Observing GW signature from supermassive BH mergings

Postnov K.

Sternberg Astronomical Institute

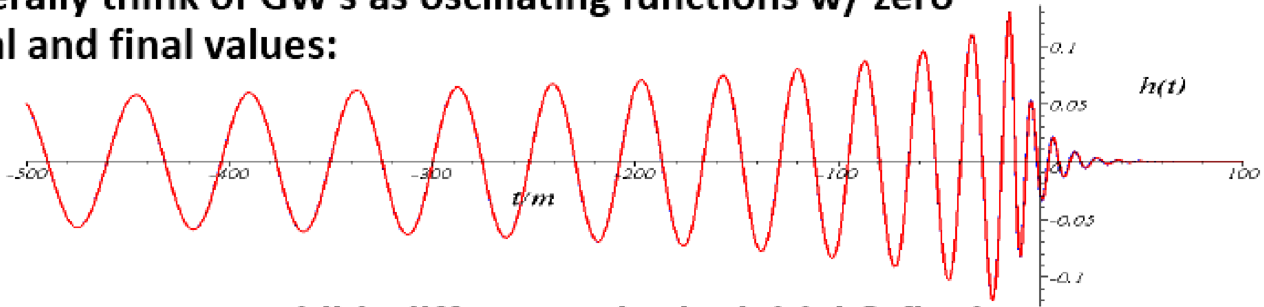
0909.0742, MNRAS 2010

Outline

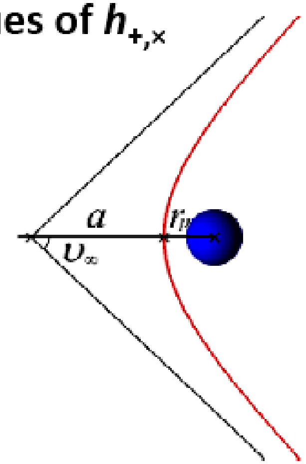
- GW bursts with memory
- SMBH mergings
- Signatures in pulsar timing
- S/N ratio
- Expected event rate
- Conclusions

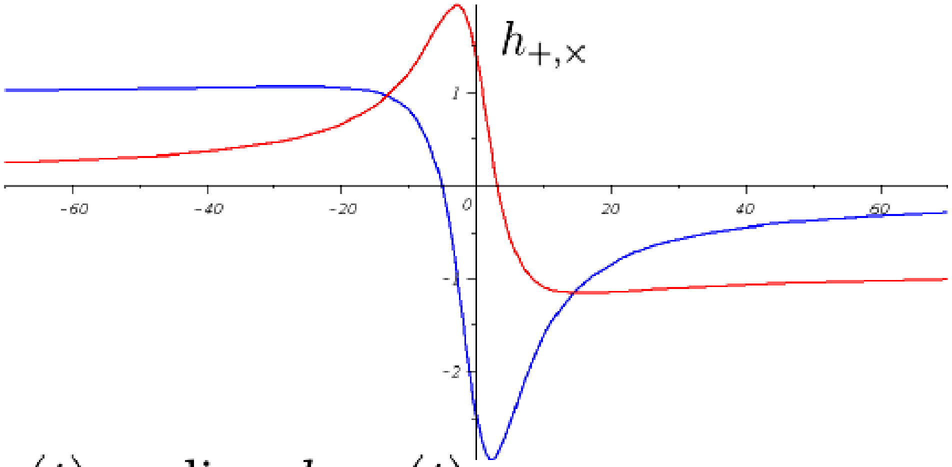
GW bursts with memory

- Generally think of GW's as oscillating functions w/ zero initial and final values:



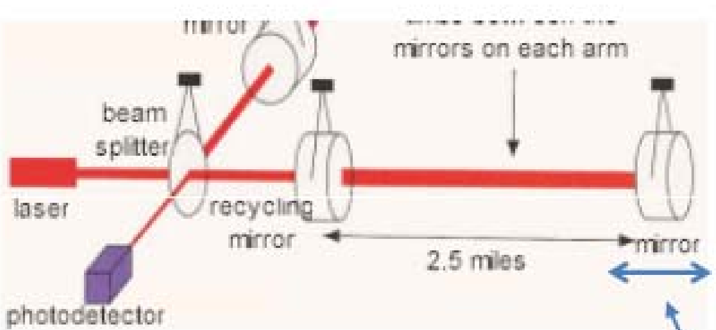
- But some sources exhibit differences in the initial & final values of $h_{+,\times}$





$$\Delta h^{\text{mem}}_{+,\times} = \lim_{t\to+\infty} h_{+,\times}(t) - \lim_{t\to-\infty} h_{+,\times}(t)$$

