

# Scalar Leptoquark Contributions into $l_i \rightarrow l_j \gamma$ Processes

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## Plan

1. Introduction
2. Model
3. Widths of  $l_i \rightarrow l_j \gamma$  decays
4. Numerical results and discussion
5. Conclusions

## 1. Introduction

Physics Beyond the SM → Four-color symmetry →

[J.C. Pati and A.Salam, PRD **10** (1974)]

→ Minimal Quark-Lepton Symmetry Model →

$$SU_V(4) \times SU_L(2) \times U_R(1).$$

[A.D. Smirnov, Phys. Lett. B **346** (1995); Ya.F. **58** (1995)]

→ Higgs mechanism → Scalar Leptoquarks →

$$S^{(\pm)} = \begin{pmatrix} S_{1\alpha}^{(+)} \\ S_{2\alpha}^{(+)} \end{pmatrix}; \begin{pmatrix} S_{1\alpha}^{(-)} \\ S_{2\alpha}^{(-)} \end{pmatrix}; \quad Q_{em} = \begin{pmatrix} 5/3 \\ 2/3 \end{pmatrix}; \begin{pmatrix} 1/3 \\ -2/3 \end{pmatrix}.$$

→ LFV Processes  $l_i \rightarrow l_j \gamma$

## 1. Introduction

Current limits from direct search:  $M_{LQ} \sim 200 - 300$  GeV,

Indirect limits from  $K_L^0 \rightarrow \mu^\pm e^\mp$ ,  $S, T, U$  parameters,

$g - 2$  and others close to direct limits

Other source  $\rightarrow$  LFV Processes

Exist strong experimental limits

$$Br(\mu \rightarrow e\gamma) < 1.2 \cdot 10^{-11}$$

[M.Ahmed et al., (MEGA Collab) PRD **65** (2002):

$$Br(\tau \rightarrow \mu\gamma) < 4.5 \cdot 10^{-8}$$

[K.Hayasaka et al., (Belle Collab) ArXiv:0705.0650]

$$Br(\tau \rightarrow e\gamma) < 3.3 \cdot 10^{-8}$$

[E.Guido, Moriond 2010]

$$Br(l_i \rightarrow l_j\gamma) \rightarrow m_{S_a^{(\pm)}} \rightarrow ?$$

## 2. Model

# 2. Minimal Quark-Lepton Symmetry Model

$$SU_V(4) \times SU_L(2) \times U_R(1).$$

(A. D. Smirnov, Phys. Lett. B **346**, YaF**58** 1995.)

$$\psi_{iaA}^{L,R} \sim (4, 2, Y_a^R),$$

$$\psi_{ia\alpha}^{L,R} = Q'_{ia\alpha}{}^{L,R} = \sum_j \left( A_{Q_a}^{L,R} \right)_{ij} Q_{ja\alpha}^{L,R},$$

$$\psi_{ia4}^{L,R} = l'_{ia}{}^{L,R} = \sum_j \left( A_{l_a}^{L,R} \right)_{ij} l_{ja}^{L,R},$$

where  $a = 1, 2$  –  $SU_L(2)$  index,  $A=\alpha, 4$ ,  $\alpha = 1, 2, 3$  –  $SU_c(3)$  index  
 $i, j = 1, 2, 3$  – indices of generations

$$C_Q = (A_{Q_1}^L)^+ A_{Q_2}^L \equiv V_{CKM} \quad C_l = (A_{l_1}^L)^+ A_{l_2}^L \equiv U_{PMNS}^+$$

