Breaking statistical isotropy

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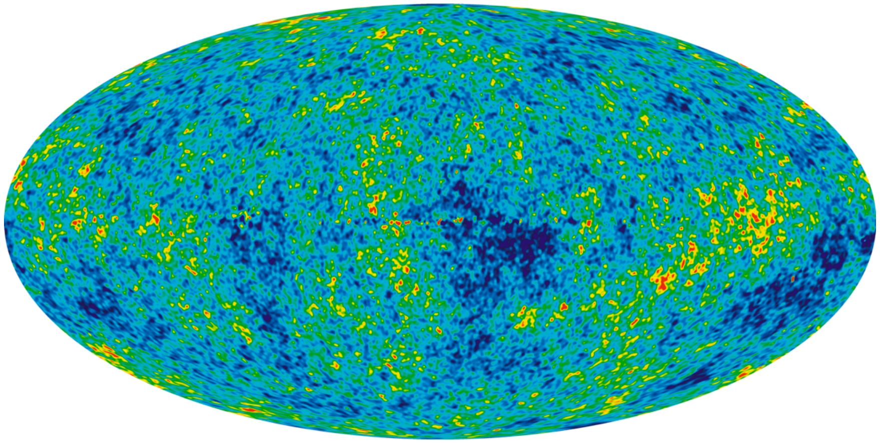
Gumrukcuoglu, Contaldi, MP, JCAP '07

Gumrukcuoglu, Kofman, MP, JCAP '08

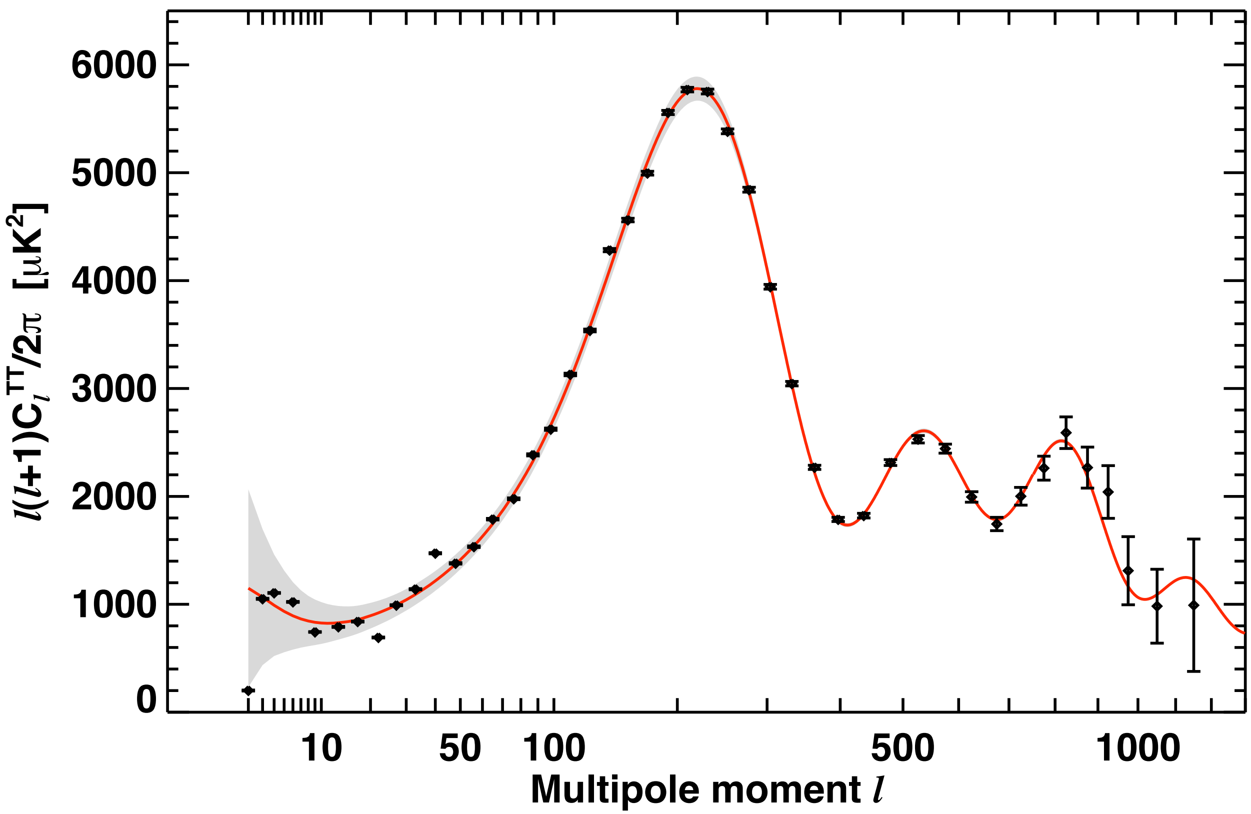
Himmetoglu, Contaldi, MP, PRL '09; PRD '09; PRD '09

Gumrukcuoglu, Himmetoglu, MP, PRD '10



$$T = \sum_{\ell m} a_{\ell m} Y_{\ell m}$$

$$\langle a^*_{\ell m} a_{\ell' m'} \rangle = C_\ell \, \delta_{\ell\ell'} \, \delta m m'$$



CMB anomalies ?

- Low quadrupole
- Lack of correlations at $\theta > 60^0$
- Quadrupole-octupole alignment (low l alignment)
- North-south asymmetry
- Broken rotational invariance
- Cold spot

(hard to assess a-posteriori significance; recent overview, with negative assessments, in WMAP-7 arXiv:1001.4758)

$$\boxed{\text{Observed anisotropy}} = \boxed{\text{Transfer function}} \times \boxed{\text{Primordial perturbations}}$$

Power spectrum

$$\langle\, \delta_{\vec{k}}\, \delta_{\vec{k}'} \,\rangle = (2\pi)^3\, \delta^{(3)} \left(\vec{k} + \vec{k}'\right) \frac{2\pi^2}{k^3}\, P\,(k)$$

$$P = P\left(|\vec{k}|\right) \quad \Rightarrow \quad \langle a^*_{\ell m}\, a_{\ell' m'} \rangle = C_\ell\, \delta_{\ell \ell'}\, \delta_{m m'}$$

