$B^0 - \bar{B}^0$ mixing at NLO of $1/m_b$ expansion

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Quarks-2010 - Kolomna, June 6-12

Outline

- Quark flavors in Standard Model
- Phenomenology of $B^0 - \bar{B}^0$ system
- Theory of $B^0 - \bar{B}^0$ mixing in SM:
  expansions, operators, bag parameters B_B
- Matrix elements in sum rules approach
- Summary

Quark flavors in SM

SM gauge sector

$$\mathcal{L}_{\rm gauge} = \bar{Q}'_L i\not{D} Q'_L + \bar{U}'_R i\not{D} U'_R + \bar{D}'_R i\not{D} D'_R$$

Flavor group:

$$F = SU(3)_{Q_L} \otimes SU(3)_{U_R} \otimes SU(3)_{D_R}$$

Yukawa couplings Y_D, Y_U with Higgs boson H

$$-\mathcal{L}_{\rm Yuk} = \bar{Q}'_L H Y_D D'_R + \bar{Q}'_L H Y_U U'_R + h.c.$$

Diagonalisation by a transformation from F

$$U'_L = V_{u_L} U_L\,, \quad U'_R = V_{u_R} U_R\,, \quad D'_L = V_{d_L} D_L\,, \quad D'_D = V_{d_D} D_D$$

defines masses ($\langle H\rangle = \langle \phi_0 \rangle = v \neq 0$)

$$V^\dagger_{u_L} Y_U V_{u_R} = m_U^{\rm diag}/v\,, \quad V^\dagger_{d_L} Y_D V_{d_R} = m_D^{\rm diag}/v$$

Then a mass term emerges

$$-\mathcal{L}_{\text{Yuk}} \to -\mathcal{L}_{\text{m}} = \bar{Q}_L m_D D_R + \bar{Q}_L m_U U_R + h.c.$$

while

$$\mathcal{L}_{\text{gauge}} = \text{diag} + (\bar{u}_L V_{\text{CKM}} \not{W} d_L) + h.c.$$

with the mismatch given by CKM matrix

$$V_{u_L}^{\dagger} V_{d_L} = V_{\text{CKM}}$$

that induces charged flavor transitions:

$$b \to c, \quad t \leftarrow b, \quad s \to u, \quad c \leftrightarrow d, \quad \ldots$$

CKM parameters V_{CKM}^{ij} and quark masses m_f are (Yukawa) coupling constants to be found from data.

Hierarchies:

- CKM is driven by $\lambda = V_{us} = \sin\theta_C \approx 0.22$.
  In Wolfenstein parameterization

$$V_{\rm CKM} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda(1+iA^2\lambda^4\eta) & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

- Masses - no pattern for numerical values

$$\begin{array}{l|l|l} m_u = 0.005\ \mathrm{GeV} & m_c = 1.30\ \mathrm{GeV} & m_t = 175\ \mathrm{GeV} \\ m_d = 0.010\ \mathrm{GeV} & m_s = 0.13\ \mathrm{GeV} & m_b = 4.2\ \mathrm{GeV} \end{array}$$

A variety of flavor changing processes are allowed.

Quark transitions = hadron transitions

In contrast to leptons – neutrino mixing, no “free” quarks detected in experiments.
Flavor transitions are between flavored hadrons:
$b \to s$ means $B \to K$ or $B \to X_s$.
QCD enters the game: most difficult part of the analysis of EW flavor structure of quark sector in SM.
$\Delta F = 2$ transitions: mixing of different flavor mesons

$$sd:\ K^0 - \bar{K}^0; \quad cu:\ D^0 - \bar{D}^0; \quad bd, bs:\ B^0 - \bar{B}^0$$

is a primary source of CP violation studies

$(B^0, \bar{B}^0)$ phenomenology

Time evolution

$$i\frac{d}{dt}\begin{pmatrix} B^0(t) \\ \bar{B}^0(t) \end{pmatrix} = \left(M - \frac{i}{2}\Gamma\right)\begin{pmatrix} B^0(t) \\ \bar{B}^0(t) \end{pmatrix}$$

M - mass matrix, Γ - decay matrix

Observables (or physics):

- $\Delta m = M_{heavy} - M_{light} = 2\,|M_{12}|$
- CP phase: $\phi = \arg(-M_{12}/\Gamma_{12})$
- $\Delta\Gamma = \Gamma_L - \Gamma_H = 2\,|\Gamma_{12}|\cos\phi$