- Ideal GW detector ('free masses') would have permanent displacement ("memory")

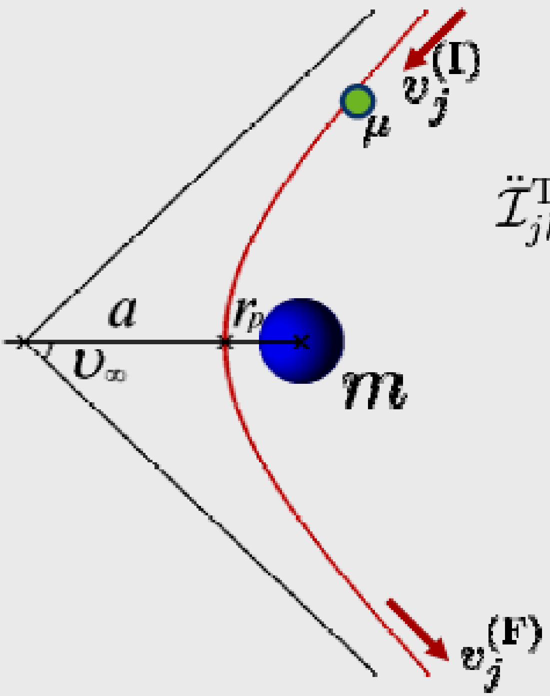


$$\delta x(t) = \frac{L}{2} h_+(t) \quad \Rightarrow \quad \delta x^{\rm mem} = \frac{L}{2} h_+^{\rm mem}$$

- Biuld-up of the displacement is measurable (difficult by ground-based LIGO, but can be done from space by LISA)

GWM: linear effect

- Non-oscillatory change of quadrupole and higher multipole moments (Zeldovich & Polnarev 74, Braginsky & Grishchuk 78, Braginsky & Thorne 87). For example, gravitational scattering (hyperbolic orbit)



$$h^{\mathrm{TT}}_{jk} \approx \frac{2}{R}\ddot{\mathcal{I}}^{\mathrm{TT}}_{jk} \qquad \mathcal{I}^{\mathrm{TT}}_{jk} = \mu[x_j x_k]^{\mathrm{TT}}$$

$$\ddot{x}_j = -\frac{M}{r^3}x_j$$

$$\ddot{\mathcal{I}}^{\mathrm{TT}}_{jk} = \mu[x_j\ddot{x}_k + \ddot{x}_j x_k + 2\dot{x}_j\dot{x}_k]^{\mathrm{TT}}$$

$$= 2\mu\left[\dot{x}_j\dot{x}_k - \frac{M}{r^3}x_j x_k\right]^{\mathrm{TT}}$$

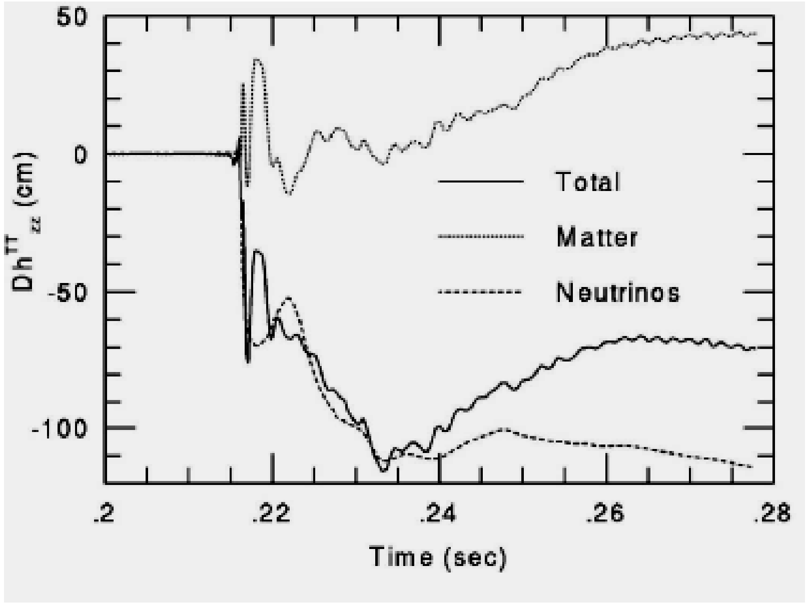
Turner 77,
Velocities before and after scattering are the same but sign is opposite

$$\Delta h^{\mathrm{TT}}_{jk} = \frac{4\mu}{R}\Delta[\dot{x}_j\dot{x}_k]^{\mathrm{TT}}$$