statistical isotropy

$\widehat{v}$

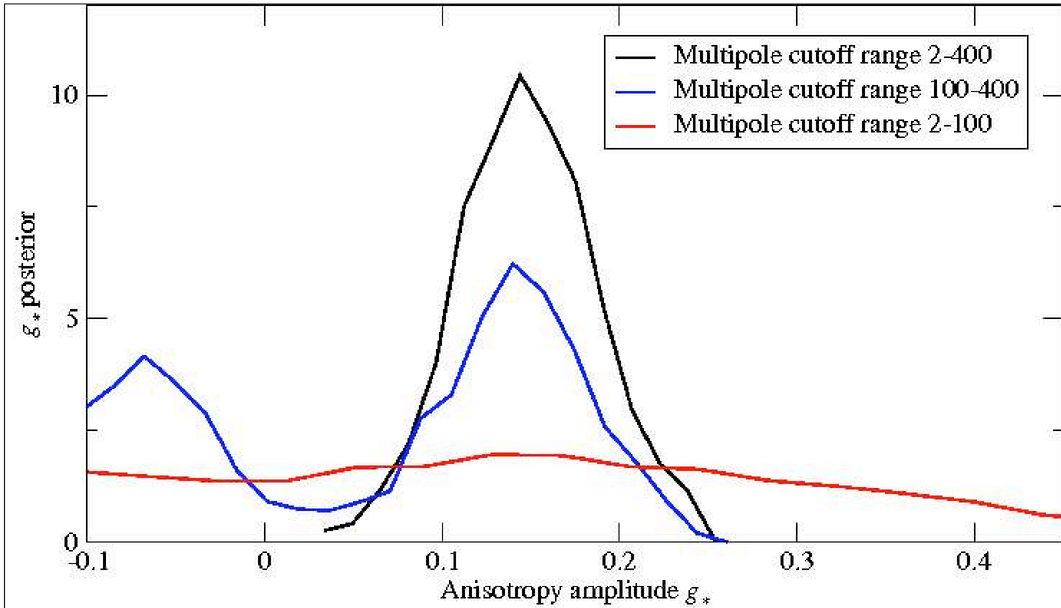
No rotational invariance during inflation

$$P = P\,(k) \left[1 + g_* \left(\widehat{k} \cdot \widehat{v}\right)^2 + \ldots\right]$$

(small anisotropy, 2d symmetry, parity)

Ackerman, Carroll, Wise '07

For simplicity, take universe isotropic after inflation

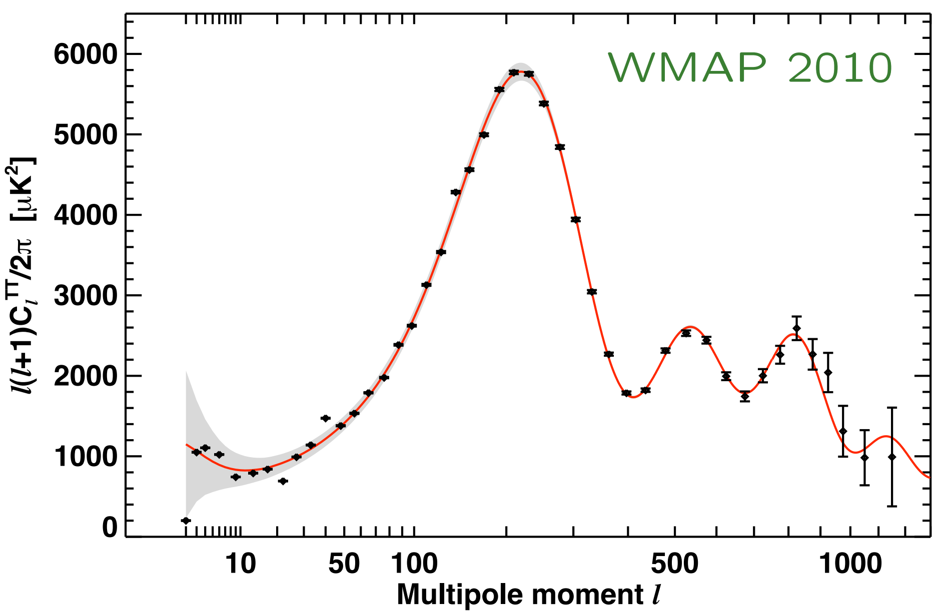


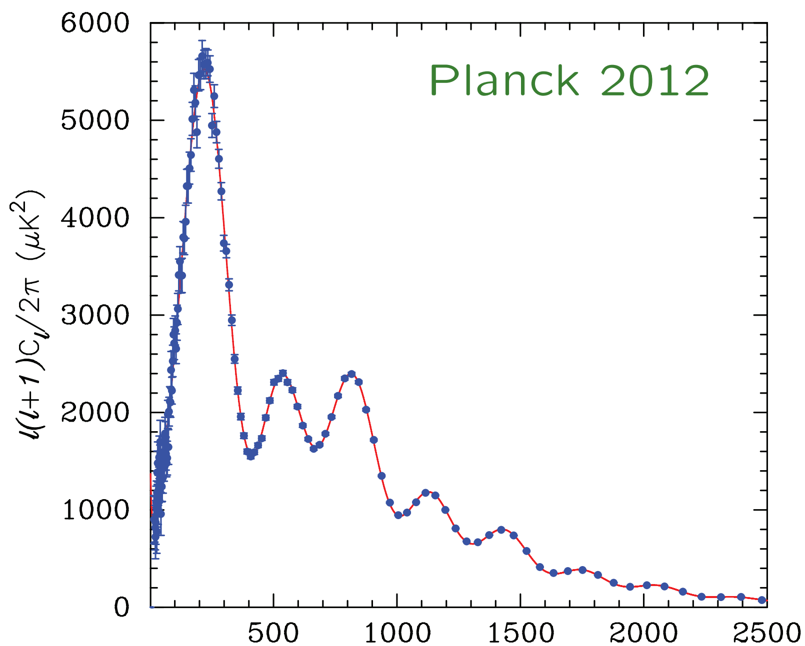
Groeneboom and Eriksen '08

WMAP 5 years (W band)

$g_* = 0.15 \pm 0.04$ (3.8σ)

Increase of significance with ℓ Pullen, Kamionkowski '07

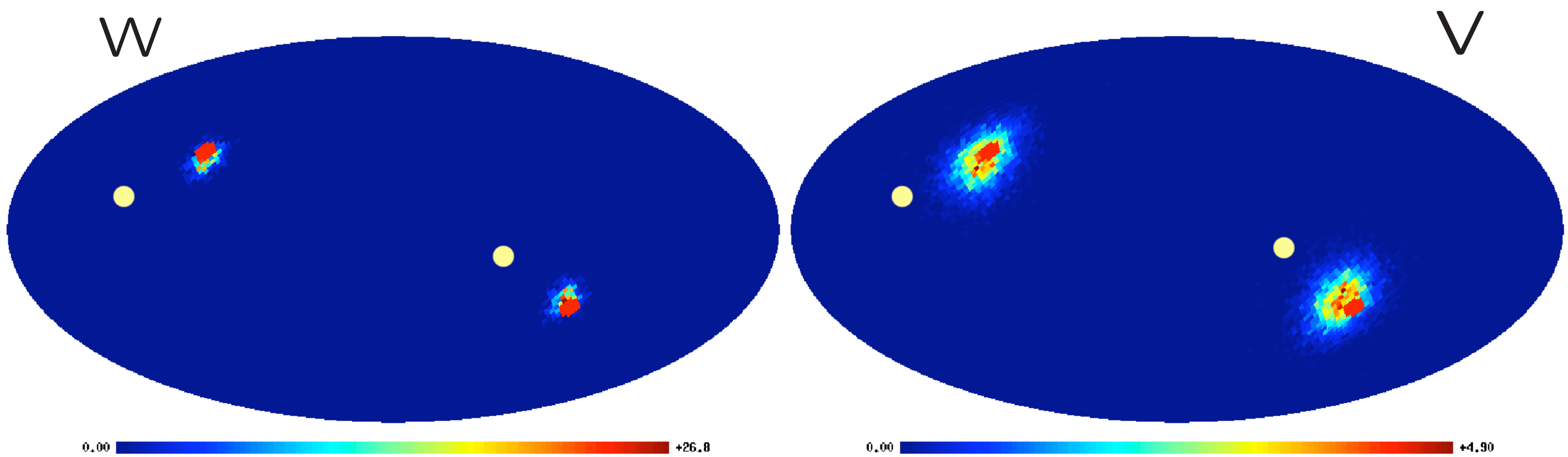




Hanson and Lewis '09 and Groeneboom et al. '09 corrected a missing factor in the '08 analysis