$$K_a^{L,R} = (A_{Q_a}^{L,R})^+ A_{l_a}^{L,R}$$

## 2. Model: Scalar Sector

$$(4, 1, 1), \quad (1, 2, 1), \quad (15, 2, 1), \quad (15, 1, 0)$$

 $\eta_1$  $\eta_2$  $\eta_3$  $\eta_4$ 

– VEV,

$$\tan \beta = \eta_3 / \eta_2$$

$$\eta = \sqrt{\eta_2^2 + \eta_3^2} \approx 250 \text{ GeV}$$

– SM VEV

## 2. Model

The interactions give contributions into  $l_i \rightarrow l_j \gamma$

$$\mathcal{L}_{S_1^{(+)} u_i l_j} = \bar{u}_{i\alpha} \left[ (h_+^L)_{ij} P_L + (h_+^R)_{ij} P_R \right] l_j S_{1\alpha}^{(+)} + \text{h.c.}$$

$$\mathcal{L}_{S_m d_i l_j} = \bar{d}_{i\alpha} \left[ (h_{2m}^L)_{ij} P_L + (h_{2m}^R)_{ij} P_R \right] l_j S_{m\alpha} + \text{h.c.}$$

where  $P_{L,R} = (1 \pm \gamma_5)/2$  are projection operators and  $(h^{L,R})_{ij}$  are generation-matrix Yukawa coupling in the MQLS model.

$$S_2^{(+)} = \sum_m c_m^{(+)} S_m, \quad S_2^{*(-)} = \sum_m c_m^{(-)} S_m,$$

where  $c_m^{(\pm)}$ ,  $m = 0, 1, 2, 3$  are the elements of the unitary scalar leptoquark mixing matrix.

## 2. Model

Because of Higgs origin the coupling constants are proportional to the ratios of fermionic masses to the SM VEV

$$\begin{aligned} m_u/\eta \sim m_d/\eta &\sim 10^{-5} & m_s/\eta &\sim 10^{-3} \\ m_c/\eta \sim m_b/\eta &\sim 10^{-2} & m_t/\eta &\sim 0.7 \end{aligned}$$

The dominant coupling constants are

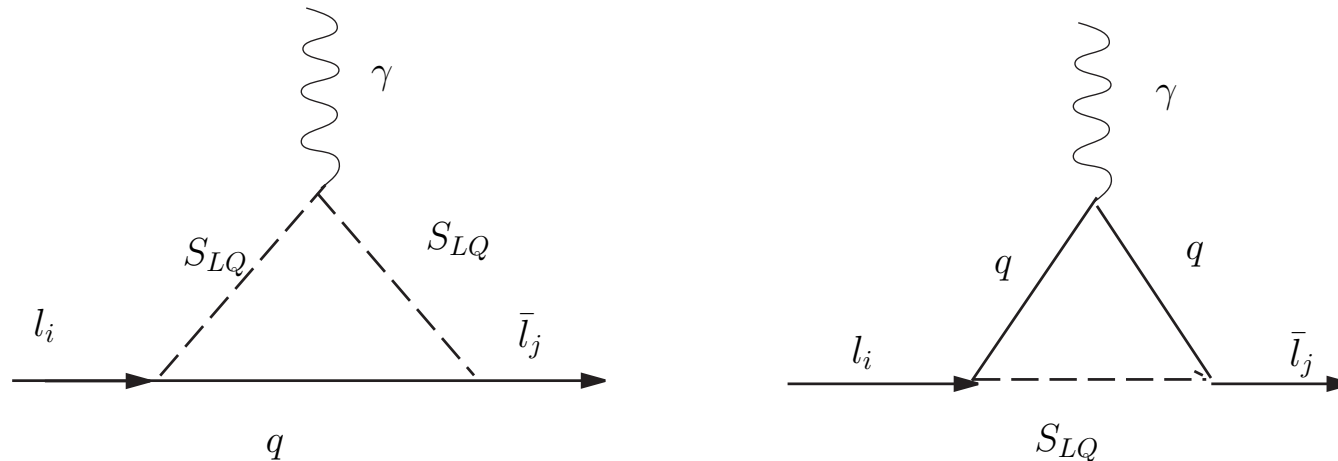
$$\begin{aligned} (h_+^L)_{3j} &= \frac{\sqrt{6} m_t}{2\eta \sin \beta} (K_1^L C_l)_{3j}, \\ (h_+^R)_{3j} &= -\frac{\sqrt{6} m_b}{2\eta \sin \beta} (C_Q)_{33} (K_2^R)_{3j}, \\ (h_{2m}^{L,R})_{3j} &= -\frac{\sqrt{6} m_b}{2\eta \sin \beta} (K_2^{L,R})_{3j} c_m^{(\mp)}, \end{aligned}$$

i.e. SLQs of MQLS model are like LQ of 3- $d$  generation.