- For any system of N gravitationally unbound bodies with velocities v_A before and after GW burst emission (Braginsky & Thorne 87, Thorne 92)

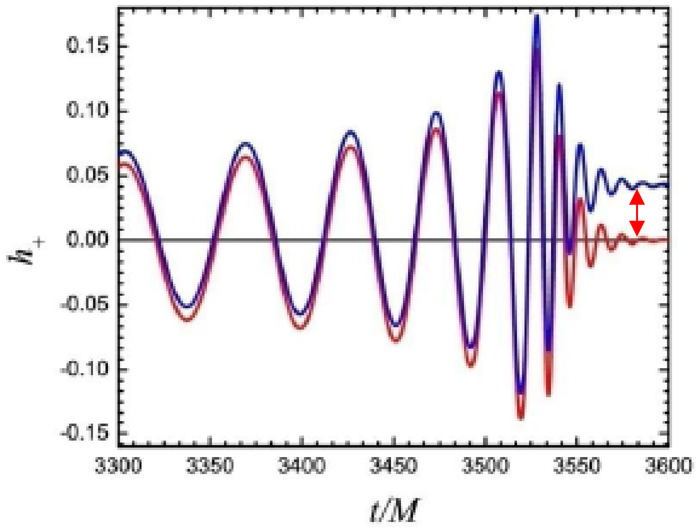
$$\Delta h_{jk}^{TT} = \Delta \sum_{A=1}^{N} \frac{4 M_A}{R\sqrt{1 - v_A^2}} \left[\frac{v_A^j v_A^k}{1 - v_A \cdot N} \right]^{TT}$$

- Examples: hyperbolic orbits (Turner 77), asymmetric neutrino emission (Epstein 78), asymmetric SN explosions (Burrows & Hayes 96, Ott 08), GRB jets…



Nonlinear effect (Cristodoulou memory) Cristodoulou 91, Blanchet & Damour 92

- Contribution to the distant GW field sourced by the emission of GWs
- Recall previous form of the Einstein's equations:



$$\Box h_{\alpha\beta} = 16\pi \det(g_{\mu\nu}) T_{\alpha\beta} + \mathcal{F}[h, h]$$

Grav'l wave stress-energy tensor...

$$T^{\rm gw}_{\alpha\beta} \propto \frac{dE^{\rm gw}}{dt d\Omega} \sim O(h^2)$$

$$\ddot{\mathcal{I}}_{jk} \to \ddot{\mathcal{I}}_{jk} + U^{\rm gw}_{jk}$$

...contributes to the changing multipole moments...

(Favata 09)

$$h^{\rm TT}_{jk} \approx \frac{2}{R} \ddot{\mathcal{I}}^{\rm TT}_{jk}$$

...which determines the GW field...

$$\Delta h^{\rm (mem)} \sim \frac{\Delta E^{\rm gw}}{R}$$

...which has a slowly-growing, non-oscillatory piece related to the radiated GW energy.

- Hereditary nature: memory piece of the GW field depends on the entire history:

$$\delta h^{TT}_{jk} = \frac{4}{R}\int_{-\infty}^{T_R} dt' \left[\int \frac{dE^{gw}}{dt'\, d\Omega'} \frac{n'_j n'_k}{(1 - n' \cdot N)}\, d\Omega'\right]^{TT}$$