Band	ℓ range	Mask	Amplitude g_*	Direction (l, b)
W1-4	$2-400$	KQ85	0.29 ± 0.031	$(94^\circ, 26^\circ) \pm 4^\circ$
V1-2	$2-400$	KQ85	0.14 ± 0.034	$(97^\circ, 27^\circ) \pm 9^\circ$
Q1-2	$2-300$	KQ85	-0.18 ± 0.040	$(99^\circ, 28^\circ) \pm 10^\circ$

NOTE. — The values for g_* indicate posterior mean and standard deviation. The ecliptic poles are located at $\pm(96^\circ, 30^\circ)$.



Systematics ? Negative answer in Groeneboom et al. '09,

Hanson et al '10: asymmetric beams can account for the anomaly

As we wait for Planck, what about theory ?

In the past few years, explosion of models with

$$ds^2 = -dt^2 + a(t)^2\, dx^2 + b(t)^2 \left[dy^2 + dz^2\right]$$

Homogeneity, parity, 2d isotropy

Claim $\Rightarrow$

$$P\left(\vec{k}\right) = P(k)\left[1 + g_*\left(\vec{k}\cdot\vec{v}\right)^2\right]$$

If true, no strong theoretical bias for statistical isotropy

Extend FRW formalism

Bardeen '80

Hawking '82

Guth and Pi '82

Starobinsky '82

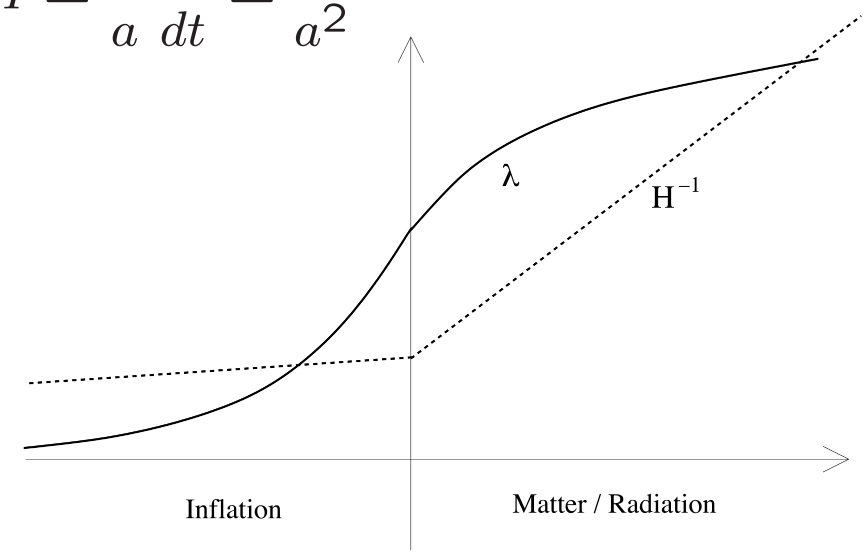
Mukhanov '85

Sasaki '86

Cosmological perturbations

$$ds^2 = a^2(\eta)\left[-d\eta^2 + d\vec{x}^2\right]$$

$$H = \frac{1}{a}\frac{da}{dt} = \frac{a'}{a^2}$$



$$\delta g_{\mu\nu}\,,\ \delta\phi \ \leftrightarrow \ v,\ h_+,\ h_\times$$

$$\delta^{(2)}S \ = \int d\eta\, d^3k\ \left\{ \left|v'\right|^2 - \left[k^2 + f(a)\right]|v|^2 + \left|h_i'\right|^2 - \left[k^2 + g(a)\right]|h_i|^2 \right\}$$

Initially deep inside the horizon,

$k/a \gg H$, and f, g negligible

Adiabatic vacuum

$$v_{\text{in}} = h_{\text{in}} = \frac{\mathrm{e}^{-i\,k\,\eta}}{\sqrt{2\,k}}$$

Decoherence as $k/a \ll H$: deterministic evolution

amplitude drawn from a gaussian distribution
(zero mean, variance only depends on $|\vec{k}|$)

Polarski,
Starobinsky '95

Initial sub-horizon quantization crucial for the amplitude

Perturbations of Anisotropic backgrounds

How many physical modes ?	As in FRW $\quad \delta g_{\mu\nu}\ ,\ \delta\phi$	$+11$ modes
	Gen. coord. transf.	-4
	$\delta g_{0\mu}$ non dynamical	-4
How do they behave ?	Coupled to each other already at the linear level (less symmetric background)	
Signatures ?	$\langle a_{\ell m} a^*_{\ell' m'} \rangle \not\propto \delta_{\ell\ell'}\,\delta_{mm'}$ Tensor-scalar correlation $\neq$ results for the 2 GW polarizations	

$$g_{\mu\nu} = \begin{pmatrix} -(1+2\Phi) & a\,\chi & b\left(\partial_i B + B_i^T\right) \\ & a^2\,(1-2\Psi) & a\,b\left(\cancel{\partial_i \tilde{B}} + \tilde{B}_i^T\right) \\ & & b^2\left[(1-\cancel{2\Sigma})\,\delta_{ij} + \partial_i \cancel{\partial_j E} + \partial_{(i}\cancel{E^T}_{j)}\right] \end{pmatrix}$$

seven 2d scalars + three 2d vectors, decoupled at linearized level

Choose a gauge preserving all $\delta g_{0\mu}$, since nondynamical

Dynamical Y_i and nondynamical N_i fields

$$\delta^{(2)} S = \int d^3k\,dt\,\Big[a_{ij}\,\dot{Y}_i^*\,\dot{Y}_j + \left(b_{ij}\,N_i^*\,\dot{Y}_j + \text{h.c.}\right) + c_{ij}\,N_i^*\,N_j$$
$$+ \left(d_{ij}\,\dot{Y}_i^*\,Y_j + \text{h.c.}\right) + e_{ij}\,Y_i^*\,Y_j + \left(f_{ij}\,N_i^*\,Y_j + \text{h.c.}\right)\Big]$$