[ P.Yu. Popov, A.V. Povarov and A.D. Smirnov, **MPLA20** (2005). ]



### 3. Widths of $l_i \rightarrow l_j \gamma$ decays



Diagrams representing the contribution of scalar leptoquarks (SLQ) into the  $l_i \rightarrow l_j \gamma$  Processes:  $q = u_i(d_i)$  is the up(down) quark of the  $i$ th generation and  $S_{LQ} = S_1^{(+)}(S_m)$  is the SLQ corresponding to the above quarks.

Contributions of  $S_m$  and  $b$ -quark in the amplitude  $l_i \rightarrow l_j \gamma$  suppress by  $m_b^2/m_t^2$  comparing with ones of  $S_1^{(+)}$  and  $t$ -quark

### 3. Widths

General form of the  $l_i \rightarrow l_j \gamma$  amplitude (one loop approximation)

$$M = -\frac{|e|}{64\pi^2 m_{LQ}^2} \bar{l}_j \sigma^{\mu\nu} q^\nu \left[ m_i \left( Q_k F_4(x) - Q_s F_2(x) \right) \left( (|h_+^L|^2)_{ji} P_R + (|h_+^R|^2)_{ji} P_L \right) \right. \\ \left. + 2m_k \left( Q_k F_3(x) - Q_s F_3(x) \right) \left( (h_+^L h_+^R)_{ji} P_R + (h_+^R h_+^L)_{ji} P_L \right) \right] l_i \epsilon^\mu,$$

Decay probability

$$W(l_i \rightarrow l_j \gamma) = \frac{9\alpha m_i}{256(4\pi)^4} \left( \frac{m_i}{\eta} \right)^4 x^2 \left( B_1^2(x) k_{ij}^{(1)} + 4 \left( \frac{m_b}{m_i} \right)^2 B_2^2(x) k_{ij}^{(2)} \right. \\ \left. - 2 \frac{m_b}{m_i} B_1(x) B_2(x) \text{Re}(k_{ij}^{(12)}) \right),$$

$$B_1(x) = Q_k F_4(x) - Q_s F_2(x), \quad B_2(x) = Q_k F_3(x) - Q_s F_1(x),$$

where  $Q_k, Q_s$  are electric charge of  $q_k$ -quark and LQ, and  $x = m_k^2/m_{LQ}^2$

### 3. Widths: functions

$$F_2(x) = \frac{1}{6(1-x)^4}(1 - 6x + 3x^2 + 2x^3 - 6x^2 \ln x),$$

$$F_3(x) = \frac{1}{(1-x)^3}(1 - x^2 + 2x \ln x),$$

$$F_5(x) = \frac{1}{6(1-x)^4}(2 + 3x - 6x^2 + x^3 - 6x \ln x),$$

$$F_6(x) = \frac{1}{(1-x)^3}(-3 + 4x - x^2 - 2 \ln x).$$

where  $x = m_k^2/m_{LQ}^2$

### 3. Widths: matrices

$$k_{ij}^{(1)} = \frac{|(K_1^L C_l)_{3j}|^2 |(K_1^L C_l)_{3i}|^2}{\sin^4 \beta},$$

$$k_{ij}^{(2)} = \frac{1}{\sin^4 \beta} \left( |(K_1^L C_l)_{3j}|^2 |(K_2^R)_{3i}|^2 + (i \leftrightarrow j) \right),$$

$$k_{ij}^{(12)} = \frac{1}{\sin^4 \beta} \left( (|(K_1^L C_l)_{3j}|^2 + |(K_2^R)_{3j}|^2) \times \right. \\ \left. \times ((K_1^L C_l)_{3i}^* (K_2^R)_{3i} + (K_2^R)_{3i}^* (K_1^L C_l)_{3i}) + (i \leftrightarrow j) \right).$$