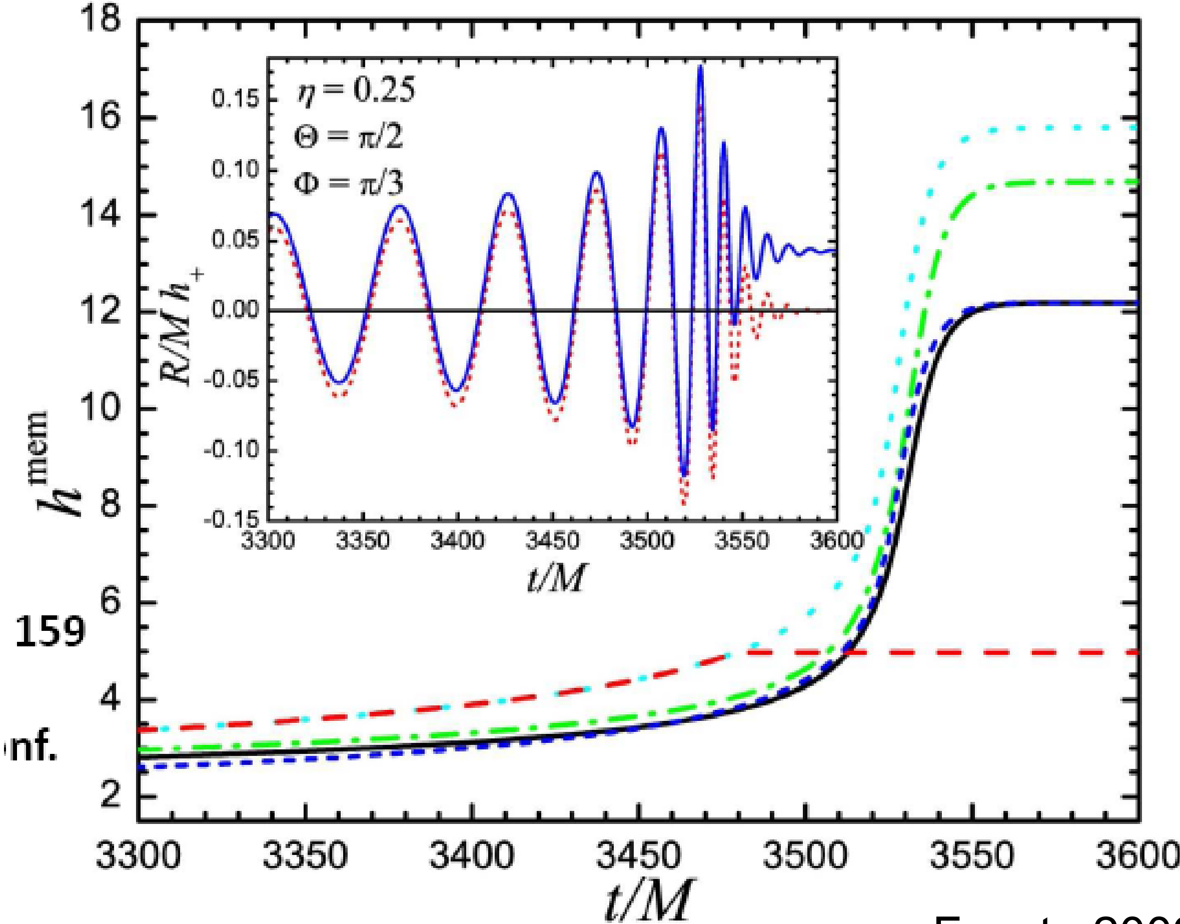
(T_R is retarded time)

- Similar to linear memory can be interpreted as arising from changes in the mass quadrupole moment of the system during emission of individual gravitons (Thorne 92) with energies $E_A = M_A/(1 - v_A^2)^{1/2}$

  and velocities $v_a^j = c n_a^j$

GWM in binary BH mergers

- Quasi-circular orbits: $h_x=0$, $h_+\neq 0$



Depends on the entire history, but roughly

$$\mathbf{h^{mem} \sim h^{insp}}$$

Favata 2009, ApJL 696 159

$$h_+^{mem} = \frac{\eta M h}{384\pi R}\sin^2\theta(17+\cos^2\theta), \quad M = M_1 + M_2,\ \eta = \frac{M_1 M_2}{M^2}$$

θ - angle between orbital ang. momentum and line of sight

$$h = \frac{16\pi}{\eta}\left(\frac{\Delta E_{GW}}{M}\right)$$

$$\left\langle h_+^{mem} \right\rangle = \frac{69}{8}\left(\frac{\Delta E_{GW}}{24R}\right) \approx \frac{\Delta E_{GW}}{3R}$$

From numerical simulations (Reisswig et al 09):

