International Seminar “Quarks-2010”

Heteron dark matter

Yu. F. Pirogov

Institute for High Energy Physics, Protvino, Russia

1 Hetero-gravity (HG)

• Motivation

DM

$\not{G}$C

UC

Massless tensor graviton $\oplus$ massive scalar graviton.

(Scalar) *heteron* < the Greek $'\varepsilon\xi\varepsilon\rho o\varsigma$ – (an)other, different.

Hetero-gravity (HG).

• Heteron field

Bi-covariance.

κ_h – a mass scale of the UC violation.

Heteron field:

$$\chi = \frac{\kappa_h}{2} \ln \frac{g}{g_0}.$$

$g_0(x)$ – a non-dynamical scalar density of the same weight as g. $\Rightarrow \chi$ is scalar.

$g_0(x)$ – the *kernel*.

- **HG Lagrangian**

$$
\begin{aligned}
L &= L_g + L_h + L_m + L_{gh} + L_{mh}, \\
L_g &= -\frac{\kappa_g^2}{2} R, \\
L_h &= \frac{1}{2}\partial\chi \cdot \partial\chi - V_h(\chi) + \mathcal{O}(\frac{1}{\kappa_h^4}(\partial\chi \cdot \partial\chi)^2)
\end{aligned}
$$

$\kappa_g = 1/(8\pi G_N)^{1/2}$ – the GC preserving mass scale of GR.

V_h – a scalar-field potential: $V_h|_{min} = \Lambda$ – cosmological constant.

L_m – some ordinary matter Lagrangian.

Minimal HG: $L_{gh} = L_{mh} = 0.$

- **HG equations**

$$
R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{1}{\kappa_g^2} T_{\mu\nu}, \quad T_{\mu\nu} = T_{m\mu\nu} + T_{\chi\mu\nu}.
$$

Heteron energy-momentum tensor:

$$
T_{\chi\mu\nu} = \partial_\mu\chi\partial_\nu\chi - \Big(\frac{1}{2}\partial\chi \cdot \partial\chi - \check{V}_h\Big) g_{\mu\nu},
$$

Hetero-potential: $\check{V}_h = V_h + \kappa_h W_h,$

Wave operator: $W_h(\chi) = \nabla \cdot \nabla\chi + \partial V_h/\partial\chi.$

Harmonic field ($W_h(\chi) = 0$) $\Rightarrow$ GR with a scalar field ($\check{V}_h = V_h$) .

Non-harmonic field ($W_h(\chi) \neq 0$) $\Rightarrow$ effects beyond GR.

• Heteron field equation

Contracted Bianchi identity: $\nabla_\mu T^{\mu\nu} = 0.$

The third-order differential equation for χ:

$$\kappa_h \partial_\mu W_h + W_h \partial_\mu \chi = -\nabla_\nu T_{m\,\mu}^{\ \ \nu},$$

Let $\nabla_\mu T_m^{\mu\nu} = 0$ (the empty space, $T_{m\mu\nu} = 0$).

The first integral: $W_h = -(w_0/\kappa_h) e^{-\chi/\kappa_h}.$

w_0 – an integration constant (the non-harmonicity parameter).

The second-order field equation:

$$\nabla \cdot \nabla \chi + \partial \check{V}_h / \partial \chi = 0.$$

Hetero-potential: $\check{V}_h = V_h - w_0 e^{-\chi/\kappa_h}.$

2 Spherical symmetry

• Polar coordinates

Invariant line element: $ds^2 = adt^2 - bdr^2 - cr^2(d\theta^2 + \sin^2\theta d\varphi^2)$.

Three independent HG equations ($R_\varphi^\varphi = R_\theta^\theta$):

$$
\begin{aligned}
R_0^0 &= \frac{1}{2\kappa_g^2}\Big(T_{m\,0}^{\;\,0} - T_{m\,A}^{\;\,A} - 2\check{V}_h\Big), \\
R_0^0 - R_r^r &= \frac{1}{\kappa_g^2}\Big(T_{m\,0}^{\;\,0} - T_{m\,r}^{\;\,r} + \frac{1}{b}\chi'^2\Big), \\
R_0^0 - R_\theta^\theta &= \frac{1}{\kappa_g^2}\Big(T_{m\,0}^{\;\,0} - T_{m\,\theta}^{\;\,\theta}\Big),
\end{aligned}
$$

Four variables: a, b, c and χ.

• Gauge fixing

Radial rescaling: $r \to \hat{r}(r) \Rightarrow ds^2$ is form-invariant.