Coefficients background-dependent

Cf. Rubakov '04

Solving for N, $\quad \dfrac{\delta S}{\delta N_i^*} = 0 \;\Rightarrow\; c_{ij}\,N_j = -b_{ij}\,\dot{Y}_j - f_{ij}\,Y_j \;\rightarrow\;$ action

Action for the dynamical (propagating) modes

$$S \;\rightarrow\; \int d^3k\, dt \left[\dot{Y}_i^* \, K_{ij} \, \dot{Y}_j + \left(\dot{Y}_i^* \, \Lambda_{ij} \, Y_j + \text{h.c.} \right) - Y_i^* \, \Omega_{ij}^2 \, Y_j \right]$$

$$K_{ij} \equiv a_{ij} - \left(b^\dagger\right)_{ik} \, \left(c^{-1}\right)_{km} \, b_{mj}$$

$$\Lambda_{ij} \equiv d_{ij} - \left(b^\dagger\right)_{ik} \, \left(c^{-1}\right)_{km} \, f_{mj}$$

$$\Omega_{ij}^2 \equiv e_{ij} - \left(f^\dagger\right)_{ik} \, \left(c^{-1}\right)_{km} \, f_{mj}$$

Eigenvalues indicate the nature of a mode

$$\frac{\delta S}{\delta Y_i^*} = 0 \;\rightarrow\; K_{ij} \, \ddot{Y}_j + \left[\dot{K}_{ij} + \left(\Lambda_{ij} + \text{h.c.}\right) \right] \, \dot{Y}_j + \left(\dot{\Lambda}_{ij} + \Omega_{ij}^2 \right) \, Y_j = 0$$

Expect divergency if K is noninvertible

This formalism $\rightarrow$ existing models of anisotropic inflation

Simplest example: universe isotropizing at the onset of inflation.

Anisotropy: Initial condition

$$\mathcal{L} = \frac{M_p^2}{2} R - \frac{1}{2} \left(\partial \phi\right)^2 - V\left(\phi\right)$$

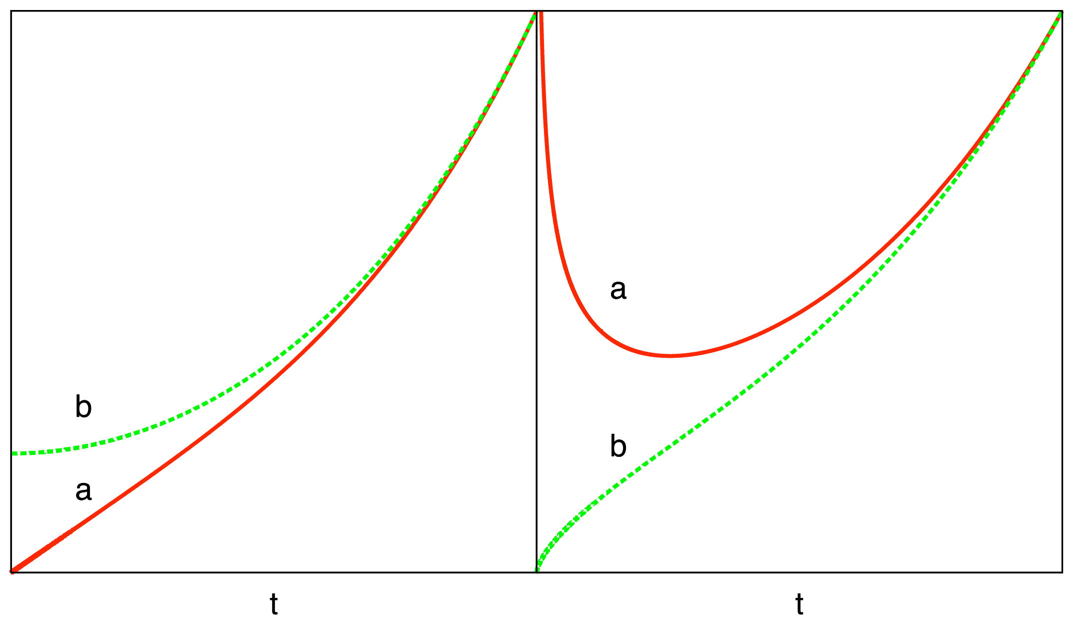
Rapid isotropization, within one Hubble time, $t_{\text{iso}} \sim \dfrac{M_p}{\sqrt{V_{\text{in}}}}$

Asymptotic Kasner (vacuum) solution in the past.

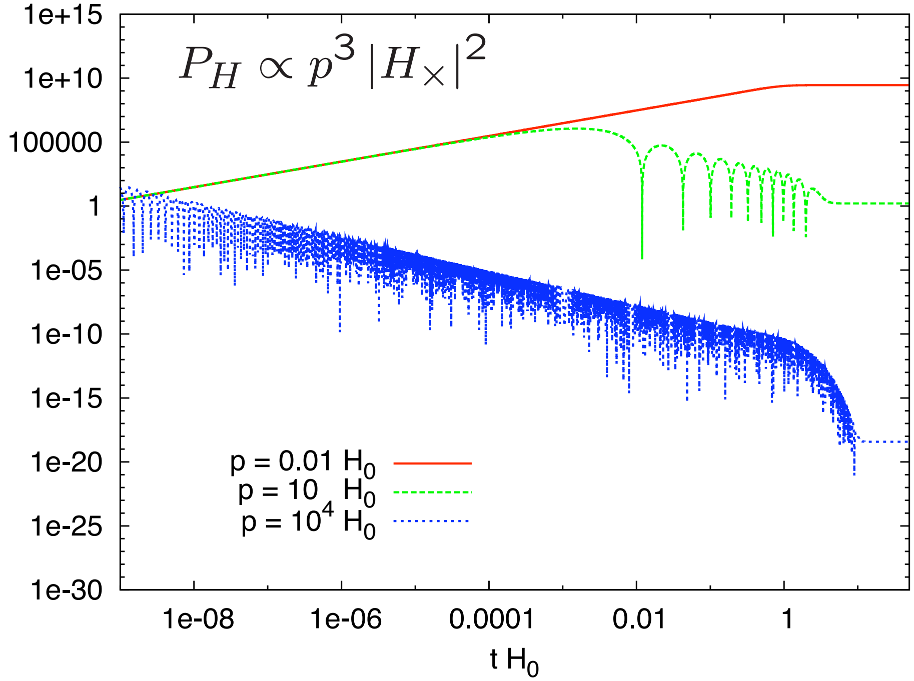
$$ds^2 = -dt^2 + t^{2\alpha}\, dx^2 + t^{2\beta}\left(dy^2 + dz^2\right) \qquad \begin{cases} \alpha + \beta + \beta = 1 \\ \alpha^2 + \beta^2 + \beta^2 = 1 \end{cases}$$

Only Kasner regular at $t = 0$.