## 4. Numerical Results and Discussion.

$$Br(\mu \rightarrow e\gamma) = 1.1 \times 10^{-4} x^2 \left( B_1^2(x) k_{\mu e}^{(1)} - \right. \\ \left. - 84 B_1(x) B_2(x) Re(k_{\mu e}^{(12)}) + 7056 B_2^2(x) k_{\mu e}^{(2)} \right),$$

$$Br(\tau \rightarrow e\gamma) = 2.2 \times 10^{-5} x^2 \left( B_1^2(x) k_{\tau e}^{(1)} - \right. \\ \left. - 4.8 B_1(x) B_2(x) Re(k_{\tau e}^{(12)}) + 20 B_2^2(x) k_{\tau e}^{(2)} \right),$$

$$Br(\tau \rightarrow \mu\gamma) = 2.2 \times 10^{-5} x^2 \left( B_1^2(x) k_{\tau\mu}^{(1)} - \right. \\ \left. - 4.8 B_1(x) B_2(x) Re(k_{\tau\mu}^{(12)}) + 20 B_2^2(x) k_{\tau\mu}^{(2)} \right).$$

#### 4. Numerical Results: Variant I

##### I) Main Contribution

$$Br(\mu \rightarrow e\gamma) = 0.7x^2 B_2^2(x) k_{\mu e}^{(2)},$$

$$k_{\mu e}^{(2)} = \frac{1}{\sin^4 \beta} \left( |(K_1^L C_l)_{3e}|^2 |(K_2^R)_{3\mu}|^2 + (\mu \leftrightarrow e) \right),$$

