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Dark halos built of scalar gravitons: numerical study

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The theory of meta-gravity violating general covariance was proposed to explain dark matter phenomenon[1].

Static spherically symmetric case in vacuum was investigated:

$$ds^2 = adt^2 - bdr^2 - cr^2(d\theta^2 + \sin^2\theta d\varphi^2) \qquad (1.1)$$

Variables of system defined by:

$$X(r) = \frac{\chi(r)}{k_h}; \quad A(r) = a(r) = \frac{1}{b(r)}; \quad C(r) = r^2 c(r) \qquad (1.2)$$

where $\chi(r)$ - new heteron[2] field, k_h - heteron field scale constant.

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[1] Yu. F. Pirogov, Graviscalar dark matter and smooth halos, arXiv:0909.3311[gr-qc].

[2] Heteron = scalar graviton.

Solution was found in form of power series:

$$X = \xi^2 - \frac{3}{10}\xi^4 + \frac{4}{35}\xi^6 \tag{1.3}$$

$$a = 1 + \varepsilon_h^2\left(\xi^2 - \frac{3}{10}\xi^4 + \frac{4}{35}\xi^6\right) \tag{1.4}$$

$$c = 1 + \varepsilon_h^2\left(-\frac{1}{10}\xi^4 + \frac{2}{35}\xi^6\right) \tag{1.5}$$

where

$$\xi^2 = \frac{r^2}{R_h^2}; \quad R_h^2 = 6\frac{k_h^2}{w_h}; \quad \varepsilon_h^2 = 2\frac{k_h^2}{k_g^2} \tag{1.6}$$

Representing the solution in form of even power series in ε_h:

$$X(r)=\sum_{n=0}^{\infty}\varepsilon_h^{2n}X_n(r);\;\; a(r)=\sum_{n=0}^{\infty}\varepsilon_h^{2n}a_n(r);\;\; c(r)=\sum_{n=0}^{\infty}\varepsilon_h^{2n}c_n(r) \quad (2.1)$$

Equations for coefficients of leading order for variable $t=\xi^2$:

$$\frac{d}{dt}X_0+\frac{2}{3}t\frac{d^2}{dt^2}X_0=\exp\left(-X_0\right) \quad (2.2)$$

$$\frac{d}{dt}a_1+\frac{2}{3}t\frac{d^2}{dt^2}a_1=\exp\left(-X_0\right) \quad (2.3)$$

$$\frac{3}{t}\frac{d}{dt}c_1+2\frac{d^2}{dt^2}c_1=-\left(\frac{d}{dt}X_0\right)^2 \quad (2.4)$$

$$\left(a_1-t\frac{d}{dt}a_1-2t^2\frac{d^2}{dt^2}a_1\right)-\left(c_1+5t\frac{d}{dt}c_1+2t^2\frac{d^2}{dt^2}c_1\right)=0 \quad (2.5)$$

Two approximations for X_0 was found:

$$\bar{X}_0 = \ln(3t) \tag{2.6}$$

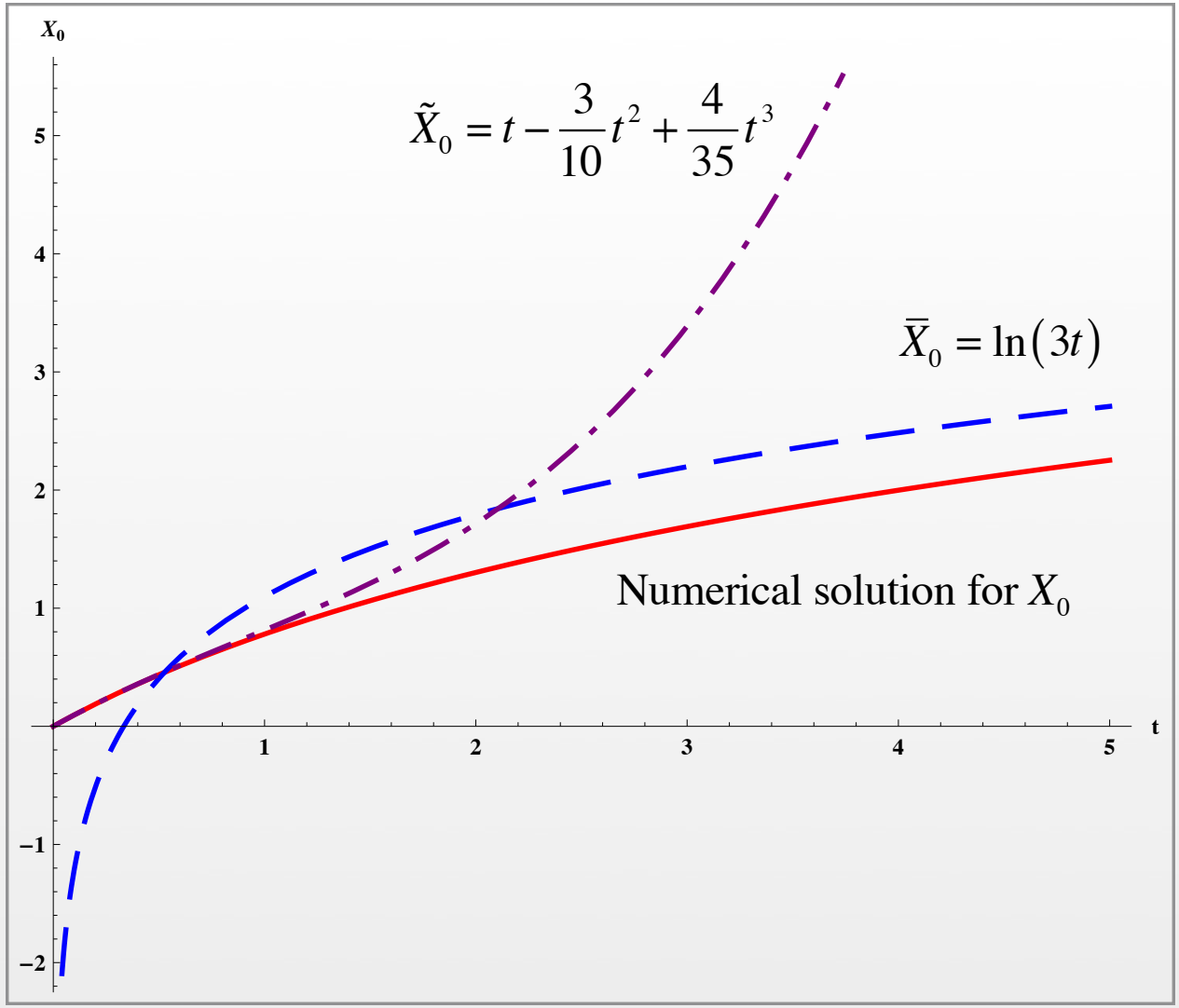
$$\tilde{X}_0 = t - \frac{3}{10}t^2 + \frac{4}{35}t^3 \tag{2.7}$$