Experiment

B_d-meson

$$\Delta m_d = 0.508 \pm 0.004 \text{ ps}^{-1}$$

$$(\Delta\Gamma/\Gamma)_d = (9 \pm 37) \cdot 10^{-3}$$

DØ and CDF results for B_s

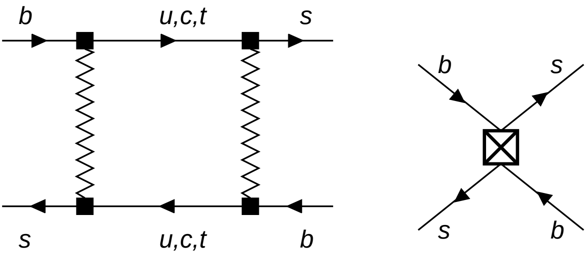
$$17 \text{ ps}^{-1} \leq \Delta m_s \leq 21 \text{ ps}^{-1} \quad 90\%\text{CL} \quad \text{DØ}$$

$$\Delta m_s = 17.77 \pm 0.10_{\text{syst}} \pm 0.07_{\text{stat}} \text{ ps}^{-1} \quad \text{CDF}$$

Theory prediction: $(\Delta\Gamma/\Gamma)_s = 0.158 \pm 0.050$ (large).

This data is used to extract CKM parameters.
What is theory in SM?

Theory in SM



Box diagram for $\Delta B = 2$ processes gives non-local transition operator (eff Hamiltonian).
Simplifications due to mass and CKM hierarchies: shrinks to a point reducing eff Hamiltonian to local operators.
Mechanisms are different for Δm and $\Delta\Gamma$.

Δm

$m_W, m_t \gg m_b, m_c$ are integrated out, loop localizes with NLO QCD result

$$M_{12} = \frac{G_F^2 M_W^2}{4\pi^2} (V_{tb}{}^* V_{td})^2 \eta_B S_0(x_t) \times \left[\alpha_s^{(5)}(\mu)\right]^{-6/23} \left[1 + \frac{\alpha_s^{(5)}(\mu)}{4\pi} J_5\right] \langle \bar{B}^0 | Q(\mu) | B^0 \rangle$$

$\eta_B = 0.55 \pm 0.1$, $J_5 = 1.627$ in the NDR scheme,
$S_0(x_t)$ is the short distance function, $x_t = m_t^2/m_W^2$
$Q(\mu) = (\bar{b}_L \gamma_\sigma s_L)(\bar{b}_L \gamma_\sigma s_L)(\mu)$ – local operator

Width difference

$$\Delta\Gamma \sim \Gamma_{12} = \langle \bar{B}_s | \mathcal{T} | B_s \rangle / 2M_{B_s}$$

Final states are (c, u) "quarks", $m_b \gg m_c, m_u$
Heavy Quark Expansion in $1/m_b$ is used

$$\langle \bar{B}_s | \mathcal{T} | B_s \rangle = \sum_n \frac{C_n}{m_b^n} \langle \bar{B}_s | \mathcal{O}_n^{\Delta B=2} | B_s \rangle$$

C_n are calculable in PT. nonPT physics is contained in ME of local operators $\mathcal{O}_n^{\Delta B=2}$.
At LO in $1/m_b$ there are two operators

$$Q = (\bar{b}_i s_i)_{V-A} (\bar{b}_j s_j)_{V-A}, \quad Q_S = (\bar{b}_i s_i)_{S-P} (\bar{b}_j s_j)_{S-P}$$

At NLO in $1/m_b$ there are more. Two important ones

$$R_2 = \frac{1}{m_b^2}(\bar{b}_i \overleftarrow{D}_\mu D^\mu s_i)_{V-A}(\bar{b}_i s_i)_{V-A}$$
$$R_3 = \frac{1}{m_b^2}(\bar{b}_i \overleftarrow{D}_\mu D^\mu s_i)_{S-P}(\bar{b}_i s_i)_{S-P}$$

Thus M_{12} and Γ_{12} reduce to evaluation of $\langle \bar{B}|Q_i|B\rangle$ in QCD that is a genuine nonPT task.

No direct techniques at present (lattice?).

