Frame-Like Action and Unfolded Formulation for Massive Higher-Spin Fields

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D.P. and M.A. Vasiliev, [arXiv:1001.0062[hep-th]].

Plan of the talk:

- Motivation
- Introduction
  - massive HS field
  - unfolded dynamics approach
- Unfolding of massive HS fields
- Action
- Conclusion

Motivation

String theory

Contains massive as well as massless
HS fields

some symmetries are broken

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Nonlinear theory of massless HS fields

[Vasiliev '90]

Contains an infinite tower of massless HS fields

unbroken HS symmetries

unfolded formalism

Motivation

String theory	Nonlinear theory of massless HS fields [Vasiliev '90]
Contains massive as well as massless HS fields	Contains an infinite tower of massless HS fields
some symmetries are broken	unbroken HS symmetries
	unfolded formalism

Is there any relation between these two theories?

Spontaneous breakdown mechanism for HS symmetries gives rise to a theory of massive HS fields.

Equations of motions

$$(\partial^n \partial_n + m^2)\phi_{a_1 a_2 \dots a_s} = 0,$$

$$\partial^n \phi_{n a_3 \dots a_s} = 0,$$

$$\phi^n{}_{n a_3 \dots a_s} = 0.$$

[Dirac ’36]

These EOM can be derived for $s > 1$ from a Lagrangian without introducing new degrees of freedom by adding auxiliary fields that are expressed in terms of (derivatives of) the dynamical fields by virtue of equations of motion.

[Fierz and Pauli ’39]

Introduction. Massive spin-s field

Metric-like formulations

Generalize metric formulation of gravity

$$g_{\mu\nu} \longrightarrow \phi_{a_1 a_2 \ldots a_s},\ \text{auxiliary fields}$$

[Singh and Hagen '74]

[Zinoviev '01]

Frame-like formulations

Generalize Cartan formulation of gravity

$$h_\mu{}^a,\ \omega_\mu{}^{a,b} \longrightarrow \omega_\mu{}^{a_1 a_2 \ldots a_{s-1}},\ \omega_\mu{}^{a_1 a_2 \ldots a_{s-1},b},\ \text{auxiliary fields}$$

[Zinoviev '08]

Introduction. Unfolded dynamics approach

$$R^{\alpha}(x) \stackrel{def}{=} dW^{\alpha}(x) + G^{\alpha}(W(x)), \qquad (1)$$

$$G^{\alpha}(W^{\beta}) \stackrel{def}{=} \sum_{n=1}^{\infty} f^{\alpha}_{\beta_1\ldots\beta_n} W^{\beta_1} \ldots W^{\beta_n},$$

$$G^{\beta}(W)\frac{\delta^L G^{\alpha}(W)}{\delta W^{\beta}} \equiv 0\,, \qquad (2)$$

where W^{α} are differential forms, α enumerates various forms, R^{α} are generalized curvatures.

$$R^{\alpha}(x) = 0. \qquad (3)$$

Gauge transformations

$$\delta W^{\alpha} = d\varepsilon^{\alpha} - \varepsilon^{\beta}\frac{\delta^L G^{\alpha}(W)}{\delta W^{\beta}}\,. \qquad (4)$$

Background Minkowski or $(A)dS$ geometry

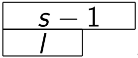
$$T^a \equiv h^a + \omega^{a,}{}_b \wedge h^b = 0,$$
$$R^{a,b} \equiv d\omega^{a,b} + \omega^{a,}{}_c \wedge \omega^{c,b} - \lambda^2 h^a \wedge h^b = 0$$

with $\lambda^2 > 0$ for AdS, $\lambda^2 < 0$ for dS and $\lambda^2 = 0$ for Minkowski space.
h^a — background vielbein,
$\omega^{a,b}$ — background connection.

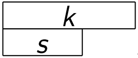
Unfolding of massless HS fields

Unfolded EOM for **massless** field of **spin**-s in $(A)dS$ and Minkowski spaces [Lopatin and Vasiliev '88].