$\Rightarrow$ Variables a, b, c are not unambiguous. $\Rightarrow$ gauge fixing

Direct problem: an external gauge. Canonical gauge ($g_0 = -1$).

$\Rightarrow \chi = \chi(a, b, c)$.

Inverse problem: internal gauges ($F(a, b, c) = 0$).

$b = c$ – isotropic gauge

$c = 1$ – astronomic gauge ($S_2 = 4\pi r^2$)

$\Rightarrow \chi$ – independent variable.

Advantages:

(i) Comparison with GR in presence of a scalar field.

(ii) χ absorbs the unknown kernel

$\Rightarrow g_0 = g_0(a, b, c, \chi)|_{F=0}$.

• Reciprocal gauge ($ab = 1$)

Variables: $1/b = a = A$, $r^2 c = C$ and $\chi/\kappa_h = X$.

Hetero-gravity equations ($V_h = 0$):

$$
\begin{aligned}
CA'' + C'A' &= \varepsilon_h^2 (ACX')', \\
CC'' - \frac{1}{2}C'^2 &= -\frac{\varepsilon_h^2}{2}(CX')^2, \\
CA'' - AC'' + 2 &= 0.
\end{aligned}
$$

Dimensionless parameter: $\varepsilon_h^2 \equiv 2\kappa_h^2/\kappa_g^2$.

Heteron field equation:

$$
(ACX')' = \frac{6}{R_0^2} C e^{-X}.
$$

New distance scale: $R_0^2 = 6\kappa_h^2/w_0 = 3\varepsilon_h^2\kappa_g^2/w_0$.

3 HG solutions

• Harmonic solution $(W_h = 0)$

$w_0 = 0$ $(R_0 \to \infty) \Leftrightarrow \nabla \cdot \nabla \chi = 0.$

Reciprocal gauge $(b = 1/a)$

$(ACX')' = 0$. Exact solution:

$$\begin{aligned} \phi_p \equiv \varepsilon_h X_p &= \pm\sqrt{1-\sigma_p^2}\ln(1 - r_p/r), \\ a_p &= (1 - r_p/r)^{\sigma_p}, \quad c_p = (1 - r_p/r)^{1-\sigma_p}. \end{aligned}$$

Two-parameter family $\supset$ one-parameter family of BHs $(\sigma_p = 1,\ X_p = 0)$

Astronomic objects – the *dark pits*.

Canonical parameters: r_p - pit radius, σ_p – steepness.

Pit kernel:

$$\sqrt{-g_0} = (1 - r_p/r)^{\beta_p}, \quad \beta_p = 1 - \sigma_p \mp \sqrt{1-\sigma_p^2}.$$

Isotropic gauge $(\hat{b} = \hat{c})$

To compare with observations:

$$\begin{aligned} \hat{\phi}_p &= -\frac{r_0}{\hat{r}} + \mathcal{O}\Big(\frac{1}{\hat{r}^3}\Big), \\ \hat{a}_p &= 1 - \frac{r_g}{\hat{r}} + \frac{1}{2}\frac{r_g^2}{\hat{r}^2} + \mathcal{O}\Big(\frac{1}{\hat{r}^3}\Big), \\ \hat{c}_p &= 1 + \frac{r_g}{\hat{r}} + \frac{3}{8}\frac{r_g^2 - r_0^2/3}{\hat{r}^2} + \mathcal{O}\Big(\frac{1}{\hat{r}^3}\Big). \end{aligned}$$

Phenomenological parameters

$$r_g = \sigma_p r_p, \quad r_0 = \eta_p\sqrt{1-\sigma_p^2}\,r_p,$$

Dark pits reproduce BHs up to the first post-Newtonian approximation.

• Non-harmonic solutions ($W_h \neq 0$)

Exceptional solution

$w_0 \neq 0$ (R_0 – finite): $\xi = r/R_0$, $a = e^{\alpha}$, $c = e^{\gamma}$

$\varepsilon_h^2 \ll 1$

Weak gravity field: $|\alpha|, |\gamma| \ll 1$. Arbitrary heteron field X.

HG equations:

$$\begin{aligned} \frac{d}{d\xi}\Big(\xi^2 \frac{d\alpha}{d\xi}\Big) &= \varepsilon_h^2 \frac{d}{d\xi}\Big(\xi^2 \frac{dX}{d\xi}\Big), \\ \frac{d}{d\xi}\Big(\xi^2 \frac{d\gamma}{d\xi}\Big) &= -\frac{\varepsilon_h^2}{2}\Big(\xi \frac{dX}{d\xi}\Big)^2. \end{aligned}$$

Heteron field equation in the leading order:

$$\frac{d}{d\xi}\Big(\xi^2 \frac{dX}{d\xi}\Big) = 6\xi^2 e^{-X}.$$

Factorization. Master equation.