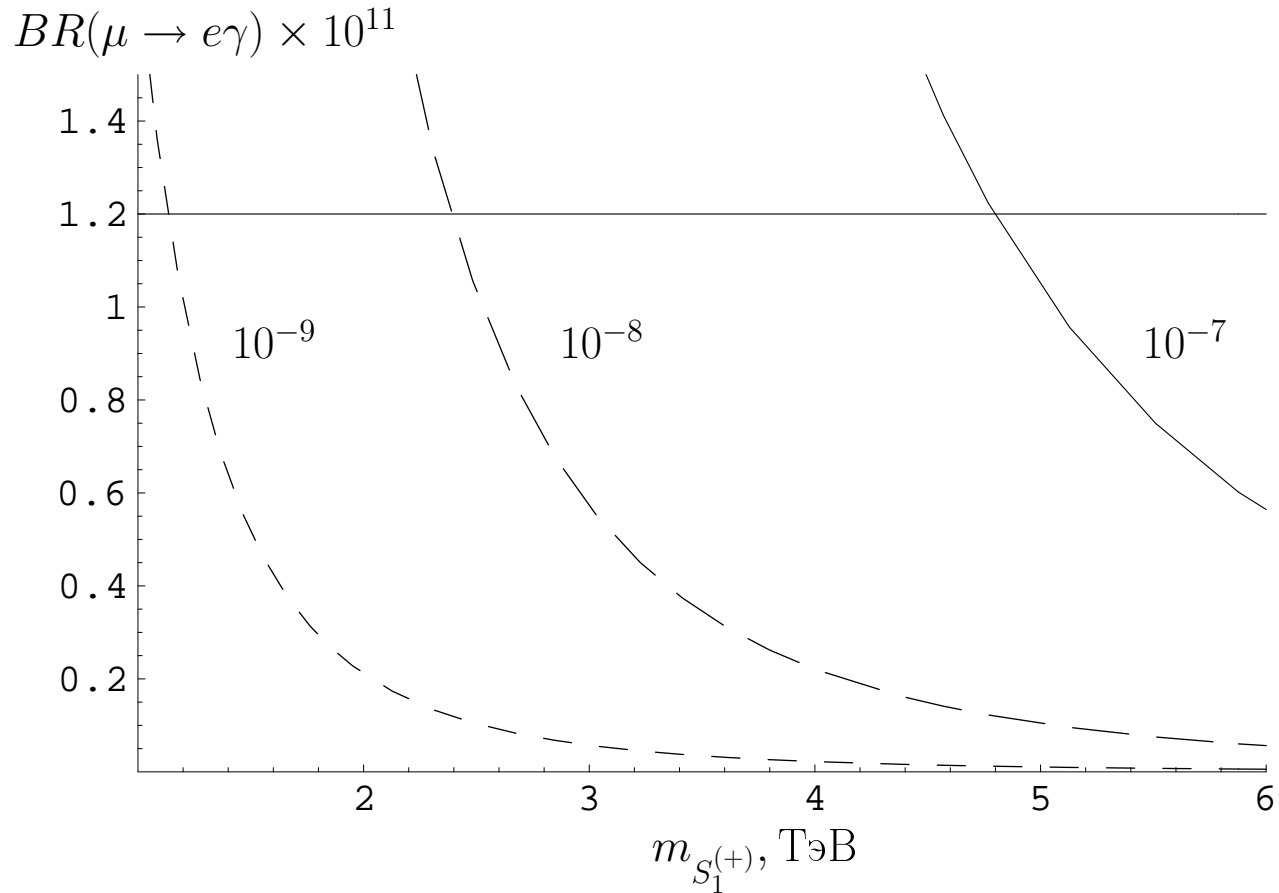
The Lower Limit on Mass Scalar Leptoquark  $S_1^{(+)}$  from Decay  $\mu \rightarrow e\gamma$  for Different Value of Parameter  $k_{\mu e}^{(2)}$ .

$k_{\mu e}^{(2)}$	$2.5 \times 10^{-12}$	$10^{-11}$	$10^{-10}$	$10^{-6}$
$m_{S_1^{(+)}} \text{ TeV}$	0.2	0.5	1	16

$$(K_2^R)_{13}, (K_1^L C_l)_{13} \sim 10^{-3}, \quad (K_2^R)_{23}, (K_1^L C_l)_{23} \sim 10^{-2}$$

$$m_{S_1^{(+)}} \sim 1 \text{ TeV}$$

## 4. Numerical Results: Variant I



The Lower Limit on Mass Scalar Leptoquark  $S_1^{(+)}$  resulting from  $BR(\mu \rightarrow e\gamma)$  with Value of Parameter  $k_{\mu e}^{(2)} = 10^{-9}, 10^{-8}, 10^{-7}$  (Horizontal line corresponds experimental limit  $Br(\mu \rightarrow e\gamma) < 1.2 \cdot 10^{-11}$ )

#### 4. Numerical Results: Variant II

II) Variant with  $K_2^R = I$

$$Br(\mu \rightarrow e\gamma) = 1.1 \times 10^{-4} x^2 B_1^2(x) k_{\mu e}^{(1)},$$

$$k_{\mu e}^{(1)} = \frac{|(K_1^L C_l)_{3e}|^2 |(K_1^L C_l)_{3\mu}|^2}{\sin^4 \beta}.$$

Lower limit on scalar leptoquark  $S_1^{(+)}$  mass resulting from  $\mu \rightarrow e\gamma$  with  $K^R = I$  for different values of parameter  $k_{\mu e}^{(1)}$