Predicted features of regular solution for X_0:

1. There is only one regular solution exists.

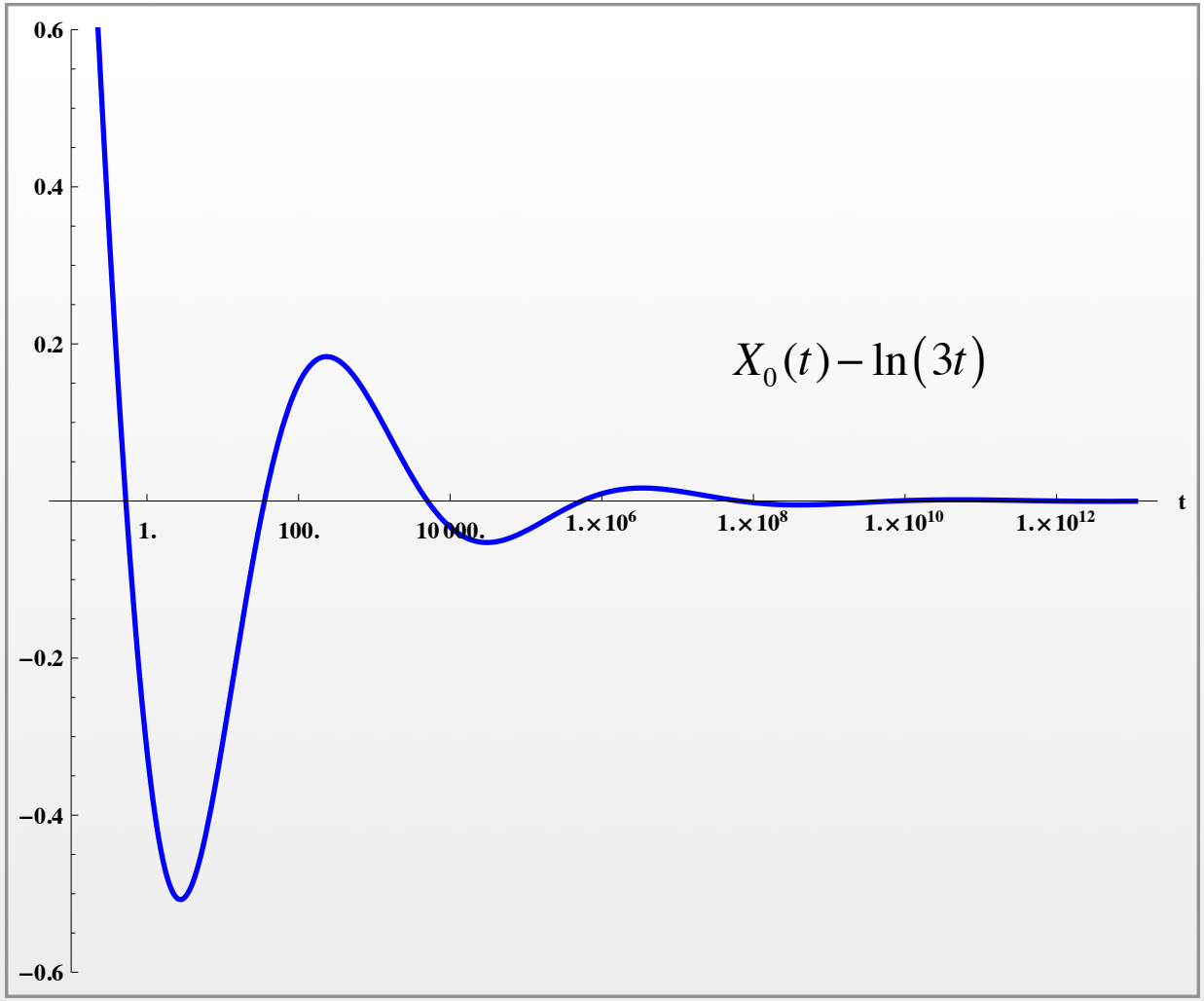
2. Solution acts like (2.7) in region $0 \le t < 1$.

3. Solution oscillates around (2.6) approaching the latter at $t \to \infty$.



(2.8)

Solution in form of power series approximates numerical one very good in interval $0 \le t < 1$.



(2.9)

Numerical solution oscillates around logarithm approaching the latter at large t.

