New Renormalization Group Equations and the Naturalness Problem

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OUTLINE

Physical Consequences of Quadratic Divergencies

Wilson, 1971; Susskind, 1979; 't Hooft, 1980

- Scalar mass term does not brake any symmetry
- $M^2 = M_0^2 - \Lambda^2 P(g_0) + \dots$
- Either $P(g_0) = 0$ (technicolor, supersymmetry, etc.), or ...
- $\frac{M_0^2}{\Lambda^2} \approx P(g_0)$, which means that
- Parameters of the effective high-energy theory should be fine tuned...
- Are the dots in the second item important?

Naturalness Problem

Scalar mass is oversensitive to tiny changes in the strength of scalar self coupling measured at high energies

OUTLINE

Higher Order Corrections to Mass and Naturalness

V. Kim & G.P., 2008

- $M^2 = M_0^2 - \Lambda^2 (P_0(g_0) + P_1(g_0)\log(\frac{\Lambda^2}{M^2}) + \ldots)$
- $\frac{\partial M^2}{\partial g_0} \approx M^2 \frac{P_0'(g_0)}{P_1(g_0)}$
- New renormalization group equation should resum powers of scale logarithms in the presence of powers of the scale

Possible Solution to Naturalness Problem

Resummation of logarithmic quantum corrections to the scalar mass

OUTLINE

1. INTRODUCTION
   - The Naturalness Problem
   - The Logarithms are Important
   - Problems with Dimensional regularization
2. NEW RENORMALIZATION GROUP EQUATION FOR SCALAR PROPAGATOR
   - The Unnaturalness of Scalar Fields in Terms of Observables
   - The Form of the Equation
   - Derivation of the Equation
3. THE CASE OF ϕ^4
   - Properties of the Solution
   - In Formulas...

MS Scheme for Scalar Propagator

Collins, 1974

- Nothing "unnatural" in RG functions of scalar field withing MS scheme
- UV asymptotics of the scalar propagator:
  $\frac{1}{Q^2} \to \frac{1}{(Q^2)^{1-\gamma_\phi}\mu^{2\gamma_\phi}}$
- anomalous dimension of scalar field within ϕ^4
  $\gamma_\phi = \frac{g^2}{12(16\pi^2)^2}$

MS Scheme Renormalization Group

Naturalness problem is inexistent

WHERE ARE THE QUADRATIC DIVERGENCIES WITHIN DIMENSIONAL REGULARIZATION?

Veltman, 1981

- For a diagram with m loops, quadratic divergence is related to a pole near dimension $4-2/m$
- Vanishing of the pole near dimension 2 is "Veltman condition":
  $2M_W^2 + M_Z^2 + M_H^2 - 4M_t^2 = 0$

Al-sarhi, Jack & Jones, 1992

- Quadratic divergencies poles computed up to four loops

The quadratic divergence poles are accumulated towards the physical dimension

DIMESIONAL REGULARISATION AND MINIMAL SUBTRACTIONS

Unable to treat naturalness problem conclusively

Outline

THE RUNNING MASS

- Quadratic part of the effective action for a field localized in momentum space:
  $-\frac{R'(Q^2)}{2}(\partial\phi)^2 + \frac{R(Q^2)-R'(Q^2)Q^2}{2}\phi^2$
- $R(Q^2) = \frac{1}{D(Q^2)}$
- Running mass
  $M^2(Q^2) \equiv \frac{R(Q^2)-R'(Q^2)Q^2}{R'(Q^2)}$
- Running mass in units of normalization point
  $m^2(Q^2) \equiv \frac{M^2(Q^2)}{Q^2} = \frac{R(Q^2)}{R'(Q^2)Q^2} - 1$

BARE MASS AT CUTOFF Λ

is replaced with running mass at normalization point Λ^2

For Free Theory...

- $R(Q^2) = Q^2 + M^2$
- $M^2(Q^2) \equiv \frac{R(Q^2) - R'(Q^2)Q^2}{R'(Q^2)} = M^2$
- $m^2(Q^2) \equiv \frac{M^2(Q^2)}{Q^2} = \frac{M^2}{Q^2}$

Unnaturalness in Terms of the Propagator

- $m^2(Q^2) \approx F(g)$ at large Q^2
- We demonstrated that $m^2(Q^2) \approx \gamma_\phi$
- This means that $\frac{R(Q^2)}{R'(Q^2)Q^2} \approx 1 + \gamma_\phi$
- The range of applicability in Q^2?