We disregard it



$\alpha = 1 \ , \ \beta = 0 \qquad \alpha = -\dfrac{1}{3} \ , \ \beta = \dfrac{2}{3}$



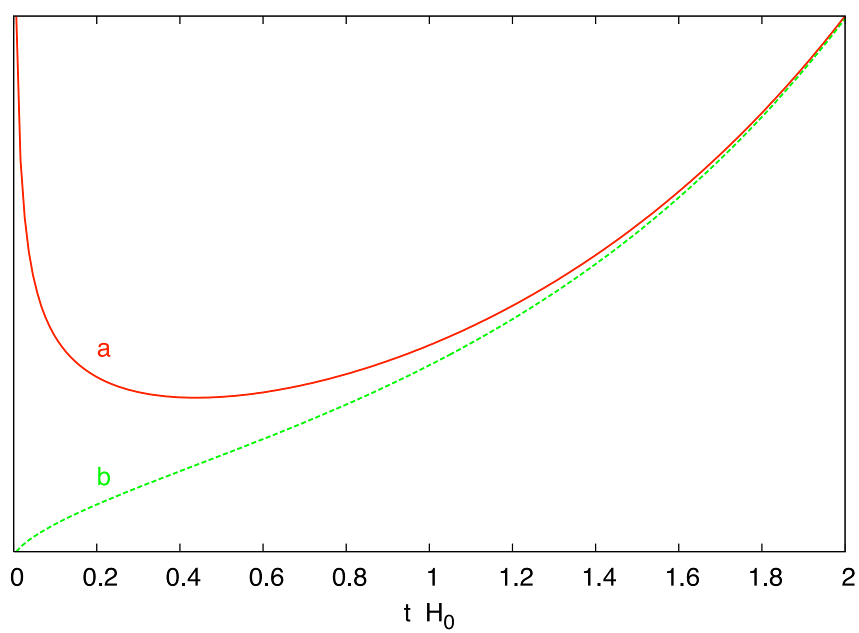
One mode (which later becomes one GW polarization) exhibits large growth at long-wavelength during anisotropic era

$$\equiv$$

Instability of contracting Kasner

(shut off as background isotropizes)

Belinsky, Khalatnikov, Lifshitz '70, '82



\+ Enhanced GW signal at large scales (requires short inflation)

– No control on initial conditions at the singularity

Search for a longer / controllable anisotropic stage

Rapid isotropization for Bianchi spaces with $\Lambda + T_{\mu\nu}$ satisfying the dominant and strong energy conditions   Wald '83

Counterexamples: Kalb-Ramond axion (Kaloper '91); higher curvature terms (Barrow, Hervik '05), Vector fields

- Potential term $V\left(A_\mu A^\mu\right)$ Ford '89
- Fixed norm $\lambda\left(A_\mu A^\mu - v^2\right)$ ACW '07
- Nonminimal coupling $A_\mu A^\mu R$ Golovnev, Mukhanov, Vanchurin '08

All these vector models have ghost instabilities

Discuss only nonminimally coupled case; ignore $\delta g_{\mu\nu}$

$$S = \int d^4x \sqrt{-g} \left[\frac{M_p^2}{2} R - V_0 - \frac{F^2}{4} + \frac{\xi}{2} R A^2 \right]$$

$$ds^2 = -dt^2 + \mathrm{e}^{2H_a t} dx^2 + \mathrm{e}^{2H_b t} \left(dy^2 + dz^2 \right) \quad , \quad \langle A_\mu \rangle = (0,\, a\, M_p\, B,\, 0,\, 0)$$

$$H = \frac{1}{3} [H_a + 2\, H_b] \qquad h = \frac{1}{3} [H_b - H_a]$$

$$h/H = \mathrm{O} \left(B^2 \right)$$

$$\ddot{B} + 3\, H\, \dot{B} + (1 - 6\, \xi)\, 2H^2\, B \simeq 0$$

Mass term causes $B \to 0$, and so $h \to 0$ during inflation unless

$$\xi = \frac{1}{6}$$

(Primordial magnetic fields, Turner, Widrow '88)

$$\mathcal{L} = -\frac{1}{4}F^2 + \frac{1}{12}\, R\, A^2 = -\frac{1}{4}F^2 + H^2\, A^2$$

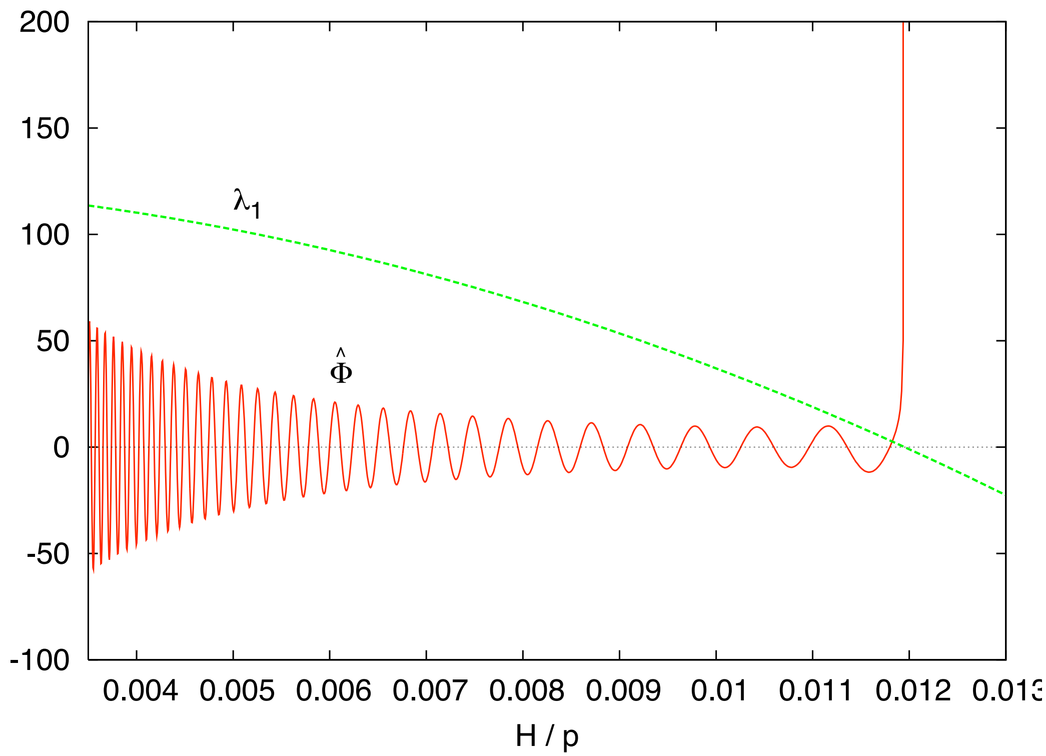
$+$ sign leads to a ghost (not a tachyon !)

Stückelberg: $A_\mu = B_\mu + \frac{1}{H}\, \partial_\mu\, \phi$ gauge $\partial^\mu B^T_\mu = 0$

$$H^2 A^2 \;\longrightarrow\; H^2\, B^T_\mu\, B^{\mu T} + \partial_\mu\phi\, \partial^\mu\phi$$

(signature $-+++$)

Instability intrinsic in the model not in the anisotropic background



Full computation, mixing

$\delta A_\mu \leftrightarrow \delta g_{\mu\nu}$, due to $\langle A_x \rangle$

One mode becomes a ghost

close to horizon crossing.