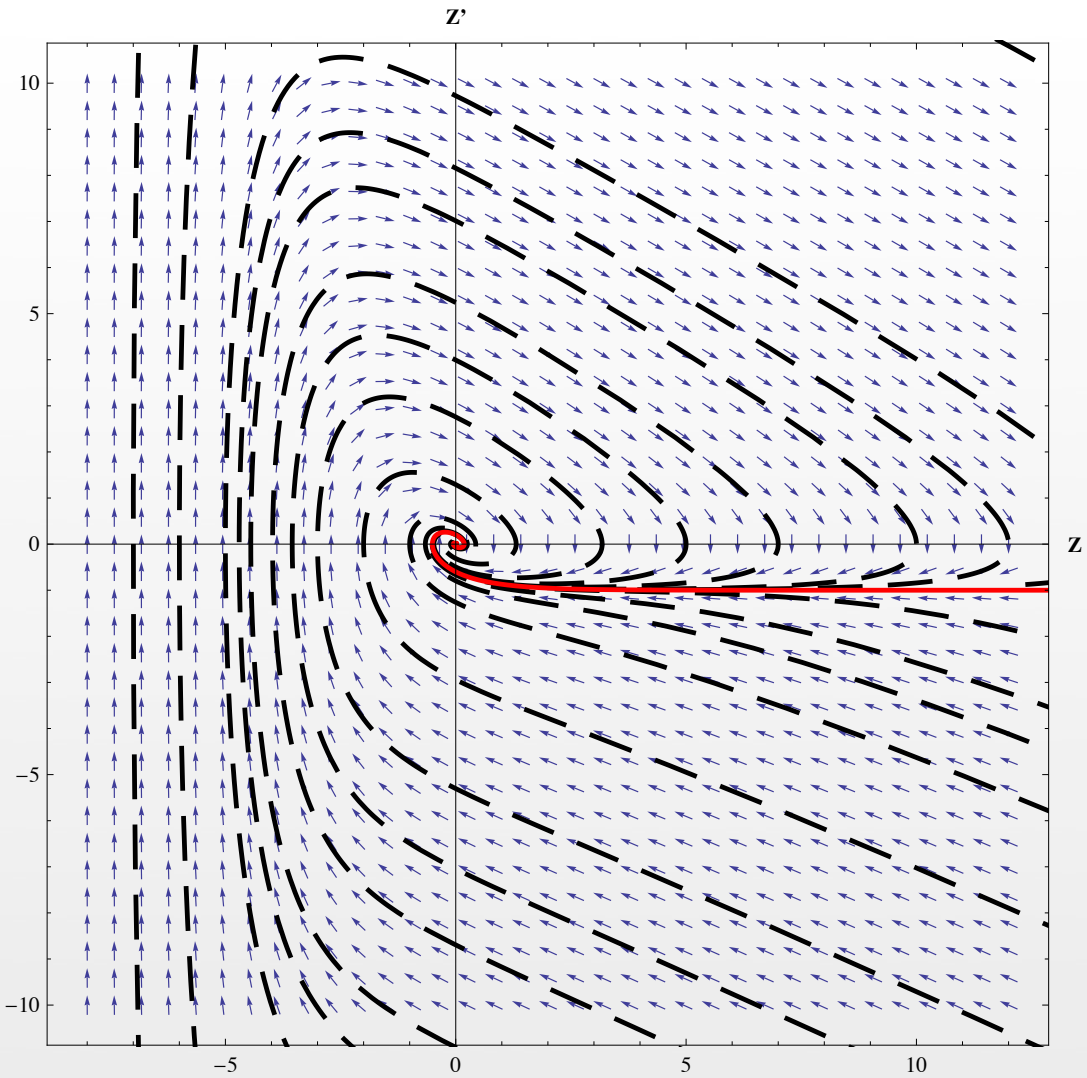
Phase plane of equation (2.2) analyzed for the function

$$Z = X_0 - \sigma; \quad \sigma = \ln(3t) \tag{2.10}$$

Equation (2.2) takes form of first order system:

$$\begin{aligned} \frac{d}{d\sigma} Z &= \dot{Z} \\ \frac{d}{d\sigma} \dot{Z} &= -\frac{1}{2}\dot{Z} + \frac{1}{2}\left(\exp(-Z) - 1\right) \end{aligned} \tag{2.11}$$

$$\begin{aligned} \sigma \to -\infty &\Leftrightarrow t \to 0 \\ \sigma \to +\infty &\Leftrightarrow t \to +\infty \end{aligned} \tag{2.12}$$



(2.13)

All trajectories are winding around exceptional one: (0;0)

The red line indicates unique trajectory finite at $t \to 0$.

Velocity of circular rotation of test particle:

$$v^2(r) = \frac{d}{dr}\ln a(r)\left(\frac{d}{dr}\left(\ln\left(r^2 c(r)\right)\right)\right)^{-1} \tag{3.1}$$