Unnaturlness of scalar fields is expressed as a relation for the scalar propagator

Treatment of Naturalness Problem

requires an evolution equation for $R(Q^2)$ describing its dependence on Q^2

Outline

Renormalization Scheme

- $R'(Q^2)$ has zero dimension (logarithmically divergent)
- $R(Q^2)$ has dimension 2 (quadratically divergent)
- Let us use $r_{Q^2} \equiv \{R'(Q^2), R(Q^2), g\}$ to parameterize our theory

Any observable O is a function of r_{Q^2} independent of the normalization point Q^2

For Scalar Fields

Derivative of the propagator at a normalization point is a natural parameter of the theory

RG Equation for Scalar Propagator

- $\frac{\partial O(r_{Q^2})}{\partial Q^2} = 0 \rightarrow$
- $R''(Q^2) = F(R'(Q^2), R(Q^2), g)$

Quadratic Divergencies

imply that RG equation for scalar propagator is second order in derivatives over momentum squared

Outline

Ingredients of the Derivation

G.P., 2010

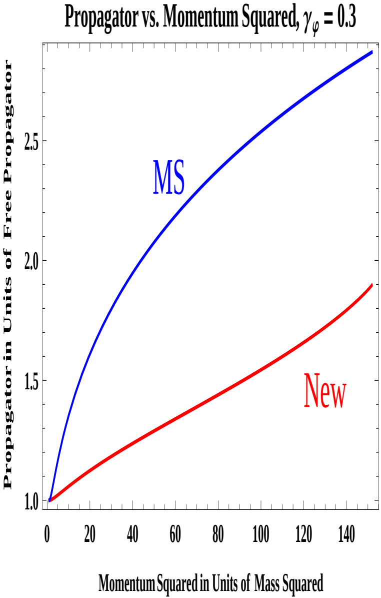
- Dominicis-Englert theorem
- Inaction equations for Green functions
- Normalized action
- Finite perturbation theory
- Evolution equation for the normalized action

The New Renormalization Group Equations

are derived for any renormalizable theory

Outline

Evolution of the Scalar Propagator

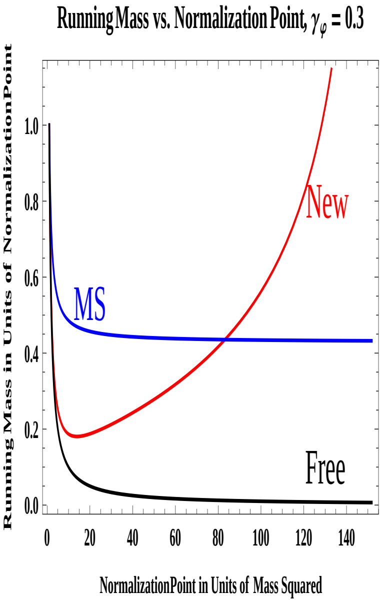


- $D_{MS} = \frac{1}{(aQ^2+bM^2)^{1-\gamma_\phi}}$
- $D_{MS} > D_{New}$
- $\frac{D_{MS}(Q^2)}{D_F(Q^2)} \sim \left(\frac{Q^2}{M^2}\right)^{\gamma_\phi}$
- $\frac{D_{New}(Q^2)}{D_F(Q^2)} \sim \left(\frac{Q^2}{M^2}\right)$
- $R'(Q^2) \to 0$

Scalar Propagator

is a nonzero constant at infinite momentum

Evolution of the Scalar Mass



- $m^2_{Free} = \frac{M^2}{Q^2}$
- $m^2_{MS} \sim \frac{\gamma_\phi}{1-\gamma_\phi}$
- m^2_{New} shoots up when R' becomes small
- m^2_{New} has a minimum
- At the minimum $m^2_{New} \approx \gamma_\phi$

Running Mass of a Scalar Field

in units of the normalization point has a minimum

Outline

SECOND ORDER EQUATION

- $R'' = -\frac{8\gamma_\phi}{(R')^3 Q^2} \int_0^\infty J_3(x)[mK_1(mx)]^3 x dx + \ldots$
- $m^2 \equiv R/(Q^2 R') - 1$
- Initial conditions
  $R(M^2) = 2M^2$, $R'(M^2) = 1$

SYSTEM OF FIRST ORDER EQUATIONS

- First order equations
  $\frac{d}{dt} m^2 = -m^2 + \frac{\gamma_\phi}{n}(1 + m^2)\Phi(m)$,
  $\frac{d}{dt} n = -4\gamma_\phi \Phi(m)$,
  where $n = (R')^4$, $t = \log(Q^2/M^2)$
- Initial conditions
  $m^2(0) = 1$, $n(0) = 1$
- $\Phi(m) \approx \frac{0.3609}{6m^2 + 0.3609}$

The Running Mass

- for $M^2/\gamma_\phi < Q^2 \ll M^2 \exp(1/(4\gamma_\phi))$
- $M^2(Q^2) \approx \frac{\gamma_\phi Q^2}{1-4\gamma_\phi \log(Q^2/M^2)}$

For high normalization points, running mass is independent of the physical mass

The Landau Pole

in the running mass invalidates perturbation theory

SUMMARY

- Unnaturalness of scalar fields does not yield a criterion for selecting consistent fundamental theories
- Unnaturalness of scalar fields is an observable effect
- New Computations for the evolution of the scalar propagator should be performed for the standard model