Exceptional exact solution:

$$\begin{aligned} \bar{\alpha}/\varepsilon_h^2 = \bar{X} &= \ln 3\xi^2, \\ \bar{\gamma}/\varepsilon_h^2 &= -\ln 3\xi^2 + 2, \end{aligned}$$

One second-order equation $\to$ autonomous system of two first-order equations.

Phase plane:

$\xi \to \infty$ is the exceptional point of the attractive fucus type.

$\Rightarrow$ Any solution X ripples around the exceptional $\bar{X}$ with attenuation at $\xi \to \infty$.

$\Rightarrow$ $\bar{X}$ – reference point.

(Graphical representation of this statements– next talk.)

Regular solution

$$\alpha_h/\varepsilon_h^2 = X_h = \begin{cases} \xi^2 - \frac{3}{10}\xi^4 + \frac{4}{35}\xi^6 + \mathcal{O}(\xi^8), & \text{at } \xi \leq 1, \\ \ln 3\xi^2, & \text{at } \xi \gg 1 \end{cases}$$

and

$$\gamma_h/\varepsilon_h^2 = \begin{cases} -\frac{1}{10}\xi^4 + \frac{2}{35}\xi^6 + \mathcal{O}(\xi^8), & \text{at } \xi \leq 1, \\ -\ln 3\xi^2 + 2, & \text{at } \xi \gg 1. \end{cases}$$

In the wide region of ξ, where $|c_h - 1| \sim \varepsilon_h^2 \ll 1$:

$$\sqrt{-g_0} \simeq e^{-X_h} = \begin{cases} 1 - \xi^2 + \mathcal{O}(\xi^4), & \text{at } \xi \leq 1, \\ 1/(3\xi^2), & \text{at } \xi \gg 1. \end{cases}$$

Kernel g_0 is localized within R_0.

$\Rightarrow$ Canonical gauge ($g_0 = -1$) is singular.

Astronomic objects – the *dark halos.* (Energy content – later in this report).

(Numerical study of the halo – next talk.)

• **Singular solution**

Put $X \equiv X_h + \Delta X$.

Look for $|\Delta X| < 1$ and $|d\Delta X/d\xi| < |dX_h/d\xi|$ (to be confirmed).

Linear in ΔX approximation. Decompose ΔX in the Laurent series in ξ:

$$\Delta X = \delta_i \begin{cases} -1/\xi + 3\xi - 2\xi^3 + \mathcal{O}(\xi^5), & \text{at} \quad \xi \leq 1, \\ (\delta_0/\sqrt{\xi}) \cos\Big((\sqrt{7}/2) \ln \xi/\xi_0\Big), & \text{at} \quad \xi \gg 1. \end{cases}$$

δ_i is to be fixed. δ_0 and ξ_0 may be found approximately.

Matching $1/\xi$-term with the $1/r$-tail of the pit implies

$$\delta_i = r_0/(\varepsilon_h R_0).$$

$\Delta X = X_p + X_i$. X_i – pit-halo interference. δ_i – interference parameter.

Three terms: $X_l = X_p + X_h + X_i$. Two length scales: $R_0 \gg r_0$.

Similarly: $\alpha \equiv \alpha_h + \Delta\alpha$ and $\gamma \equiv \gamma_h + \Delta\gamma$.

$$\begin{aligned} \Delta\alpha &= -\frac{\delta_\alpha}{\xi} + \varepsilon_h^2 \Delta X, \\ \Delta\gamma &= -\frac{\delta_\gamma}{\xi} - \varepsilon_h^2 \delta_i \begin{cases} \xi - 2\xi^3/5 + \mathcal{O}(\xi^5)\Big), & \text{at} \quad \xi \leq 1, \\ \mathcal{O}(1/\sqrt{\xi}), & \text{at} \quad \xi \gg 1. \end{cases} \end{aligned}$$

Matching with pit $\Rightarrow$ δ_α and δ_γ.

Gravitational potential $\alpha = \alpha_p + \alpha_h + \alpha_i \Rightarrow$

Hetero-gravitational confinement, with the $1/r$ Newtonian potential for the pit superseded eventually by the logarithmically growing one for the halo.

Astronomic objects – the *dark lacunas.*

The confinement scale R_0 is object-dependent.