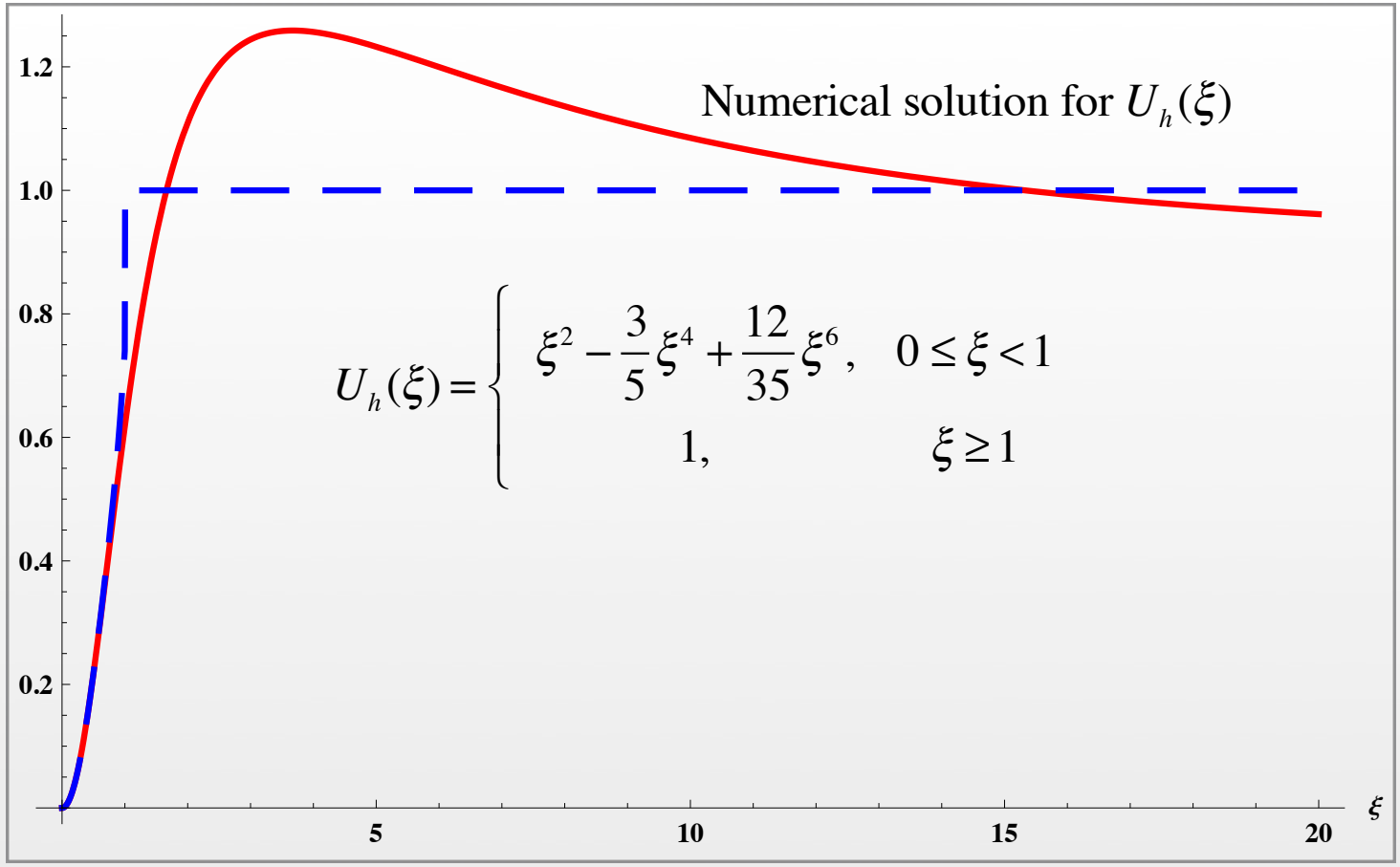
in terms of decomposition by ε_h (in variable ξ):

$$v_h^2(\xi) = \varepsilon_h^2 U_h(\xi); \quad U_h(\xi) \approx \frac{\xi}{2}\frac{d}{d\xi}X_0(\xi) \tag{3.2}$$

Asymptotical solutions (2.6), (2.7) gives profile:

$$U_h(\xi) = \begin{cases} \xi^2 - \frac{3}{5}\xi^4 + \frac{12}{35}\xi^6, & 0 \le \xi < 1 \\ 1, & \xi \gg 1 \end{cases} \tag{3.3}$$

Numerical velocity profile in comparison with asymptotical one:



$$U_h(\xi) = \begin{cases} \xi^2 - \frac{3}{5}\xi^4 + \frac{12}{35}\xi^6, & 0 \le \xi < 1 \\ 1, & \xi \ge 1 \end{cases} \qquad (3.4)$$

Qualitatively corresponds to observable data.