Fields

$W^{a_1\dots a_{s-1},b_1\dots b_l}$, 1-forms: 

, $0 \le l \le s-1$,

$C^{a_1\dots a_k,b_1\dots b_s}$, 0-forms: 

, $k \ge s$.

Traceless 2-row Young diagrams

Young symmetry condition: $W^{(a_1\dots a_k,b_1)b_2\dots b_l} = 0,$

tracelessness: $W^{a_1\dots a_k,c}{}_c{}^{b_3\dots b_l} = \dots = 0,$

$$k \ge l.$$

Unfolding of massless HS fields

Equations of motion

$$R_2^{\begin{array}{|c|}\hline s-1 \\ \hline l \\ \hline\end{array}} = DW^{\begin{array}{|c|}\hline s-1 \\ \hline l \\ \hline\end{array}} + \sigma_-^2(h)\,W^{\begin{array}{|c|}\hline s-1 \\ \hline l+1 \\ \hline\end{array}} + \sigma_+^2(h)\,W^{\begin{array}{|c|}\hline s-1 \\ \hline l-1 \\ \hline\end{array}}\,,$$

$$R_2^{\begin{array}{|c|}\hline s-1 \\ \hline s-1 \\ \hline\end{array}} = DW^{\begin{array}{|c|}\hline s-1 \\ \hline s-1 \\ \hline\end{array}} + \sigma_+^2(h)\,W^{\begin{array}{|c|}\hline s-1 \\ \hline s-2 \\ \hline\end{array}} + \kappa_{--}^{12}(h)\,C^{\begin{array}{|c|}\hline s \\ \hline s \\ \hline\end{array}}\,,$$

$$R_1^{\begin{array}{|c|}\hline k \\ \hline s \\ \hline\end{array}} = DC^{\begin{array}{|c|}\hline k \\ \hline s \\ \hline\end{array}} + \sigma_-^1(h)\,C^{\begin{array}{|c|}\hline k+1 \\ \hline s \\ \hline\end{array}} + \sigma_+^1(h)\,C^{\begin{array}{|c|}\hline k-1 \\ \hline s \\ \hline\end{array}}\,.$$

Unfolding of massless HS fields

Equations of motion

$$R_2^{\substack{s-1\\ l}} = DW^{\substack{s-1\\ l}} + \sigma_-^2(h)W^{\substack{s-1\\ l+1}} + \sigma_+^2(h)W^{\substack{s-1\\ l-1}},$$

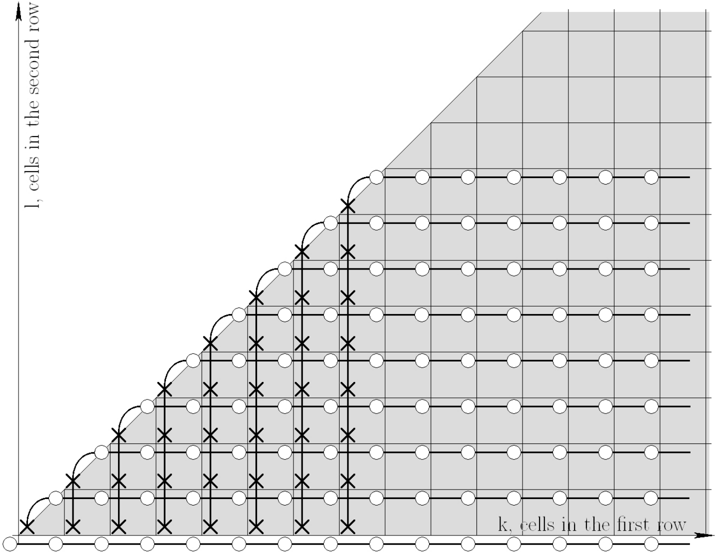
$$R_2^{\substack{s-1\\ s-1}} = DW^{\substack{s-1\\ s-1}} + \sigma_+^2(h)W^{\substack{s-1\\ s-2}} + \kappa_{--}^{12}(h)C^{\substack{s\\ s}},$$

$$R_1^{\substack{k\\ s}} = DC^{\substack{k\\ s}} + \sigma_-^1(h)C^{\substack{k+1\\ s}} + \sigma_+^1(h)C^{\substack{k-1\\ s}}.$$