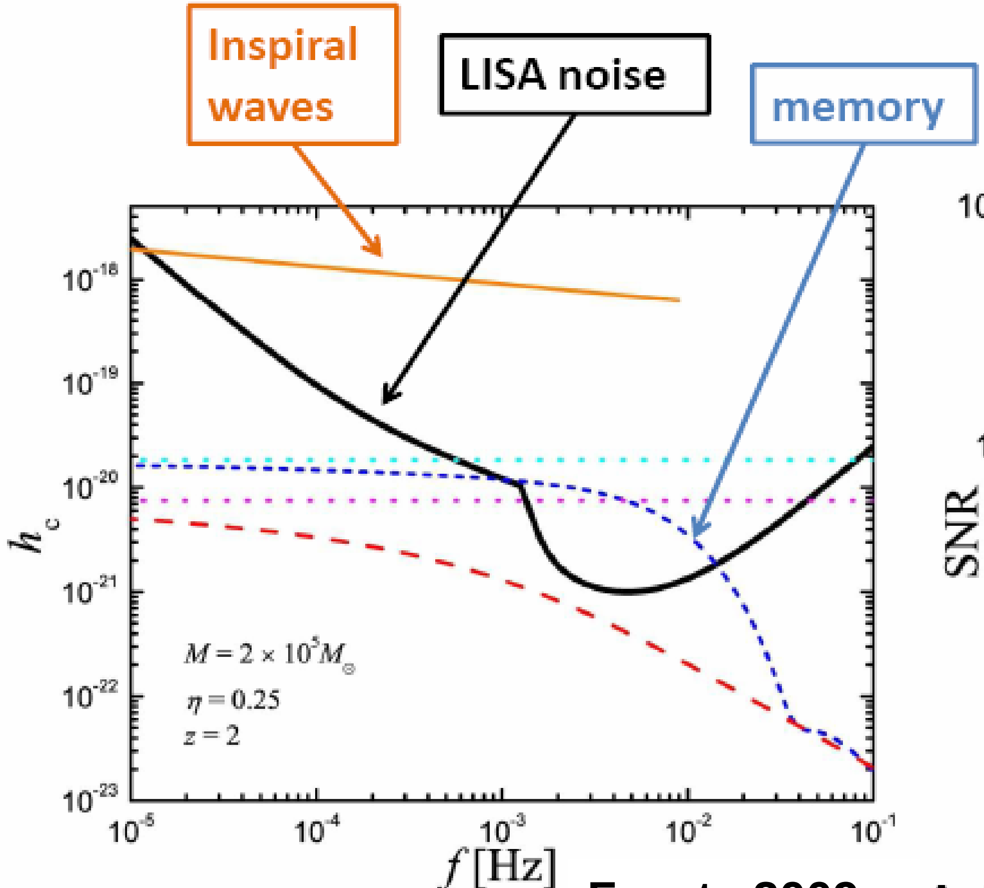
$$\Delta E_{GW} \approx (3.6-10\%)M \quad \Rightarrow$$

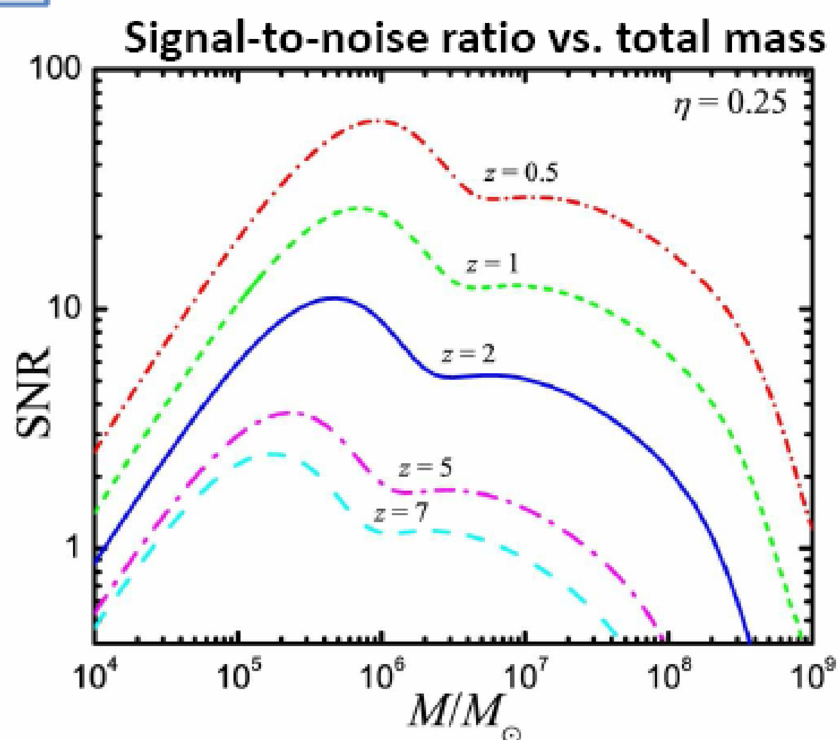
$$h^{mem} \approx 5\times10^{-16}\left(\frac{m}{10^8 M_\odot}\right)\left(\frac{1\ \mathrm{Gpc}}{R}\right)$$

Detectability of the memory:

SMBH mergers

- will be difficult to observe w/ Advanced LIGO
- likely to be visible by LISA out to redshift $z \lesssim 2$