Ultimately, the confinement should be terminated by the potential V_h at the halo edge, as well as the influence of the near-by lacunas. $\Rightarrow$

Partial confinement.

The dark lacunas, with the supermassive dark pit at the origin surrounded by the dark halo, nicely fit as a prototype model of galaxies and the cluster of galaxies. Quite similar dark matter structure on their own scales.

4 Energy content

• Static HG objects

An isolated system in the gravitational field.

$T_{\mu\nu}$ is the bare energy-momentum tensor without the gravity field contribution.

The total energy-momentum pseudo-tensor of the system:

$$\bar{T}_{\mu\nu} = T_{\mu\nu} + \bar{T}^{g}_{\mu\nu}.$$

$\bar{T}^{g}_{\mu\nu}$ is the pseudo-tensor of the gravitational field. Definition and coordinate dependent.

Take as $\bar{T}^{g}_{\mu\nu}$ the Einstein pseudo-tensor in the quasi-Galilean coordinates,

The total energy (mass) of a static isolated system:

$$M = \int \bar{T}^{0}_{0} \sqrt{-g} d^3x = \int \left(T^0_0 - T^n_n \right) \sqrt{-g} d^3x.$$

HG equations $\Rightarrow$

$$M = 2\kappa_g^2 \int R^0_0 \sqrt{-g} d^3x.$$

Partition:

$$M = M_m + M_\chi \equiv \int \left(\left(T_{m}{}^{0}_{0} - T_{m}{}^{n}_{n} \right) + \left(T_{\chi}{}^{0}_{0} - T_{\chi}{}^{n}_{n} \right) \right) \sqrt{-g} d^3x.$$

The heteron energy density in the gravitational field is

$$\rho^g_\chi = T_{\chi}{}^{0}_{0} - T_{\chi}{}^{n}_{n} = -2\check{V}_h = -2\kappa_h \nabla \cdot \nabla\chi.$$

The heteron contribution: $M_\chi = -2\kappa_h \int \nabla \cdot \nabla\chi \sqrt{-g} d^3x.$

Matter contribution: $M_m = M - M_\chi.$

• Dark pits

The three-dimensional Gauss theorem gives for the total mass of the pit interior to r:

$$M_{p<}(r) = M_p = 4\pi\kappa_g^2\sigma_p r_p = r_g/(2G_N) \geq 0.$$

The heteron contribution

$$M_{\chi<}(r) = M_\chi = 4\pi\kappa_g^2\varepsilon_h\eta_p\sqrt{1-\sigma_p^2}r_p = \varepsilon_h r_0/(2G_N).$$

Otherwise, $M_\chi = \varepsilon_h\eta_p\sqrt{1/\sigma_p^2 - 1}M$. $M_\chi \geq 0 \Rightarrow \eta_p = +1$.

The ordinary matter contribution:

$$M_{m<}(r) = M_m = M_p - M_\chi = \left(1 - \varepsilon_h\sqrt{1/\sigma_p^2 - 1}\right)M \geq 0,$$

$$\varepsilon_h \leq \sigma_p \leq 1, \quad \text{at} \quad \varepsilon_h. \ll 1$$

Marginal cases:

$\sigma_p = 1 \Leftrightarrow$ BHs ($M_\chi = 0$, $M_m = M_p$).

$\sigma_p \simeq \varepsilon_h \Leftrightarrow$ the pure heteron dark pit ($M_m = 0$, $M_\chi = M_p$).

Absolutely flat pits ($\sigma_p = 0$) are excluded ($M_m = M_\chi = M_p = 0$) .

• Dark halos

The heteron energy density in the empty space, incorporating the gravitational energy, is $\rho^g_\chi = -2\check{V}_h = 2w_0 e^{-X}$. This results in

$$\rho^g_\chi = \kappa_g^2 \frac{\varepsilon_h^2}{\rho_h^2} \frac{1}{\xi^2} \frac{d}{d\xi}\left(\xi^2 \frac{dX}{d\xi}\right).$$

For $X = X_h$, one gets the halo energy density profile:

$$\rho_h = \rho_0 \begin{cases} 1 - \xi^2 + \frac{4}{5}\xi^4 + \mathcal{O}(\xi^6), & \text{at} \quad \xi \leq 1, \\ 1/(3\xi^2), & \text{at} \quad \xi \gg 1. \end{cases}$$

The energy central density of a halo: $\rho_0 = 6\kappa_g^2 \varepsilon_h^2 / R_0^2$.

The halo possesses the soft core profile with the scale R_0.

ρ_h reproduces $\sqrt{-g_0}$.