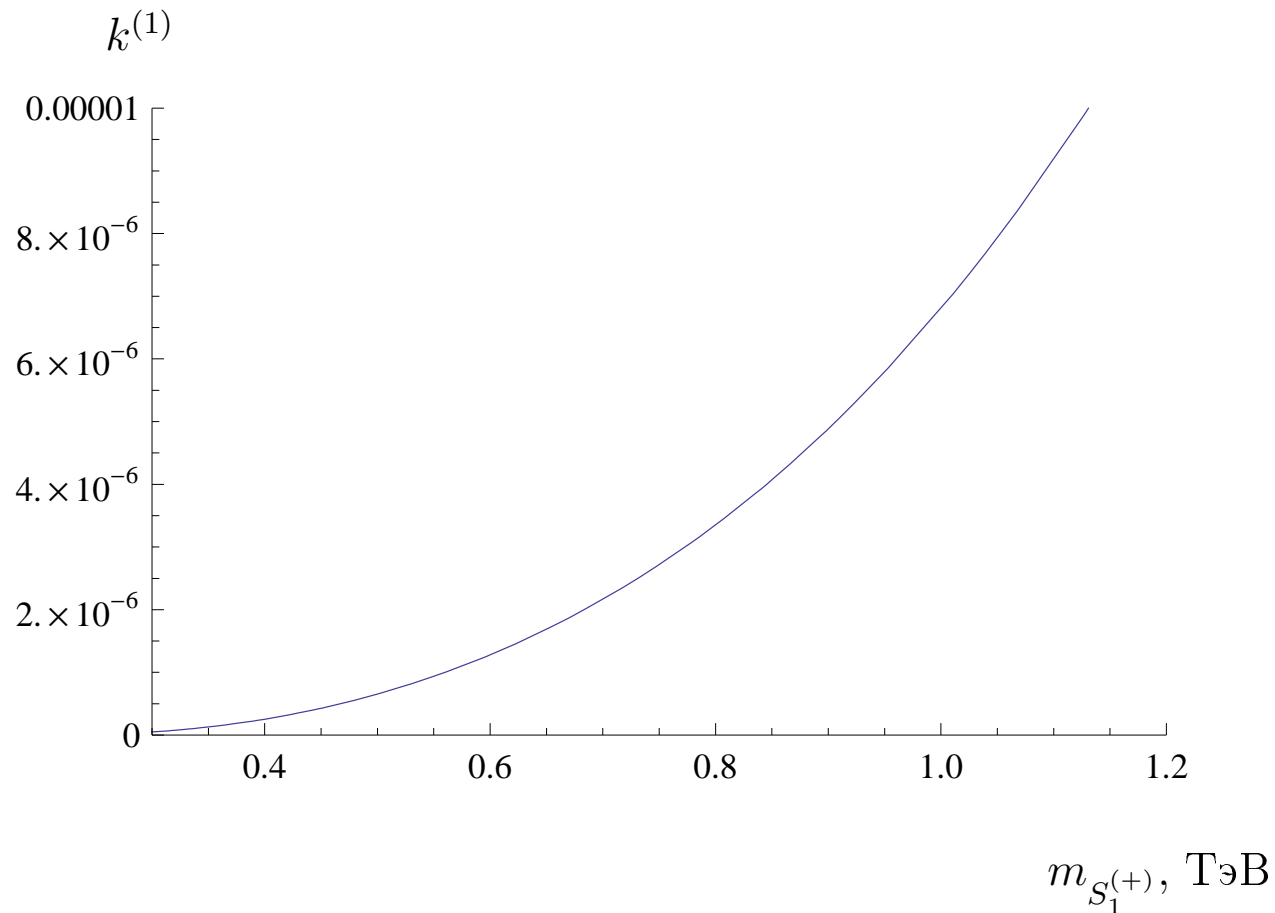
$k_{\mu e}^{(1)}$	$10^{-8}$	$10^{-6}$	$10^{-5}$	$10^{-4}$	0.01	1
$m_{S_1^{(+)}} \text{ TeV}$	0.25	0.55	1.1	2.4	9.3	34

$$(K_1^L C_l)_{13} \sim 10^{-2}, \quad (K_1^L C_l)_{23} \sim 0.1$$

$$m_{S_1^{(+)}} \sim 1 \text{ TeV}$$



## 4. Numerical Results: Variant II



The allowed region of parameters  $m_{S_1^{(+)}}$  and  $k_{\mu e}^{(1)}$  resulting from  $BR(\mu \rightarrow e\gamma)$  with  $K^R = I$  (below the curve) .