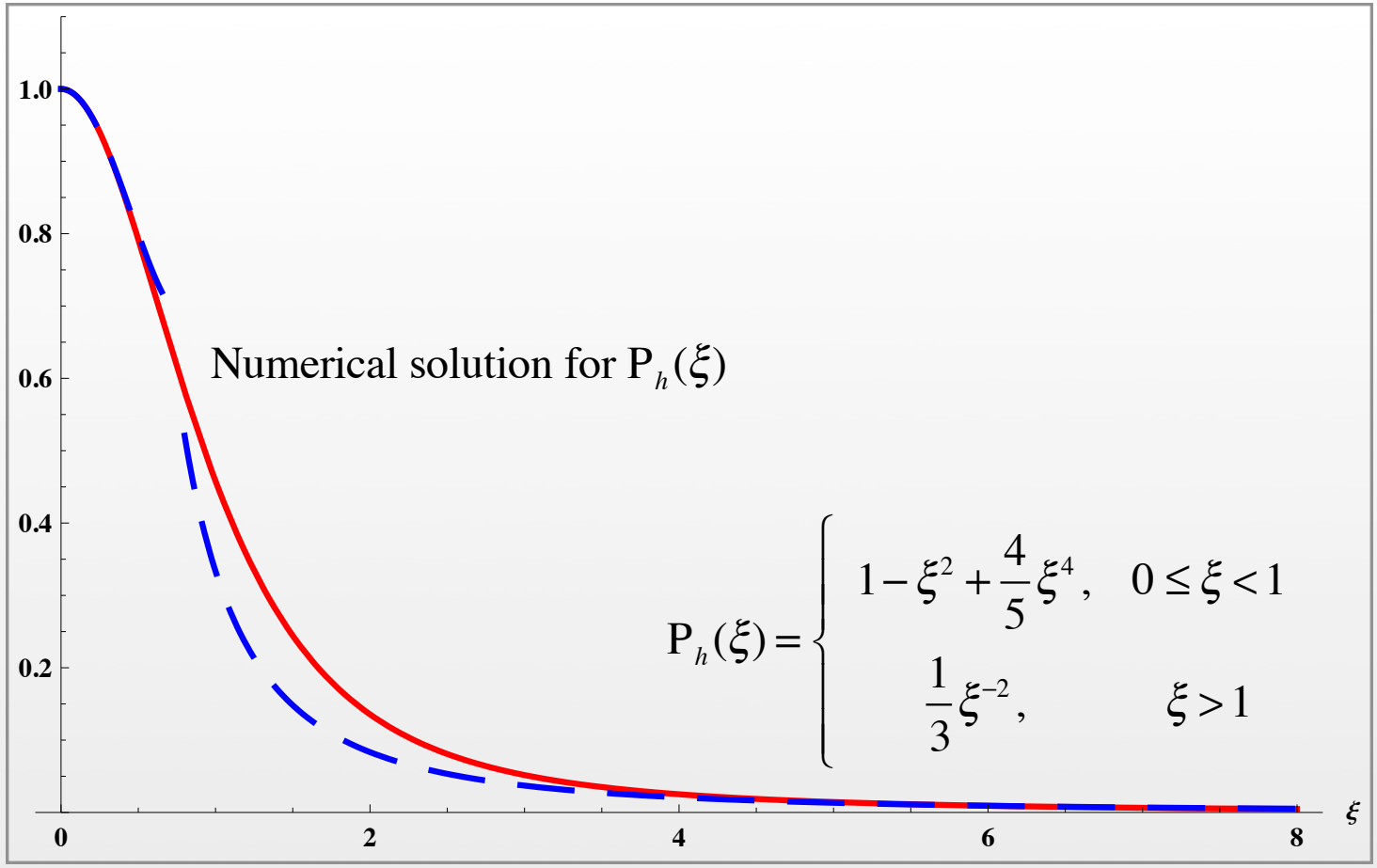
Solution results in a profile of energy density of heteron halo:

$$\rho_h(\xi) = \rho_h(0)\mathrm{P}_h(\xi); \quad \rho_h(0) = 6\frac{\varepsilon_h^2 k_g^2}{R_h^2}; \quad \mathrm{P}_h(\xi) = \exp\left(-X_0(\xi)\right) \quad (3.6)$$

Asymptotical solutions (2.6), (2.7) gives:

$$\mathrm{P}_h(\xi) = \begin{cases} 1 - \xi^2 + \frac{4}{5}\xi^4, & 0 \le \xi < 1 \\ \frac{1}{3}\xi^{-2}, & \xi \gg 1 \end{cases} \quad (3.7)$$

Numerical density profile in comparison with asymptotical one



$$P_h(\xi) = \begin{cases} 1 - \xi^2 + \frac{4}{5}\xi^4, & 0 \le \xi < 1 \\ \frac{1}{3}\xi^{-2}, & \xi > 1 \end{cases} \tag{3.8}$$

are in good correspondence.

Results of numerical analysis of static spherically symmetric case in vacuum for theory of meta-gravitation with heteron field (regular case):

1. Analytical asymptotical solution for heteron field and metric is in good correspondence with precise solution obtained numerically.

2. Test particle velocity profile, predicted by the theory, qualitatively confirmed by observable data.

Transformed full system (in variable t):

$$2t\frac{d}{dt}\left(AC\frac{d}{dt}X\right)+AC\frac{d}{dt}X=3C\exp\left(-X\right) \tag{4.1}$$

$$2t\frac{d}{dt}\left(C\frac{d}{dt}A\right)+C\frac{d}{dt}A=3\varepsilon_h^2C\exp\left(-X\right) \tag{4.2}$$

$$2C\frac{d^2}{dt^2}C+\frac{1}{t}C\frac{d}{dt}C-\left(\frac{d}{dt}C\right)^2=-\varepsilon_h^2\left(C\frac{d}{dt}X\right)^2 \tag{4.3}$$

$$2t\left(C\frac{d^2}{dt^2}A-A\frac{d^2}{dt^2}C\right)+\left(C\frac{d}{dt}A-A\frac{d}{dt}C\right)+1=0 \tag{4.4}$$

The only parameter has a physical estimation: $\varepsilon_h \sim 10^{-3}$.

Decomposition for the heteron field variable:

$$X(t) = \sum_{n=0}^{\infty} t^n \alpha_n(\varepsilon_h^2) \tag{4.5}$$

$$\alpha_0 = 0 \tag{4.6}$$

$$\alpha_1 = 1$$

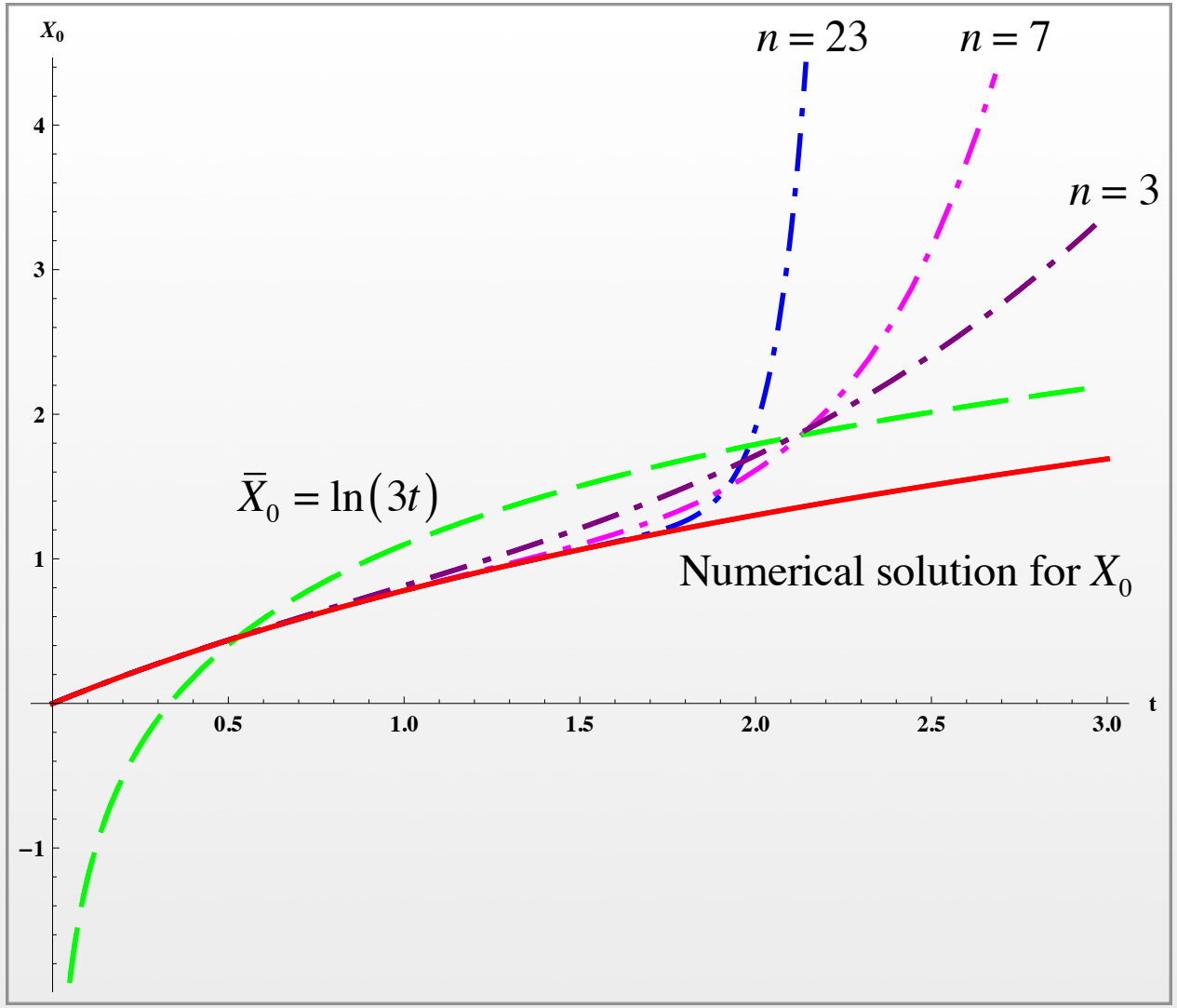
$$\alpha_2 = -\frac{1}{10}\left(3 + 5\varepsilon_h^2\right)$$

$$\alpha_3 = \frac{1}{105}\left(12 + 41\varepsilon_h^2 + 35\varepsilon_h^4\right)$$

$$\alpha_4 = -\frac{1}{6300}\left(305 + 1573\varepsilon_h^2 + 2735\varepsilon_h^4 + 1575\varepsilon_h^6\right)$$

$$\alpha_5 = \frac{1}{173250}\left(3774 + 25969\varepsilon_h^2 + 68120\varepsilon_h^4 + 79675\varepsilon_h^6 + 34650\varepsilon_h^8\right)$$

Putting $\varepsilon_h = 0$ we can compare this solution with found above:



(4.7)

EXTRA SLIDES

Initial system of differential equations:

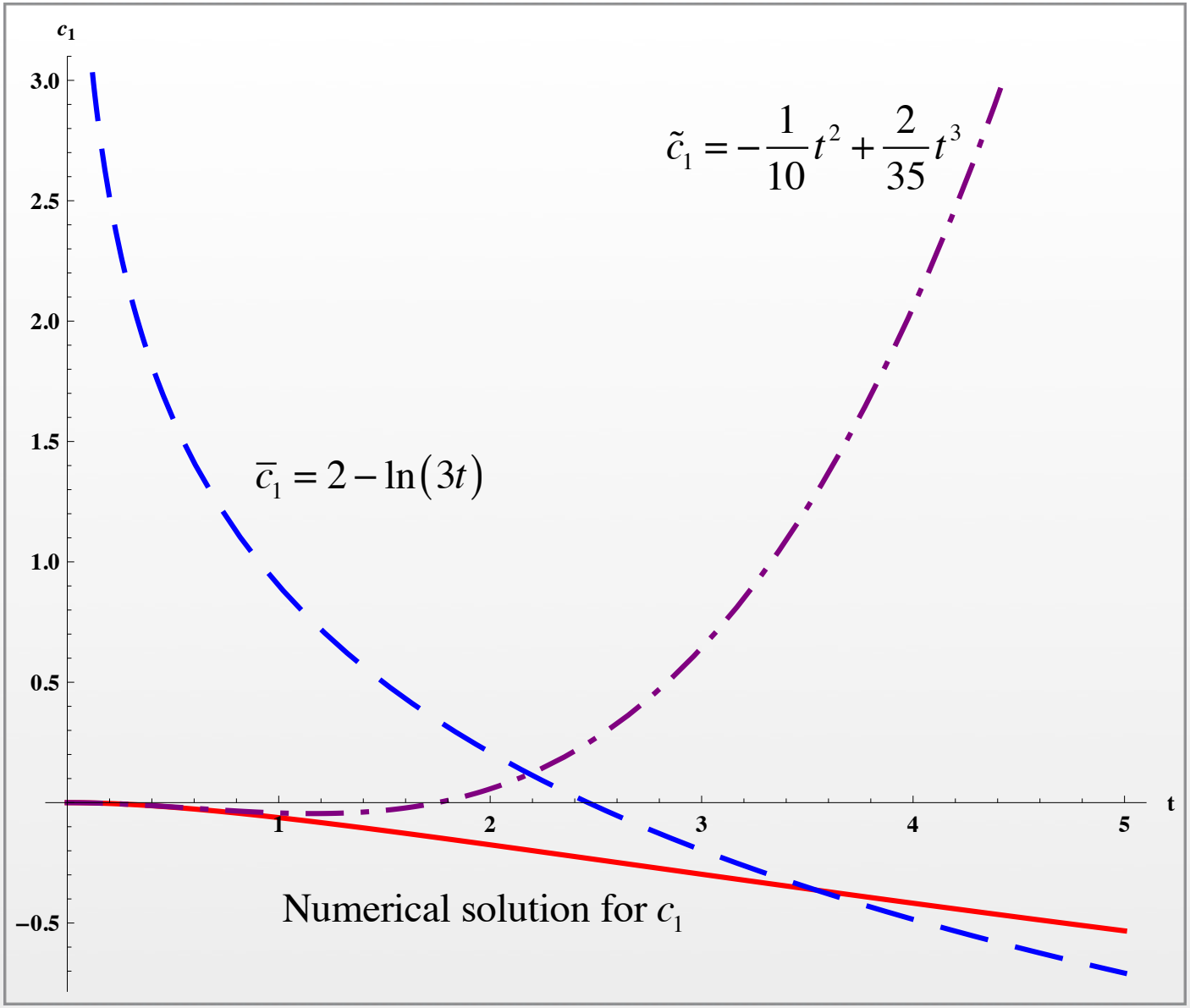
$$\frac{d}{dr}\left(AC\frac{d}{dr}X \right) = \frac{w_h}{k_h^2} C\exp(-X) \tag{1.3}$$

$$\frac{d}{dr}\left(C\frac{d}{dr}A \right) = \frac{2w_h}{k_g^2} C\exp(-X) \tag{1.4}$$

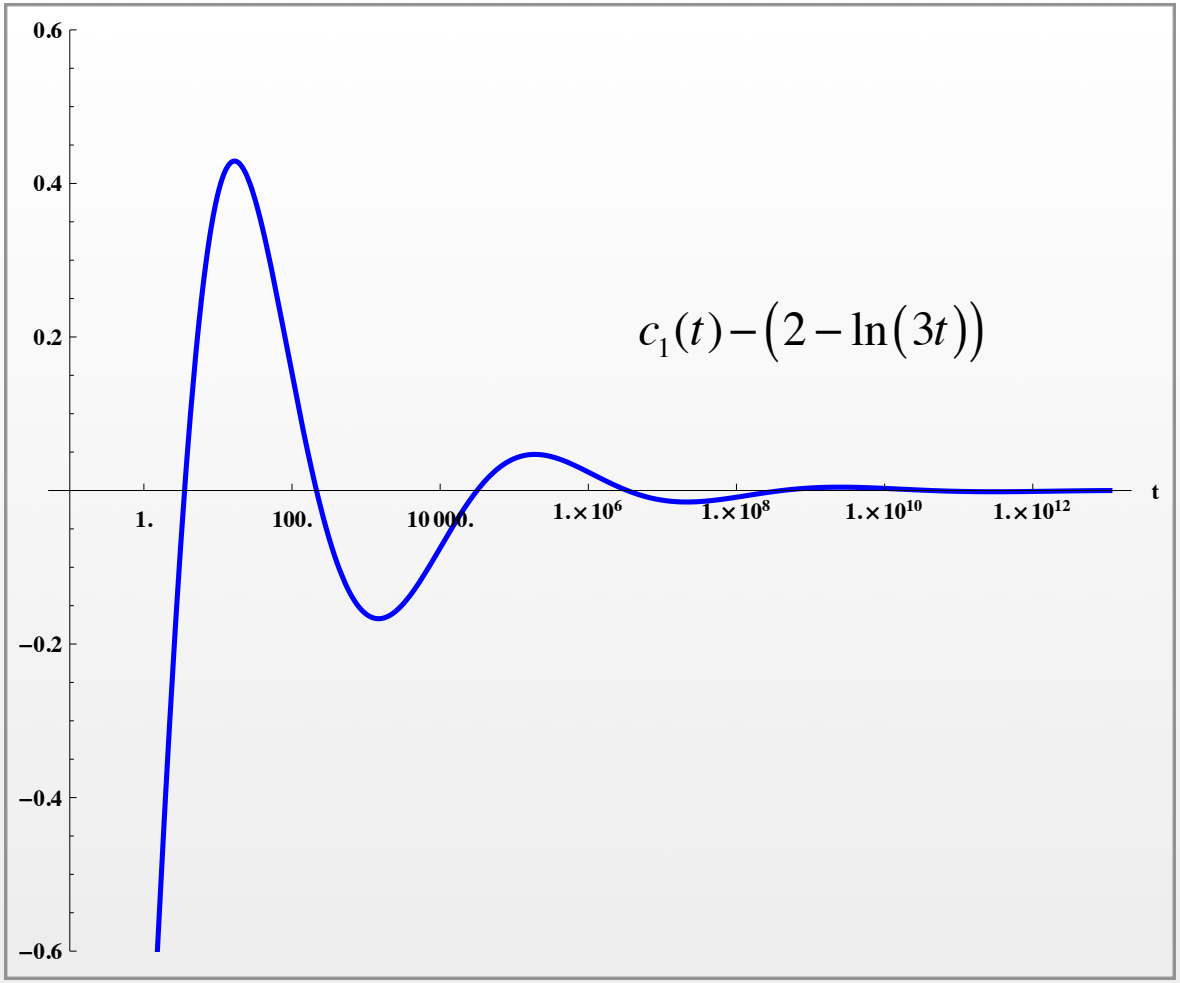
$$\frac{d}{dr}\left(C\frac{d}{dr}C \right) - \frac{3}{2}\left(\frac{d}{dr}C \right)^2 = -\frac{k_h^2}{k_g^2}\left(C\frac{d}{dr}X \right)^2 \tag{1.5}$$