Factorization

Since $Q_i \sim J \cdot J$ with $J \sim \bar{s}b$ and $\langle \bar{B}|'' ='' s\bar{b}$
it is prompting “to factorize”

$$\langle \bar{B}|Q_i|B\rangle = \langle \bar{B}|J \cdot J|B\rangle = C_{\rm comb}\langle \bar{B}|J|0\rangle\langle 0|J|B\rangle$$

For $J \sim \bar{b}_L\gamma_\mu d_L$, $\quad \langle 0|\bar{b}_L\gamma_\mu d_L|B^0(\mathbf{p})\rangle = ip_\mu f_B/2.$

Main problem: accuracy of such factorization

In general one parameterises

$$\langle \bar{B}_s|\mathcal{O}_i|B_s\rangle = B_i\langle \bar{B}_s|\mathcal{O}_i|B_s\rangle^{fac}$$

with B_i – genuine dynamical QCD quantities with normalization $B_i = 1$ in factorization approxmation

For relevant operators

$$
\begin{aligned}
\langle \bar{B}|Q|B\rangle &= f_B^2 M_B^2 2\left(1+\frac{1}{N_c}\right) B \\
\langle \bar{B}|Q_S|B\rangle &= -f_B^2 M_B^2 \frac{M_B^2}{(m_b+m_s)^2}\left(2-\frac{1}{N_c}\right) B_S \\
\langle \bar{B}|R_2|B\rangle &= -f_B^2 M_B^2 \left(\frac{M_B^2}{m_b^2}-1\right)\left(1-\frac{1}{N_c}\right) B_2 \\
\langle \bar{B}|R_3|B\rangle &= f_B^2 M_B^2 \left(\frac{M_B^2}{m_b^2}-1\right)\left(1+\frac{1}{2N_c}\right) B_3,
\end{aligned}
$$

Main theoretical uncertainties of the analysis are related to the ME of the local operators $\mathcal{O}_i \in \{Q, Q_S, R_2, R_3\}$, or equivalently, the bag parameters B_i.

OPE and QCD sum rules

- model-independent, first-principles method, close in spirit to lattice computations. QCD sum rules rely on asymptotic expansions of Green's functions (analytically in a small parameter) while on the lattice the entire function can be found (numerically)
- OPE techniques provide a consistent way of treating perturbative corrections to matrix elements which is needed to retain RG invariance of physical observables usually violated in other approximations (factorization)

OPE

The starting point is the three-point correlator

$$T(p_1,p_2) = i^2 \int d^4x d^4y e^{ip_1x - ip_2y} \langle Tj(x)\mathcal{O}(0)j(y)\rangle$$

$\mathcal{O} \in \{Q, Q_S, R_2, R_3\}$ is a generic four-quark operator and j can be either AV or PS current

$$j_5^\mu = \bar{s}\gamma^\mu\gamma_5 b \quad (\mathrm{AV}), \qquad j_5 = \bar{s}i\gamma_5 b \quad (\mathrm{PS})$$

The overlap

$$\langle 0|\bar{s}\gamma_\mu\gamma_5 b(0)|\bar{B}(p)\rangle = if_B p_\mu, \langle 0|\bar{s}i\gamma_5 b(0)|\bar{B}(p)\rangle = \frac{f_B M_B^2}{m_b + m_s}$$

For AV the correlator is a tensor, and one takes $p_1^\mu p_2^\nu$:

$$T^{\mu\nu}(p_1,p_2) = p_1^\mu p_2^\nu T(p_1,p_2) + \dots$$

Spectral density $\rho(s_1, s_2, q^2)$

$$T(p_1, p_2) = \int ds_1 ds_2 \frac{\rho(s_1, s_2, q^2)}{(s_1 - p_1^2)(s_2 - p_2^2)}$$

contains physics.

- Hadronic picture: B-meson pole plus continuum

$$\rho_{\rm AV}^{\rm had}(s_1, s_2) = f_B^2 \delta(s_1 - M_B^2)\delta(s_2 - M_B^2)\langle \bar{B}|\mathcal{O}|B\rangle + \rho_{\rm AV}^{\rm cont}$$

- Quark-gluon picture (QCD): OPE for $T(p_1, p_2)$ with a nonPT effects through condensates.