All perturbations diverge

Analogous behavior for all the models studied

(one ghost per vector in the model)

So what ?

Linearized computation blows up; maybe nonlinear evolution ok

Linearized computation $\rightarrow$ CMB

nonlinear interactions: $|0\rangle \rightarrow$ ghost-nonghost; UV ∞

For a gravitationally coupled ghost today, $\Lambda < 3\,\mathrm{MeV}$

Cline et al' 03

Hard mass, A_L interactions p/m enhanced
strongly interacting QFT for $E > m \sim H$
(sub-horizon regime)

Unclear UV completion $\quad \pm\,|DH|^2 \rightarrow \pm\,m^2\,A^2$

Preserve U(1) invariance !

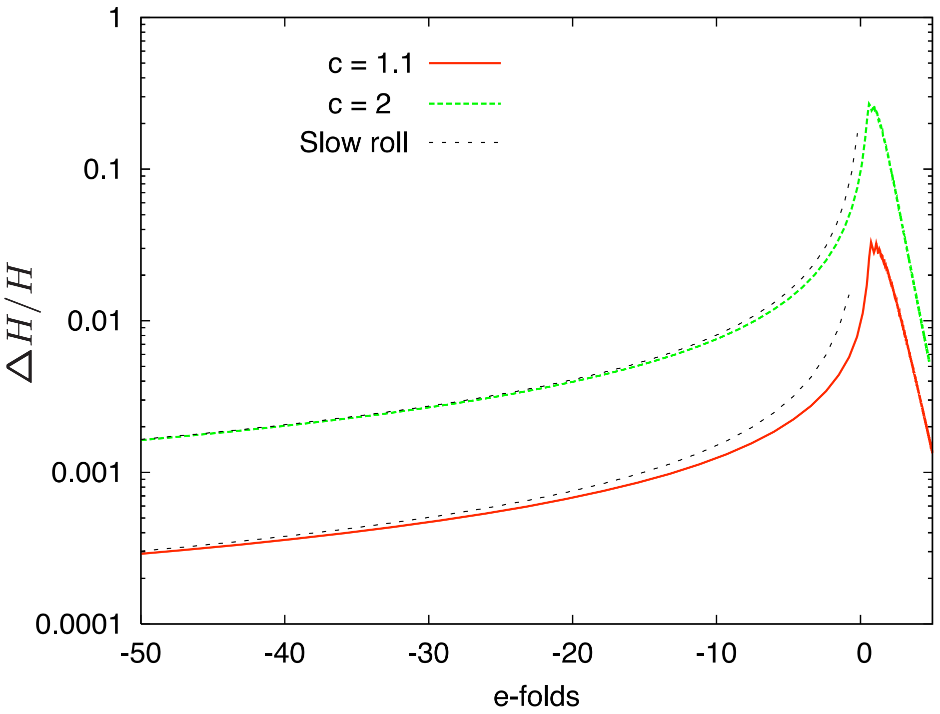
$$S = \int d^4x \sqrt{-g}\left[\frac{M_p^2}{2} R - \frac{1}{2}(\partial\phi)^2 - V(\phi) - \frac{1}{4} f(\phi)^2 \, F^2\right]$$

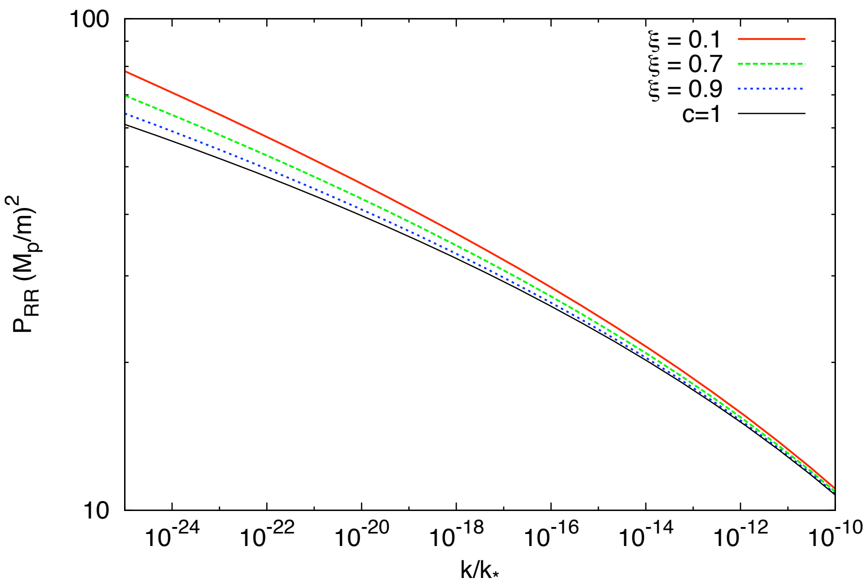
Watanabe, Kanno, Soda '09

slow roll $\Rightarrow \quad \dfrac{\rho_A}{\rho_\phi} \propto \dfrac{1}{a^2 f(\phi)^2}$

$$\Rightarrow \begin{cases} V = m^2 \phi^2/2 \\ f = \exp\left(c\,\phi^2/2\,M_p^2\right) \end{cases}$$