• Dark lacunas

For $X = X_l = X_p + X_h + X_i \Rightarrow \rho_l = \rho_h + \rho_i$ (the $1/r$-term in X_p does not contribute), with the interference contribution:

$$\rho_i = \delta_i \rho_0 \begin{cases} 1/\xi - 4\xi + \mathcal{O}(\xi^3), & \text{at} \quad \xi \leq 1, \\ \mathcal{O}(1/\xi^{5/2}), & \text{at} \quad \xi \gg 1. \end{cases}$$

The effective halo density, $\rho_{heff} \equiv \rho_h + \rho_i$, complies the cuspy $1/r$-correction compared to the soft core profile ρ_h.

Dark lacunas $\sim$ dark lumps.

5 Rotation curves

• RC profiles

Rotation velocity:

$$v^2 = \frac{(\ln a)'}{(\ln cr^2)'}.$$

The total velocity squared is the sum of three components,

$$\begin{aligned}
v_t^2 &= v_p^2 + v_h^2 + v_i^2, \\
v_p^2 &= \frac{1}{2}\frac{r_g}{r}, \\
v_h^2 &= \varepsilon_h^2 \begin{cases} \xi^2 - \frac{3}{5}\xi^4 + \frac{12}{35}\xi^6 + \mathcal{O}(\xi^8), & \text{at} \quad \xi < 1, \\ 1, & \text{at} \quad \xi \gg 1, \end{cases} \\
v_i^2 &= \frac{3\varepsilon_h^2 \delta_i}{2} \begin{cases} \xi - 2\xi^3 + \mathcal{O}(\xi^5), & \text{at} \quad \xi < 1, \\ \mathcal{O}(1/\sqrt{\xi}), & \text{at} \quad \xi \gg 1. \end{cases}
\end{aligned}$$

The effective halo RC profile $v_{heff} \equiv (v_h^2 + v_i^2)^{1/2}$ behaves at the very small r as $\sqrt{r}$, similar to v_i, while it remains flat asymptotically, similar to v_h.

The exceptional solution $\bar{X}$ would result in the precisely flat profile $\bar{v} = \varepsilon_h$, around which all the profiles $v_{heff}(r)$, with various ρ_h, ripple approaching $\bar{v}$ at $r \gg \rho_h$.

Observationally for galaxies $v_h(\infty) \sim 10^{-3}$ (in units of the speed of light).

$\Rightarrow \varepsilon_h \sim 10^{-3}$, or the HG mass scale $\kappa_h \sim 10^{15}$ GeV $\sim M_{\rm GUT}$.

• DM profiles

Interpret the heteron field in terms of DM. The Newton's dynamics in the flat space ($a = c = 1$) with the DM halo results in the rotation velocity:

$$\frac{v_d^2}{r} = \frac{GM_{d<}(r)}{r^2}.$$

$M_{d<}(r) = 4\pi \int_0^r \rho_d(r) r^2 dr$ is the DM energy interior to r. $\Rightarrow$
DM energy density:

$$\rho_d = \frac{1}{4\pi G} \frac{(r v_d^2)'}{r^2}.$$

Equating $v_d^2 = v_{heff}^2 = v_h^2 + v_i^2$ we get

$$\rho_d = \rho_{heff} = \rho_h + \rho_i,$$

The exceptional solution $\bar{X}$ would result in the $1/r^2$-cuspy profile:

$$\bar{\rho}_d = \frac{\rho_0}{3\xi^2} = \frac{2\varepsilon_h^2 \kappa_g^2}{r^2}.$$

The family $\rho_d(r)$, with various R_0, ripples around $\bar{\rho}_d$ approaching it at $r \gg R_0$.

RCs admit a complementary interpretation:

in terms of a coherent heteron field in HG

or in terms of an effective DM in the Newtonian dynamics.

This justifies treatment of the heteron field as DM.

As for the gravitational manifestations, both descriptions are equivalent in the non-relativistic weak-field approximation.

6 Conclusion

• HG is a theoretically viable extension to GR, with the (scalar) heteron field as DM.

• HG predicts the new astronomic objects – the dark lacunas, with the property of the (partial) hetero-gravitational confinement.

• Dark lacunas, with the singular dark pit at the origin surrounded by the extended dark halo, nicely suite as a prototype model of galaxies and the cluster of galaxies.

• Due to the coherent-field nature of the heteron DM, the pit-halo interference is important. This widely extends the pattern of the energy density profiles of lacunas.

• For further refining the HG applications to the astrophysical observations, studying the lacuna solution in the whole parameter region is urgent.