Quark-hadron duality

QCD sum rules = duality

$$\int ds_1 ds_2\, \rho_i^{\rm had}(s_1,s_2) = \int ds_1 ds_2\, \rho_i^{\rm OPE}(s_1,s_2).$$

Two practical techniques:

1. Finite Energy sum rules:

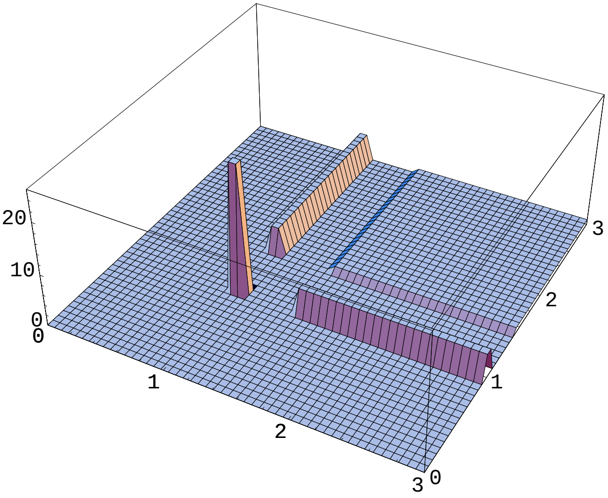
Δ being a square $m_b^2 < s_i < s_0$ in (s_1, s_2) plane

$$f_B^2\langle \bar{B}|\mathcal{O}|B\rangle = \int_\Delta ds_1 ds_2\, \rho_{\rm AV}^{\rm OPE}(s_1,s_2)$$

2. Borel sum rule: the OPE prediction model for the hadronic continuum and Borel transform

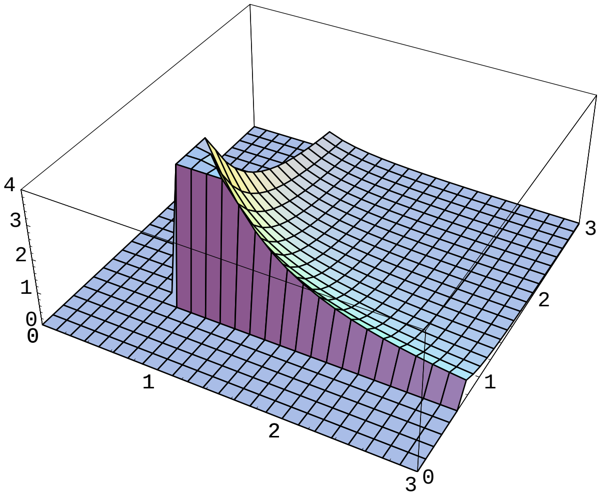
$$f_B^2\langle \bar{B}|\mathcal{O}|B\rangle e^{-\frac{M_B^2}{M_1^2}-\frac{M_B^2}{M_2^2}} = \int_\Delta ds_1 ds_2\, e^{-\frac{s_1}{M_1^2}-\frac{s_2}{M_2^2}}\, \rho_{\rm AV}^{\rm OPE}(s_1,s_2)$$

Illustration: a model for physical spectrum



One-resonance hadronic spectrum

Spectrum of the OPE in QCD



Spectrum in OPE

Factorization in the OPE

OPE diagrams show that one can split three-point correlator into two pieces

$$T(p_1,p_2) = T_{\rm fac}(p_1,p_2) + \Delta T(p_1,p_2)$$

The factorized part has an explicit form

$$T_{fac}(p_1,p_2) = {\rm const} \times \Pi(p_1)\Pi(p_2)$$

"const" and $\Pi(p_i)$ specific to the operator involved.
For the operators of V-A structure

$$T^{\rm AV}_{fac}(p_1,p_2) = 2\left(1+\frac{1}{N_c}\right)\Pi^V(p_1)\Pi^V(p_2)$$

with

$$p^\alpha \Pi^V(p) = i\int dx e^{ipx}\langle Tj(x)\bar{b}\gamma^\alpha(1-\gamma_5)s(0)\rangle.$$

Sum rule for the factorized $T_{\rm fac}$ yields $B = 1$.

Deviation from factorization in the OPE

Then one finds a sum rule for $\Delta B = B - 1$ directly

$$f_B^2 \Delta B e^{-\frac{M_B^2}{M_1^2} - \frac{M_B^2}{M_2^2}} = \int ds_1 ds_2 \Delta\rho_{\mathrm{AV}}^{\mathrm{OPE}}(s_1, s_2) e^{-\frac{s_1}{M_1^2} - \frac{s_2}{M_2^2}}$$

(given for AV current)