$c \gtrsim 1 \quad \longrightarrow \quad$ anisotropic attractor



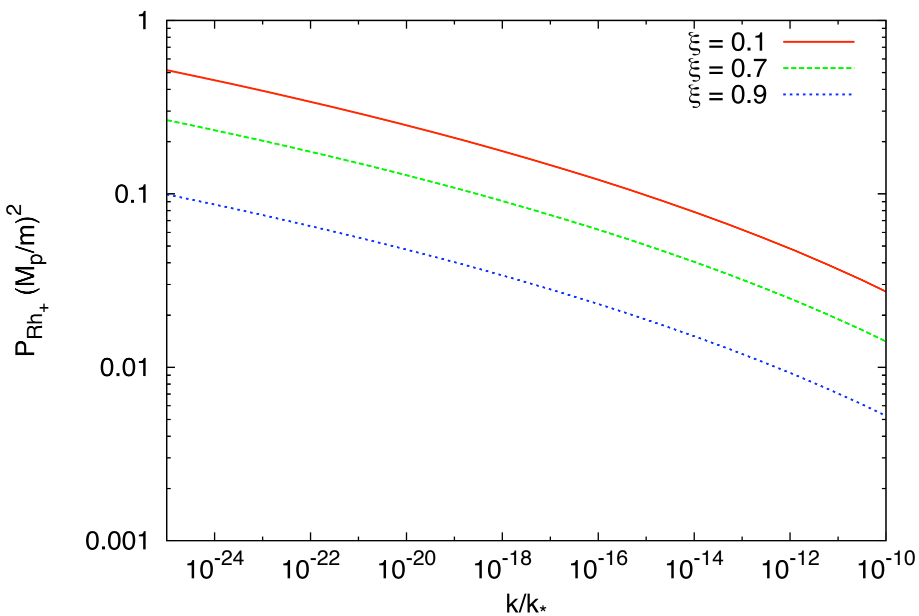


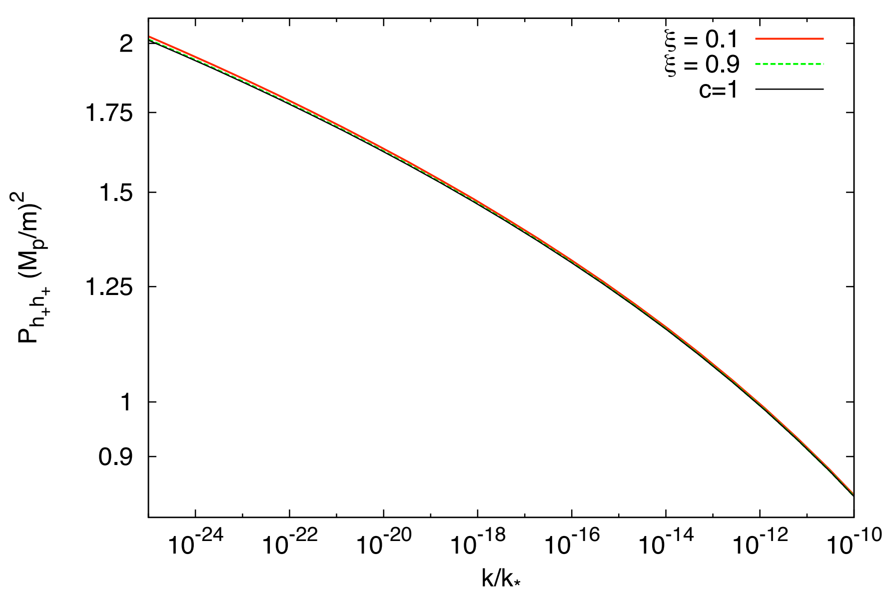
10% anisotropy in power ($\xi = \widehat{k} \cdot \widehat{v}$)

for $\delta H/H \approx 10^{-6}$

For such small anisotropy,

$P_{ST} < P_{TT}$





Agreement with Dulaney, Gresham '10; Watanabe, Kanno, Soda '10

Conclusions

- Motivated by "CMB anomalies", several models realizing breaking of statistical isotropy have been proposed

- How strong is the theoretical bias ? Explicit computations

  $+$ direct arguments arguments, show that it is a nontrivial task

- We know little of inflation. Not many "hairs" in standard picture. Any hint form data should be studied.

  Issue to be re-assessed in 2012, from Planck

Vector inflation

Golovnev, Mukhanov, Vanchurin '08

Kanno, Kimura, Soda, Yokoyama '08

$$\mathcal{L} = \sum_a \; -\frac{1}{4} F^{(a)}_{\mu\nu} F^{(a)\mu\nu} - \frac{1}{2}\left(m^2 - \frac{R}{6}\right) A^{(a)}_\mu A^{(a)\mu}$$

$$\vec{A}^{(a)} = a\, M_p\, \vec{B}^{(a)} \qquad \rightarrow \qquad \ddot{B} + 3\,H\,\dot{B} + m^2\,B = 0 \;\; , \quad \frac{h}{H} \sim \frac{1}{\sqrt{N}}$$

$$\begin{cases} \delta g_{\mu\nu} \;\rightarrow\; 10 - 4 = 2\,\text{dyn} + 4\,\text{non dyn} \\ \delta A^{(a)}_\mu \;\rightarrow\; 4\,N = 3N\,\text{dyn} + N\,\text{non dyn} \end{cases}$$

Simplest case, 3 mutually orthogonal vectors with equal vev

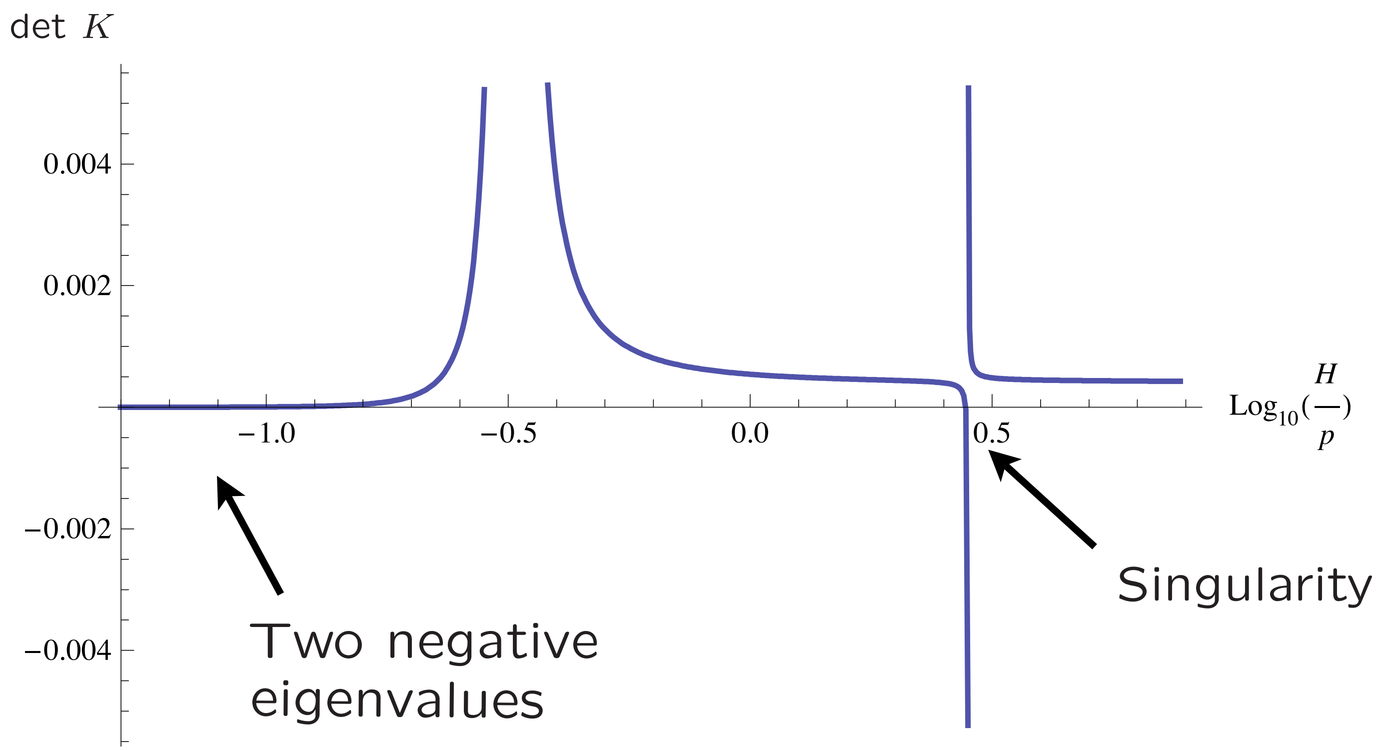
$\rightarrow$ 18 coupled modes

$$\delta_2 S \supset a^2\, M_p \left[\left(\dot{B} + H\,B\right) \delta F^{(i)}_{0j} + \left(m^2 - 2H^2 - \dot{H}\right) B\, \delta A^{(i)}_j \right] h^{TT}_{ij}$$