#### 4. Numerical Results: Variant IIa

IIa) Case with  $K_2^L = K_2^R = I$ ,  $\hat{C}_l^\dagger = U_{PMNS} \equiv U$

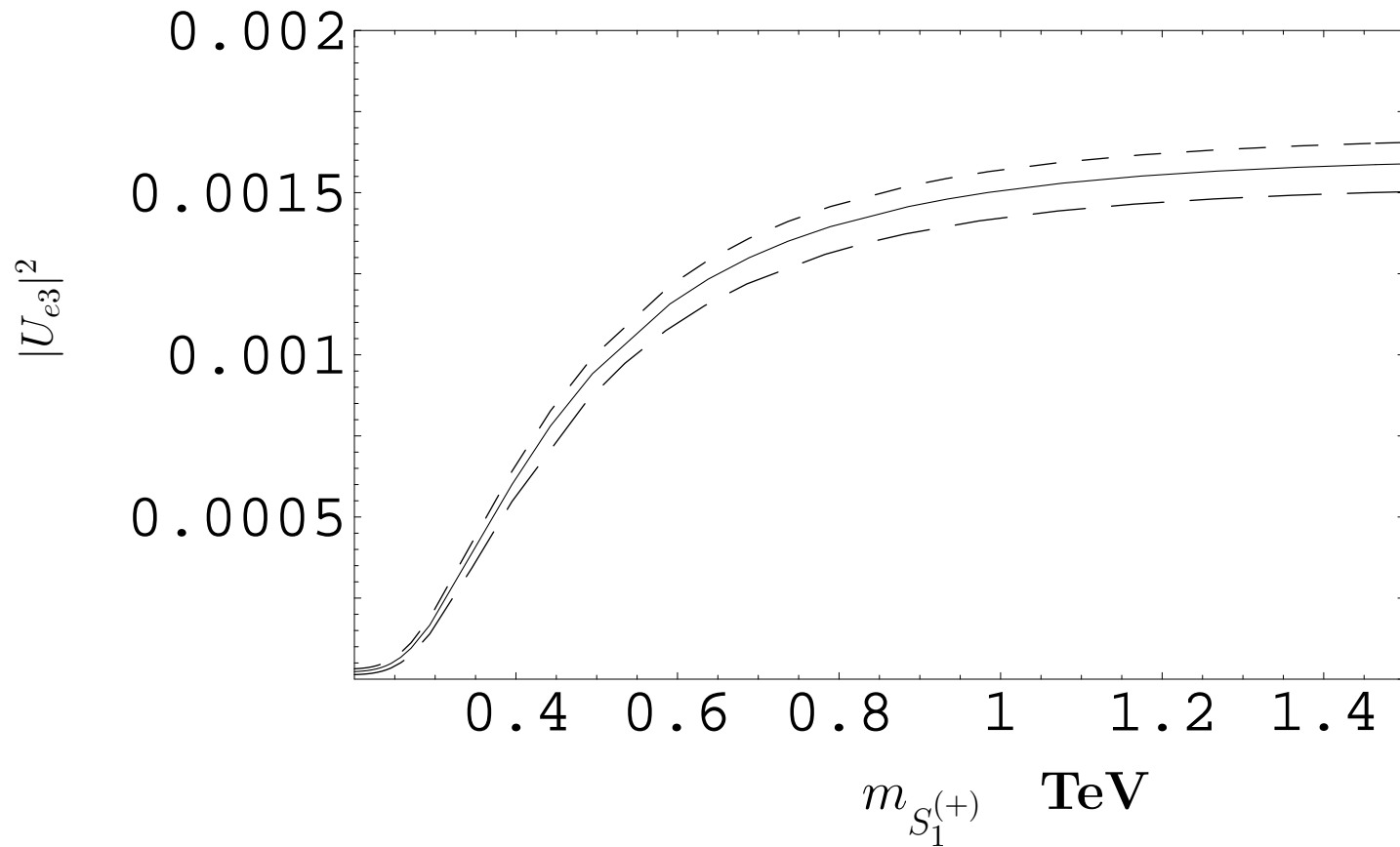
$$k_{\mu e}^{(1)} = \frac{|(U)_{13}|^2 |(U)_{23}|^2}{\sin^4 \beta}.$$

