

Then we obtain that

$$N_{L1} \sim N_{L2} \sim N_{L3}.$$

The most natural way to satisfy this condition is to assume that all the lepton doublets are localized near the UV-brane.

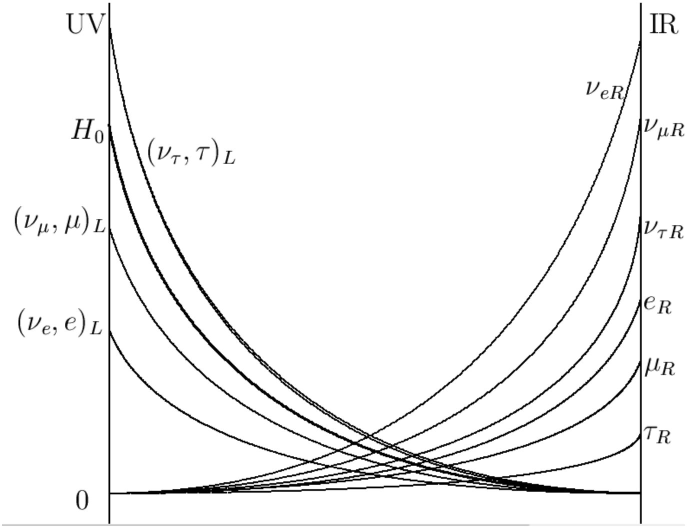
Lepton parameters in the 5D SM with warp factor $k\pi R = 10$

c	L	ν	e
c_{L1}	1.0	-	-
c_{L2}	2.0	-	-
c_{L3}	3.0	-	-
c_{R1}	-	< -2.1	-0.8
c_{R2}	-	-2.1	-0.3
c_{R3}	-	-1.8	0.1

Table: Lepton parameters in the 5D SM

With these parameters c we obtain the right pattern of lepton masses. We choose the normal hierarchy of neutrino masses and assume no degeneracy, i.e. $m_1 \ll m_2$, $m_2 \approx \sqrt{\Delta m^2_{sol}} \approx 0.008$ eV and $m_3 \approx \sqrt{\Delta m^2_{atm}} \approx 0.05$ eV.

Effective profiles of leptons in the 5D space-time



Neutrino masses

An interesting possibility would be that neutrinos obtain their masses predominantly via excited modes of the Higgs field. This would occur, if the condition

$$|e^{(2-\alpha-c_L-c_R^\nu)k\pi R}-1| \ll e^{(1-c_L-c_R^\nu)k\pi R}\frac{v_n}{v_0}$$

was satisfied. This condition (16) is equivalent to the following two,

$$e^{(c_L+c_R^\nu-1)k\pi R} \ll \frac{v_n}{v_0},$$

$$e^{(1-\alpha)k\pi R} \ll \frac{v_n}{v_0}\ .$$

In the best case, the contribution of the Higgs KK excitations is suppressed by the small factor $\frac{v_{SM}^2}{m_n^2}$ as compared to the zero mode. We remind the estimate for the VEVs of the excited Higgs field:

$$v_n \sim \left(\frac{v_{SM}}{m_n}\right)^2 \left(\frac{m_n}{k}\right)^{\alpha-1} v_{SM}.$$

FCNC problem

The IR localization of light fermions introduces FCNC problem. The KK excitations of the Higgs field and bulk gauge bosons also live near the IR brane. Thus, their wave functions have large overlaps with the wave functions of light fermions.

$$S^{eff}_{quark} = \int d^4x \Big(\sum_{n=1}^{\infty} y^d_{nij} \bar{d}_{Li}(x) d_{Rj}(x) h_n(x) + h.c. \Big) \ .$$

Here $y^{u,d}_{nij}$ are the effective 4D Yukawa couplings. They are given by

$$y_{nij} = \lambda^d_{ij} I^d_{nij} \ , \quad I^d_{nij} = \int dy \sqrt{g} Q_{0i}(y) H_n(y) d_{0j}(y) \ .$$

$$y_{nij} \approx \lambda^d_{ij} \sqrt{\frac{(1-2c_{Li})(1-2c^d_{Rj})k}{\int_0^{\beta_n} s J^2_{\alpha}(s) ds}} \int_0^{\beta_n} \Big(\frac{s}{\beta_n} \Big)^{1-c_{Li}-c^d_{Rj}} J_{\alpha}(s) ds \ .$$

$$y_{nij} \sim 1.$$

Kaon mixing

$$H_{eff}^{\Delta F=2} = \sum_{a=1}^{5} C_a Q_a^{q_i q_j} + \sum_{a=1}^{3} \widetilde{C}_a \widetilde{Q}_a^{q_i q_j} \ .$$

$$Q_2^{q_i q_j} = \bar{q}_{jR}^{\alpha} q_{iL}^{\alpha} \bar{q}_{jR}^{\beta} q_{iL}^{\beta}, \quad Q_4^{q_i q_j} = \bar{q}_{jR}^{\alpha} q_{iL}^{\alpha} \bar{q}_{jL}^{\beta} q_{iR}^{\beta} \ .$$