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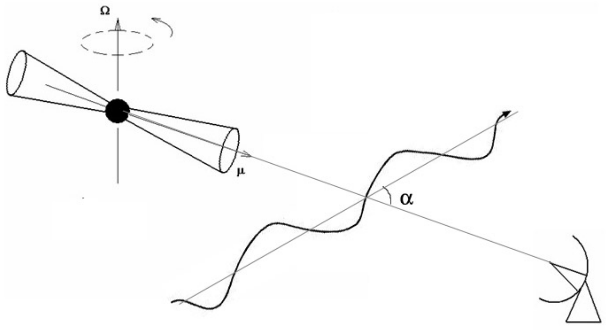
Detection of GWM by pulsar timing

- GWM leaves unique signature in pulsar timing: linear growth of rms residuals with time

$$s_{mem} \sim h_+^{mem} T_{obs}$$

$$s_{insp} \sim h^{insp} / \omega_{insp}$$



$$\omega_{insp}^{-1} \approx 7.5 \times 10^3 (M / 10^8 M_\odot)\,\mathrm{s}$$

$$\mathrm{T_{obs}} \approx 10\, yrs$$

$$\frac{\mathrm{SNR_{GWM}}}{\mathrm{SNR_{insp}}} \sim \frac{h_+^{mem} T_{obs} \omega_{insp}}{h^{insp}} \sim T_{obs} \omega_{insp} \sim 2 \times 10^4$$

Open prospects for detection!

Pulsar frequency modulation (Sazhin 78, Detweiler 79)

$$\frac{\Delta\nu}{\nu_0} = \frac{1}{2}\int_0^D d\lambda \left(e^i e^j \frac{\partial h^{ij}}{\partial t} \right)\Bigg|_{path}$$

Timing residuals

$$s(t) = \int_0^t d\tau \frac{\Delta\nu(\tau)}{\nu_0}$$

For plain gravitational wave

$$h_{ij}(x^i,t) = h_+(t - n_i x^i)\, p_{ij}^+ + h_\times(t - n_i x^i)\, p_{ij}^\times$$

$$\frac{\Delta\nu(t)}{\nu_0} = \frac{1}{2}(1+\mu)\left\{ \begin{matrix} \left[h_+(t)\cos 2\phi + h_\times(t)\sin 2\phi\right] - \\ \left[h_+(t - D(1-\mu))\cos 2\phi + h_\times(t - D(1-\mu)\sin 2\phi\right] \end{matrix} \right\}$$

Physical sense: frequency variation is determined by difference between the GW strength at the place and time of observations (first [.]) and its strength at the site and time of signal emission

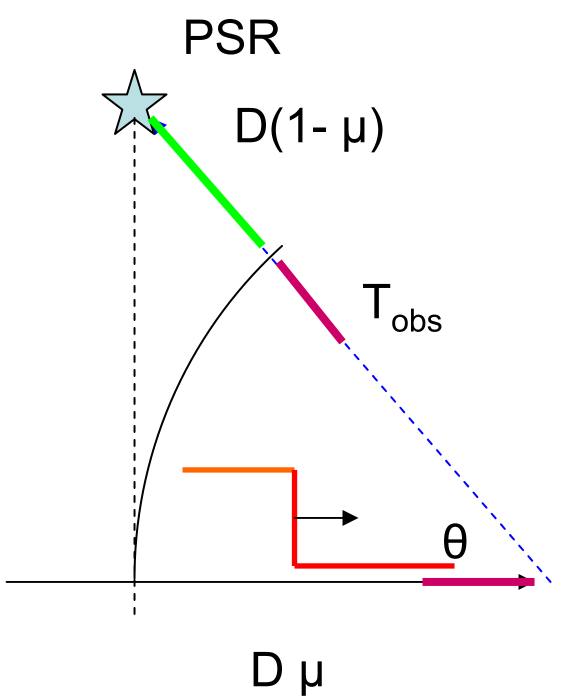
For GWBM from SMBH $h_\times = 0$,

$h_+(t - D(1-\mu)) = 0$ at the site of PSR

if $D(1-\mu) > T_{obs}$.