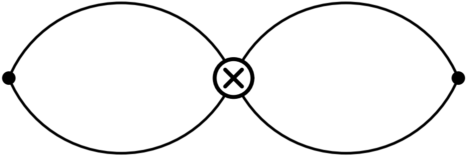


Figure: PT diagram at LO

At LO in pQCD the three-point function factorizes

$$T(p_1,p_2) = T_{\rm fac}(p_1,p_2), \quad \Delta T(p_1,p_2) = 0$$

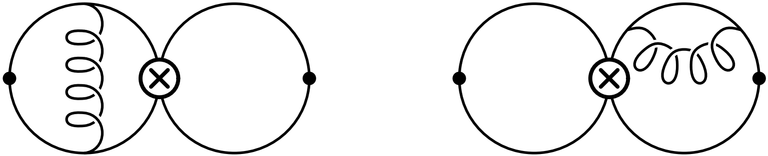
and

$$T^{\rm LO}(p_1,p_2) = T^{\rm LO}_{fac}(p_1,p_2) = {\rm const} \times \Pi^{\rm LO}(p_1)\Pi^{\rm LO}(p_2)$$

Then we have $B = 1$. But this is only LO analysis. Higher order diagrams build up the full function $\Pi^{\rm LO}(p_1) \to \Pi(p_1)$

$$T^{\rm LO}_{fac}(p_1,p_2) \to T_{fac}(p_1,p_2) = {\rm const} \times \Pi(p_1)\Pi(p_2)$$

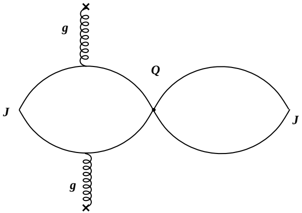
Indeed, NLO pQCD gives



NLO factorizable contributions are given by the product of two-point correlation functions

$$\Pi^f_{\mathrm{NLO}} = \frac{8}{3}(p_1.p_2)\{\Pi_{\mathrm{LO}}(p_1^2)\Pi_{\mathrm{NLO}}(p_2^2) + \mathrm{symm}(p_1, p_2)\}$$

Condensate factorizable contributions



factorizable nonPT GG diagram

• Factorizable diagrams form an important subset of all contributions, they are gauge and RG invariant.

•• Classification of diagrams in terms of their factorizability is consistent and gives a powerful technique in the quantitative analysis.

Non-factorizable contributions. pQCD diagram:

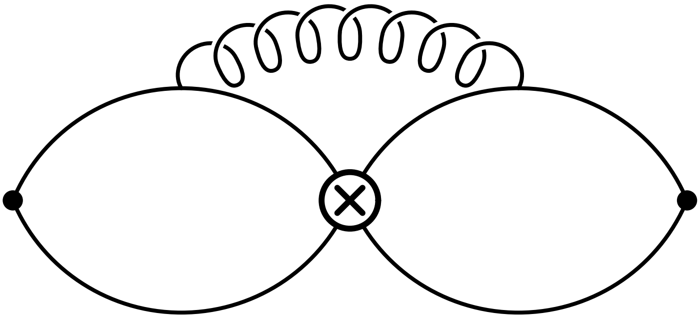
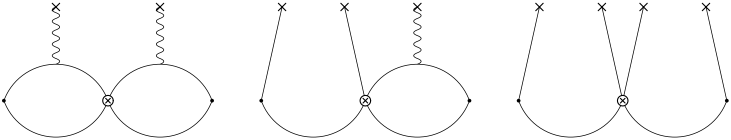


Figure: A non-factorizable diagram at NLO

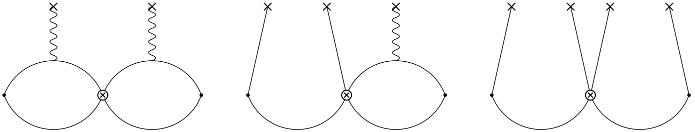
The NLO analysis of non-factorizable contributions within perturbation theory amounts to the calculation of a set of three-loop diagrams.

Non factorizable condensate contributions:



(a) (b) (c)

Results for the operators in NLO of $1/m_b$ are obtained.



(a) (b) (c)

OPE result for the spectral density is

$$\Delta\rho_i(s_1,s_2) = \Delta\rho_i^{\mathrm{GG}}(s_1,s_2)\langle GG\rangle + \Delta\rho_i^{\mathrm{sGs}}(s_1,s_2)\langle \bar{s}Gs\rangle + \ldots$$

for each of the eight cases:
AV or PS current for Q, Q_S, R_2, R_3 operators.