$$\frac{d}{dr}\left(C\frac{d}{dr}A \right) - \frac{d}{dr}\left(A\frac{d}{dr}C \right) + 2 = 0 \tag{1.6}$$

where k_g - gravitation field scale constant, w_h - parameter of solution.


c_1
3.0
2.5
2.0
1.5
1.0
0.5
−0.5
1
2
3
4
5
t
$\tilde{c}_1 = -\frac{1}{10}t^2 + \frac{2}{35}t^3$
$\bar{c}_1 = 2 - \ln(3t)$
Numerical solution for c_1

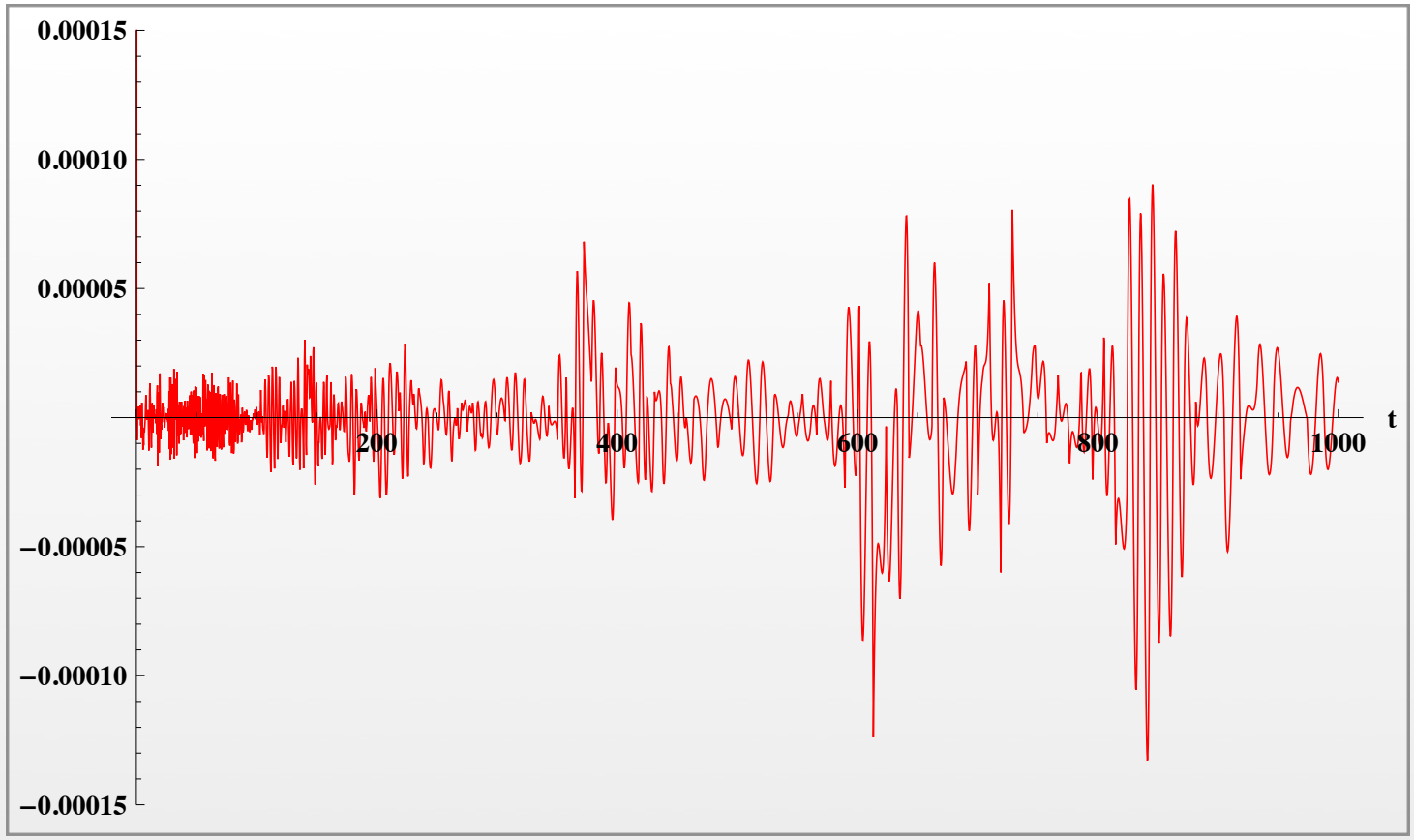
(2.16)



(2.17)

At large t solution for c_1 also oscillates around logarithm and approaches the latter.

Relative errors for equation (2.5):



(2.19)