Typically D~kpc >>$\mathrm{T_{obs}}$ $\sim 10\,yrs$

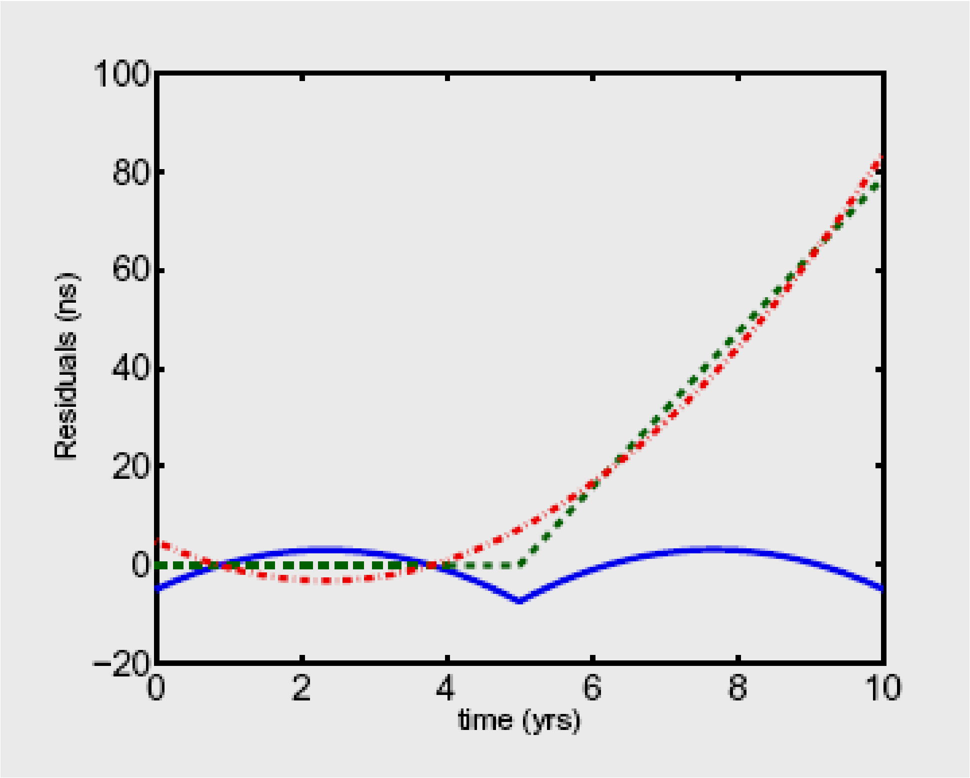
μ=cos θ



Net result:

$$s(t) = \frac{1}{2}(1+\mu)\int_0^t d\tau h_+(\tau)\cos 2\phi$$

Timing residuals



Pshirkov, Baskaran, PK, 2010 MNRAS

- Calculate expected signal
- Extract quadratic fit to obtain postfit residuals
- Check whether they can be measured at a given SNR by pulsar timing array (PTA)

Detectability: 1. SNR

Residuals (t) = signal residuals (t) + noise (t)

Gaussian stationary noise uncorrelated for each pulsar

$$\overline{n_\alpha(t_i) n_\beta(t_j)} = \sigma_n^2 \delta_{ij} \delta_{\alpha\beta}$$