18 coupled modes, 11 dynamical and 7 non dynamical

We computed the kinetic matrix for the dynamical modes

$$\delta_2 S = \int d^3k\, dt\, \left[\dot{Y}_i^*\, K_{ij}\, \dot{Y}_j + \ldots\right]$$



$k_1 : k_2 : k_3 = 100 : 80 : 60$

Fixed norm vector fields

Kostelecky, Samuel '89; Jacobson, Mattingly '04; Carroll, Lim '04

$$S = \int d^4x\,\sqrt{-g}\left[\frac{M_p^2}{2}\,R - \frac{1}{4}F_{\mu\nu}\,F^{\mu\nu} + \lambda\,\left(A^2 - m^2\right) - V_0\right]$$

Ackerman, Carroll, Wise '07

$$ds^2 = -dt^2 + \mathrm{e}^{2\,H_a\,t}\,dx^2 + \mathrm{e}^{2\,H_b\,t}\left(dy^2 + dz^2\right)$$

$$\langle A_x \rangle = m \quad \Rightarrow \quad H_b = \left(1 + \frac{m^2}{M_p^2}\right) H_b$$

Test field χ

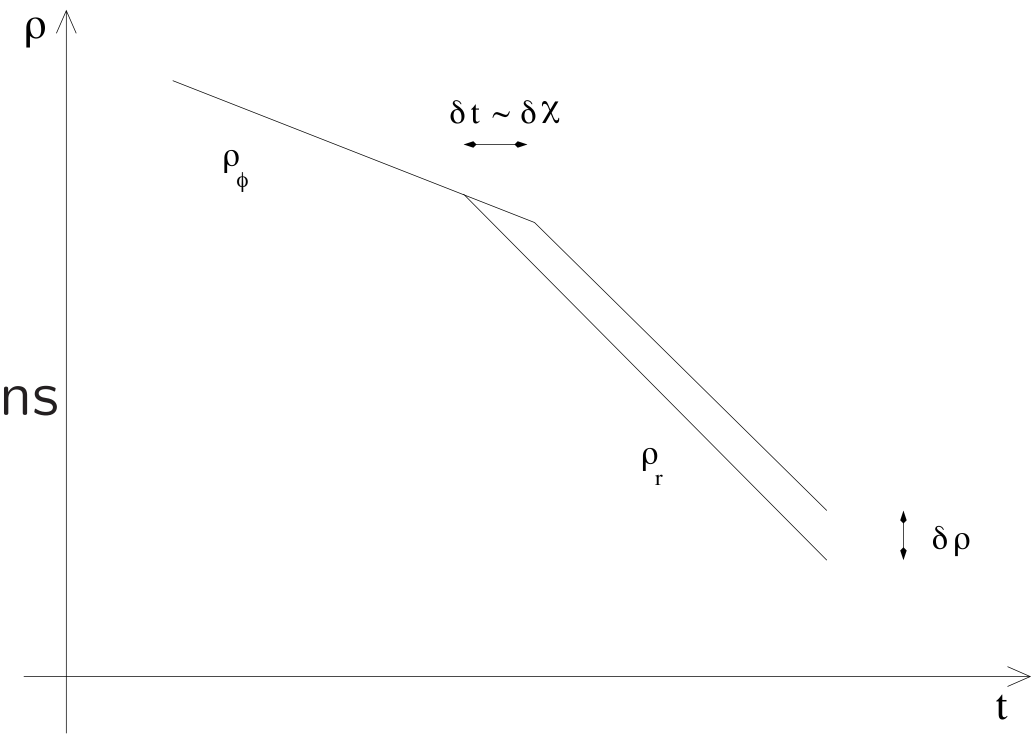
$$P_{\delta\chi} = P\left(|\vec{k}|\right)\,\left(1 + g_*\,k_x^2\right)$$

Assumed $\delta\chi \rightarrow \delta g_{\mu\nu}$

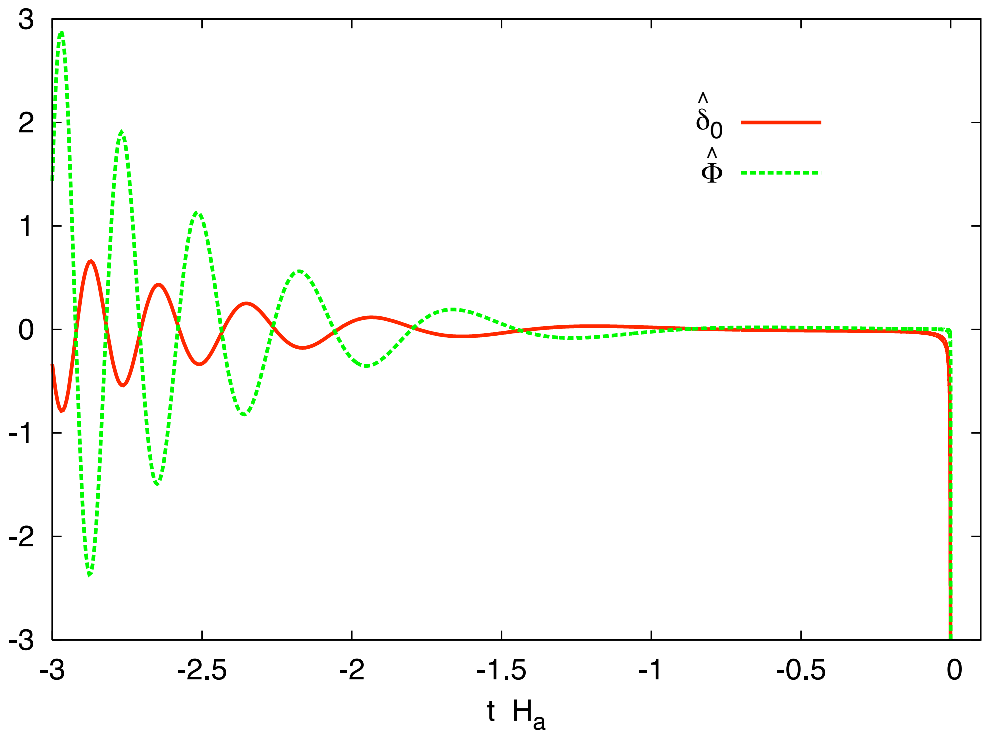
through modulated pertrbations

Dvali, Gruzinov, Zaldarriaga '03

Kofman '03



Complete study ($\delta g_{\mu\nu}$, δA_μ) shows that this model is unstable



Divergency at

"horizon crossing"

$$p_x^2 = \left(2 + \frac{m^2}{M_p^2}\right) H_a\, H_b$$

One mode becomes a ghost at that point

Kinetic term vanishes $\rightarrow$ perturbations diverge

Problems in theories with fixed A^2 also pointed out by Clayton '01