Example (short): expression for Q_S with PS current

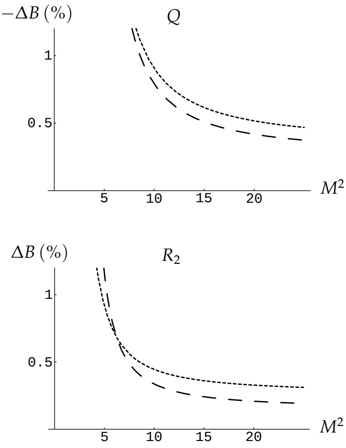
$$\Delta\rho_{\mathrm{PS}}(s_1,s_2) = \frac{1}{48\pi^2}\langle\frac{\alpha_s}{\pi}GG\rangle\frac{1}{s_1 s_2}(\frac{s_1 s_2}{2}(6-3z_1-3z_2+z_1 z_2)$$

$$+(p_1 p_2)^2 z_1 z_2)$$

$$+\frac{m_b}{16\pi^2}\langle\bar{s}Gs\rangle\left((-2+z_1)\delta(s_2-m_b^2)+(-2+z_2)\delta(s_1-m_b^2)\right)$$

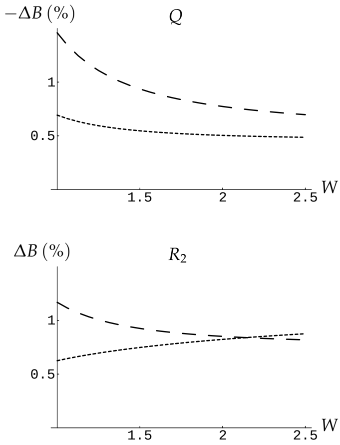
Here $z_i = m_b^2/s_i$.

Borel method (T.Mannel,B.Pecjak, AAP (2007))



$|\Delta B| = 0.5\%$ in all cases

Borel sum rules results in HQET approximation



Quite consistent. $|\Delta B| = 0.5 - 1\%$ in all cases.
Formal procedure:

$$M_B = m_b + \bar{\Lambda}, \; s = (m_b + E)^2, \quad s_0 = (m_b + E_0)^2$$

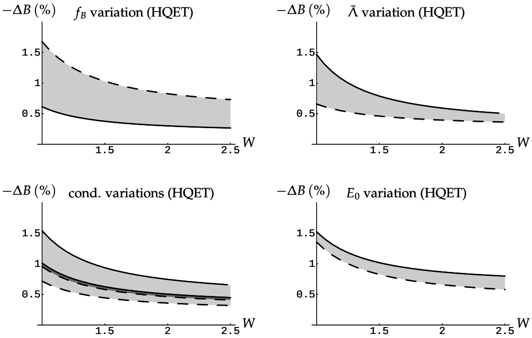
and expand QCD sum rules in $1/m_b$.

Numerical results and uncertainties

Operator	$\Delta B(\%)$ QCD	$\Delta B(\%)$ HQET
Q	-0.6 ± 0.5	-0.6 ± 0.5
Q_S	-0.5 ± 0.4	-0.6 ± 0.4
R_2	0.3 ± 0.3	0.8 ± 0.7
R_3	0.3 ± 0.2	0.3 ± 0.2

Parameters: $210\ \mathrm{MeV} < f_{B_s} < 270\ \mathrm{MeV}$
QCD: $m_b = 4.2 \pm 0.2\ \mathrm{GeV}$, $32 < s_0 < 40\ \mathrm{GeV}^2$
HQET: $m_b = 4.8\ \mathrm{GeV}$, $1\ \mathrm{GeV} < E_0 < 1.5\ \mathrm{GeV}$
Condensates are varied by $\pm 30\%$.
The largest errors are associated with the value of the decay constant f_B. The dependence on $\bar{\Lambda}$ and E_0 is moderate.

Uncertainty due to each parameter variation:
an example – the Q_S operator in HQET.
Uncertainty due to the condensates is comparable with that due to f_B.



$-\Delta B$ for Q_S operator, AV current, HQET sum rule.
Dark-gray band – gluon condensate,
larger light-gray band – quark-gluon condensate.

Summary

- SR is a powerful tool for analysing ME of local operators relevant to flavor physics.
- Factorization results are reproduced at diagram level (not only LO)
- Non-factorizable contributions due to nonPT condensates to bag parameters are small

$$\Delta B_i = (0.5 - 1)\%$$

  for all operators $\{Q, Q_S, R_2, R_3\}$
- The computation of the width difference for $B^0 - \bar{B}^0$ is under solid theoretical control