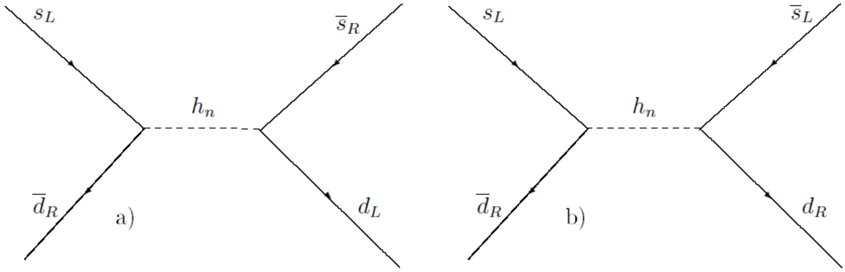


Figure: Excited Higgs mediated processes leading to kaon mixing.

Constraints on the masses of KK excited modes

$$C_2 = \sum_{n=1}^{\infty} \frac{(y'^{\star}_{n21})^2}{m_n^2}\,, \quad \widetilde{C}_2 = \sum_{n=1}^{\infty} \frac{(y'_{n12})^2}{m_n^2}\,, \quad C_4 = \sum_{n=1}^{\infty} \frac{2y'_{n12}y'^{\star}_{n21}}{m_n^2}\,.$$

$$-5.1 \times 10^{-17}\ \text{GeV}^{-2} \lesssim\ \text{Im}\,C_2,\ \text{Im}\,\widetilde{C}_2 \lesssim 9.3 \times 10^{-17}\ \text{GeV}^{-2}\,,$$
$$-1.8 \times 10^{-17}\ \text{GeV}^{-2} \lesssim\ \text{Im}\,C_4 \lesssim 0.9 \times 10^{-17}\ \text{GeV}^{-2}\,.$$

$$m_n \gtrsim 5 \times 10^5\ \text{TeV}\,.$$

Other sources of kaon mixing

A way to avoid FCNC mediated by the Higgs KK excitations is to localize the Higgs field on the UV brane. In this case, the major source of kaon mixing is the interaction of down-quarks with the KK excitations of the bulk gauge fields. Constraints coming from this interaction are much weaker by the following reasons:

- Effective 4D Yukawa couplings are the $O(\frac{1}{100})$ constants.
- Interaction of d- and s-quarks with gauge fields leading to kaon mixing is described by the operator $Q_1^{q_i q_j} = \bar{q}^{\alpha}_{jL}\gamma_{\mu} q^{\alpha}_{iL}\bar{q}^{\beta}_{jL}\gamma^{\mu} q^{\beta}_{iL}$. Thus it gives contribution to the parameters C_1 and $\widetilde{C}_1$.

$$-4.4\times 10^{-15}\ \text{GeV}^{-2} \lesssim\ \text{Im}\, C_1,\ \text{Im}\,\widetilde{C}_1 \lesssim 2.8\times 10^{-15}\ \text{GeV}^{-2}\ .$$

$$m_n^{\gamma} \gtrsim 700\ \text{TeV}\ .$$

Conclusions

- Masses and mixings in both quark and lepton sectors are reproduced without introducing large or small parameters.
- In the particular scenarios we consider neutrino masses are generated via the interaction with the zero mode of the Higgs field. VEVs of the excited modes of the Higgs field are too small to generate the main contribution to neutrino masses.
- The interaction of fermions with the Higgs and gauge boson KK excitations gives rise to FCNC with no built-in suppression mechanism. This strongly constrains the scale of KK masses. In the bulk Hggs scenario the masses of the KK excitations are constrained as:

  $$m_n \gtrsim 5 \times 10^5 \text{TeV}.$$

  In the brane-localized Higgs scenario this constraint is much weaker:

  $$m_n \gtrsim 700 \text{TeV}.$$