PTA includes $\mathrm{N}_\alpha = 20$ PSRs, $\mathrm{T_{obs}} \approx 10\,yrs$,

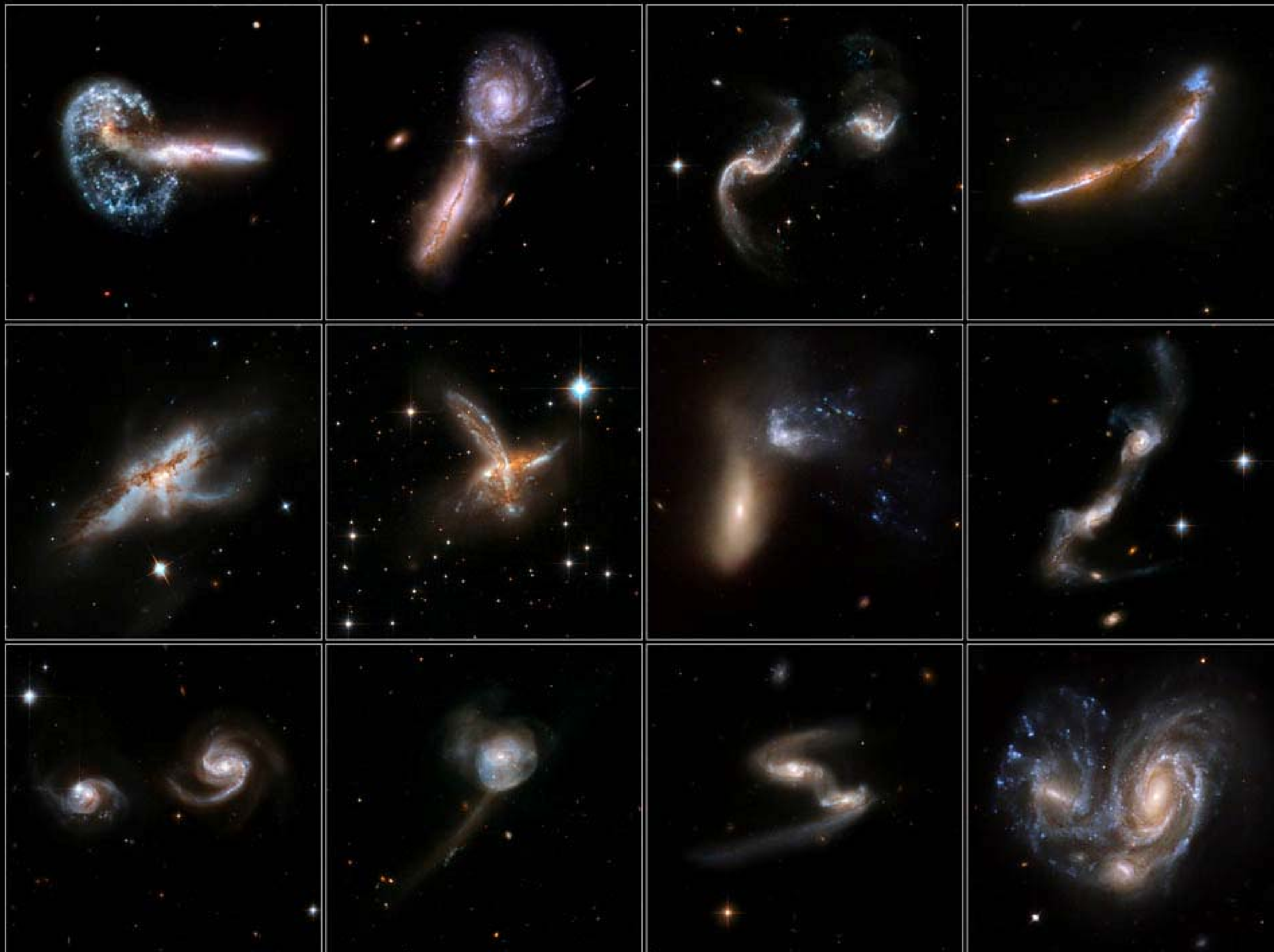
# of individual observatins $\mathrm{N_t} = 250$

Expected sensitivity: $\sigma_n = 100\,ns$

$$\mathrm{SNR_{GWM}} \approx 1.6 \left(\frac{h^{mem}}{10^{-15}}\right)\left(\frac{N_t}{250}\right)^{1/2}\left(\frac{N_t}{250}\right)^{1/2}\left(\frac{T_{obs}}{10\,yrs}\right)\left(\frac{100ns}{\sigma_n}\right)$$

Interacting Galaxies

Hubble Space Telescope • ACS/WFC • WFPC2



NASA, ESA, A. Evans (University of Virginia, Charlottesville/NRAO/Stony Brook University), and the Hubble Heritage (AURA/STScI)-ESA/Hubble Collaboration

STScI-PRC08-16a

Detectability: 2. Rate of events

- Rough estimate: ~ 10^{-3} Gpc^{-3} yr^{-1} as major galaxy mergers (Concelice et al 09) ➔ ~ 0.5 event per year up to z~0.5. Consistent with numerical simulations
- More accurate analysis: from observed SMBH mass function $N(>M)=0.07(M/10^7)^{-2}$ and galaxy merger rate evolution $n(z)\sim(1+z)^{(2\ldots3)}$

$$N \approx 0.1\left(\frac{N_t}{250}\right)\left(\frac{N_\alpha}{20}\right)\left(\frac{T_{obs}}{10\,yrs}\right)^3\left(\frac{100ns}{\sigma_n}\right)^2\left(\frac{3}{SNR}\right)$$

Conclusions

- GW bursts with memory leave unique imprint in pulsar timing residuals
- Gravitational bursts with memory and amplitude $h \sim (1.5\text{-}2) \times 10^{-15}$ can be potentially detected in 10yrs of Pulsar Timing Array observations at the SNR=3 at the current timing noise level 100 ns.
- With account for observed SMBH mass function and evolution of galaxy major mergers with redshift, PTA detection rate can be around 1 in a 10yrs run
- For merging SMBH, this method is complimentary to LISA for $M > 10^8 M_\odot$ to which LISA sensitivity decreases

THANK YOU!