Upper limit on the matrix element  $U_{13}$  resulting from process  $\mu \rightarrow e\gamma$  with  $K^{R,L} = I$  in dependence on  $\sin \beta$  and  $S_1^{(+)}$  mass

$m_{S_1^{(+)}} \text{ TeV}$	0.55	1.3	9.3
$\sin \beta = 0.2$	$6 \times 10^{-5}$	$2 \times 10^{-4}$	$6 \times 10^{-3}$
$\sin \beta = 1$	$1 \times 10^{-3}$	$5 \times 10^{-3}$	0.14

$U_{23} = 0.7$  [ W.L. Guo and Z.Z. Xing, PRD**67**, (2003); M.C. Gonzalez-Garcia and C. Pena-Garay, PRD.**68**, (2003). ]

## 4. Numerical Results: Variant IIa



Upper limit on the matrix element  $U_{13}$  resulting from  $\mu \rightarrow e\gamma$  with  $K^{R,L} = I$  as function of  $S_1^{(+)}$  mass at  $U_{23} = 0.7^{+0.12}_{-0.16}$  and  $\sin \beta = 1$ . Current experimental limit is  $|U_{13}|^2 < 0.032$  (G. Fogli et al., Prog. Part. Nucl. Phys. 57 (2006)) .

#### 4. Numerical Results: Variants III-IV

III) Variant with  $(K_1^L C_l)_{13} = 0$ ,  $(K_2^R)_{13} = 0$

$$Br(\tau \rightarrow \mu\gamma) < 4.5 \cdot 10^{-8}$$

Lower Limit on the  $S_1^{(+)}$  Mass resulting from  $\tau \rightarrow \mu\gamma$  in dependence on  $k_{\tau\mu}^{(1)} = k_{\tau\mu}^{(2)} = k_{\tau\mu}^{(12)}$

$k_{\tau\mu}^{(1)} = k_{\tau\mu}^{(2)} = k_{\tau\mu}^{(12)}$	$10^{-4}$	$10^{-3}$	$10^{-2}$
$m_{S_1^{(+)}} \text{ TeV}$	0.3	0.7	1.4

IV) Interaction of scalar leptoquark  $S_m$  ( $Q = 2/3$ ) gives the lower limit on  $m_{S_m}$  weaker than the current experimental ones

#### 4. Numerical Results: Variant V

V) The Case of the chiral interaction of scalar leptoquarks  $S_1^{(+)}$  with fermions gives the limits which coincide with those of the [Variant II]

## Conclusions

The contributions of scalar leptoquarks  $S_1^{(+)}$ ,  $S_m$  from the MQLS model in  $l_i \rightarrow l_j \gamma$  decays are analysed in comparison with experimental data on  $\mu \rightarrow e \gamma$ ,  $\tau \rightarrow \mu \gamma$ ,  $\tau \rightarrow e \gamma$  decays

It is shown that in the appropriate region of the mixing parameters relatively light scalar leptoquarks (with masses of order **1 TeV** or below) do not contradict current experimental restrictions on LFV processes