Example

$$\sigma_-^2(h)W^{a_1\ldots a_{s-1},b_1\ldots b_{l+1}} = F(s-1,l+1)h_m W^{a_1\ldots a_{s-1},b_1\ldots b_l m}.$$

Combining massless fields to obtain a massive field

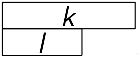


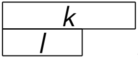
"×" denote 1-forms and "○" denote 0-forms.

Unfolded EOM for **massive** field of **spin**-s in $(A)dS$ and Minkowski spaces

[D.P. and Vasiliev '10].

Fields

$W^{a_1\ldots a_k, b_1\ldots b_l}$, 1-forms: , $\quad k \leq s-1, \quad l \leq k,$

$C^{a_1\ldots a_k, b_1\ldots b_l}$, 0-forms: , $\quad l \leq s, \quad k \geq l.$

Unfolding of massive HS fields

Equations

$$R_2 = dW + \sigma_{(1)}(h)W + \kappa(h)C,$$
$$R_1 = dC + \sigma_{(0)}(h)C + \alpha W.$$

Gauge transformations

$$\delta W = d\xi + \sigma_{(1)}(h)\xi, \qquad \delta C = -\alpha\xi.$$

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Example

$$\alpha W^{a_1\ldots a_k,b_1\ldots b_l} = \alpha(k,l)W^{a_1\ldots a_k,b_1\ldots b_l},$$
$$\kappa^{12}_{--}(h)C^{a_1\ldots a_k,b_1\ldots b_k} = A(k,k)h^c h^d C^{a_1\ldots a_{k-1}}{}_{c,}{}^{b_1\ldots b_{k-1}}{}_{d}.$$

Unfolding of massive HS fields

Compatibility conditions

$$\alpha\sigma_{(1)} = \sigma_{(0)}\alpha,$$

$$\alpha\kappa = D^2 + (\sigma_{(0)})^2,$$

$$\kappa\alpha = D^2 + (\sigma_{(1)})^2,$$

$$\sigma_{(1)}\kappa = \kappa\sigma_{(0)}.$$

Field redefinition ambiguities

$$W = \beta_{(1)}\tilde{W}, \quad C = \beta_{(0)}\tilde{C}, \quad W = \tilde{W} + \tilde{\sigma}C.$$

Action

Gauge invariance:

$$\delta_\xi R = 0, \quad \Rightarrow \quad \delta_\xi S = 0 \quad \text{for} \quad S = f(R).$$

The extra field decoupling condition

$$\frac{\delta S}{\delta W^{a(k),b(l)}} \equiv 0, \quad l \geq 2, \quad \frac{\delta S}{\delta C^{a(q),b(r)}} \equiv 0, \quad q \geq 2,$$

The massless decomposition condition.
Decomposition of kinetic term.

Action

The scalar product

$$\langle\psi_{\{p\}}^{a_1\dots a_k,b_1\dots b_l}|\phi_{\{q\}}^{a_1\dots a_k,b_1\dots b_l}\rangle =$$

$$= \int \epsilon_{l_1\dots l_d} e_0^{l_5}\dots e_0^{l_d}\psi_{\{p\}}{}^{l_1 a_2\dots a_k,l_2 b_2\dots b_l}\phi_{\{q\}}{}^{l_3}{}_{a_2\dots a_k}{}^{,l_4}{}_{b_2\dots b_l}.$$

Final action

$$S = \sum\langle R_2|R_2\rangle - \sum\langle R_1|\kappa R_1\rangle.$$

Conclusion

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- In terms of these curvatures we constructed the manifestly gauge invariant action as well as full unfolded field equations for free symmetric massive HS fields.
- Good starting point for investigation of unfolded equations for general massive mixed symmetry fields.
- Nice background for establishment of Nonlinear massive HS theory.