Neutrino magnetic moment in a magnetized plasma

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Introduction

Investigations of the influence of an active medium on neutrino dispersion are based on calculations of the neutrino self-energy operator $\Sigma(p)$

$$M_{(\nu \to \nu)} = - \bar{U}(p) \, \Sigma(p) \, U(p),$$

where $M_{(\nu \to \nu)}$ is the amplitude of the neutrino forward scattering.

The operator $\Sigma(p)$ was studied in the previous literature, for example:

- J.C. D'Olivo, J.F.Nieves, and P.B.Pal, *Phys.Rev. D40*, 3679, (1989).
- V.B. Semikoz and J.W.F.Valle, *Nucl.Phys. B425*, 651, (1994); *485* , 545 (Erratum), (1997).
- G. G. Raffelt, *Phys. Rep. 198*, 1 (1990).
- P. Elmfors, D. Grasso and G.Raffelt, *Nucl.Phys. B479*, 3, (1996).
- E. Elizalde, E.J.Ferrer, V. de la Incera, *Phys. Rev. D70*, 043012, (2004).

The additional interest to the $\Sigma(p)$ is caused by the possibility of extraction from one the data on neutrino magnetic moment.

V. Ch. Zhukovskii, T. L. Shoniya, and P. A. Aminov, Zh. Eksp. Teor. Fiz. 104 (4), 3269 (1993) [JETP 77 (4), 539 (1993)].

under physical conditions

$$E_\nu \ll \frac{m_W^2}{\mu}$$

in the strong magnetic field limit, $2eB > \mu^2 - m_e^2$,

$$\Delta\mu_\nu = \frac{16\pi^3}{3}\,\frac{n_e\,(E_\nu - q_3)}{m_W^2\,\mathbf{m}_\nu^2}\,\mu_\nu^{(0)}, \qquad \mu_\nu^{(0)} = \frac{3eG_F}{8\sqrt{2}\,\pi^2}\,\mathbf{m}_\nu.$$

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in the weak magnetic field limit, $2eB \ll \mu^2 - m_e^2$,

$$\Delta\mu_\nu = -\frac{8}{3}\,\frac{(3\,\pi^2\,n_e)^{1/3}}{(E_\nu + q_3)\,\mathbf{m}_\nu}\,\mu_\nu^{(0)}, \qquad \mu_\nu^{(0)} = \frac{3eG_F}{8\sqrt{2}\,\pi^2}\,\mathbf{m}_\nu.$$

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P. Elmfors, D. Grasso and G.Raffelt, Nucl.Phys. B479, 3, (1996):
„The use of an "effective magnetic dipole moment" to describe the neutrino energy shift in a magnetized medium is somewhat misleading..."

Additional energy and magnetic moment of neutrino

In the general case in a magnetized plasma

$$\Sigma(p) = [A_L\,(p\,\gamma) + B_L\,(u\,\gamma) + C_L\,(p\tilde{F}\gamma)]\,\gamma_L + \\ + [A_R\,(p\,\gamma) + B_R\,(u\,\gamma) + C_R\,(p\tilde{F}\gamma)]\,\gamma_R + m_\nu[K_1 + iK_2(\gamma F\gamma)],$$

where u^μ is the 4-vector of medium velocity, p^μ is the neutrino 4-momentum, $\gamma_{L,R} = (1 \pm \gamma_5)/2$, $A_R, B_R, C_R, A_L, B_L, C_L, K_1, K_2$ are the numerical coefficients, $F^{\mu\nu}$ and $\tilde{F}^{\mu\nu}$ are the tensor and dual tensor of the electro-magnetic field.

A change of the neutrino energy caused by its forward scattering in a medium can be expressed via the neutrino self-energy operator as follows

$$\Delta E_\nu = \frac{1}{4E_\nu} Sp\left\{\left((p\gamma) + m_\nu\right)\left(1 + (s\gamma)\,\gamma_5\right)\Sigma(p)\right\},$$

taking into account the general expression for $\Sigma(p)$ we can rewrite ΔE_ν in the form

$$\Delta E_\nu = \frac{m_\nu^2}{2E_\nu}(A_L + A_R + 2K_1) + B_L\,\frac{1-(\vec{\xi}\cdot\vec{v})}{2} + B_R\,\frac{1+(\vec{\xi}\cdot\vec{v})}{2}$$
$$- \frac{m_\nu}{2}(C_L - C_R + 4K_2)\left[(\vec{\xi}\cdot\vec{B}_t) + \frac{m_\nu}{E}(\vec{\xi}\cdot\vec{B}_l)\right],$$

where E_ν is the neutrino energy in a vacuum,
$\vec{\xi}$ is the double average neutrino spin vector,
$\vec{B}_l$ and $\vec{B}_t$ are the longitudinal and transverse magnetic field components relative to the direction of neutrino propagation respectively,
$\vec{v}$ is the neutrino velocity vector.

The change of neutrino energy due to presence of magnetic moment μ_ν could be found from lagrangian.

$$\Delta L_{int}^{(\mu)} = \frac{i\mu_\nu}{2} \left(\bar{\Psi} \, \sigma_{\mu\nu} \, \Psi \right) F^{\mu\nu},$$

where $\sigma_{\mu\nu} = (\gamma_\mu \, \gamma_\nu - \gamma_\nu \, \gamma_\mu)/2$.

$$\Delta E_\nu^{(\mu)} = - \int dV \, < \, \Delta L_{int}^{(\mu)} \, >$$

The contribution to neutrino energy caused by μ_ν

$$\Delta E_\nu^{(\mu)} = -\mu_\nu \left[(\vec{\xi} \cdot \vec{B}_t) + \frac{m_\nu}{E}(\vec{\xi} \cdot \vec{B}_l)\right].$$

The neutrino additional energy

$$\begin{aligned}\Delta E_\nu &= \frac{m_\nu^2}{2E_\nu}(A_L + A_R + 2K_1) + B_L \frac{1 - (\vec{\xi} \cdot \vec{v})}{2} + B_R \frac{1 + (\vec{\xi} \cdot \vec{v})}{2} \\ &- \frac{m_\nu}{2}(C_L - C_R + 4K_2)\left[(\vec{\xi} \cdot \vec{B}_t) + \frac{m_\nu}{E}(\vec{\xi} \cdot \vec{B}_l)\right],\end{aligned}$$

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$$\Delta E_\nu^{(\mu)} = -\mu_\nu \, [(\vec{\xi} \cdot \vec{B}_t) + \frac{m_\nu}{E} \, (\vec{\xi} \cdot \vec{B}_l)].$$

The neutrino additional energy

$$\begin{aligned} \Delta E_\nu &= \frac{m_\nu^2}{2E_\nu} \, (A_L + A_R + 2K_1) + B_L \, \frac{1 - (\vec{\xi} \cdot \vec{v})}{2} + B_R \, \frac{1 + (\vec{\xi} \cdot \vec{v})}{2} \\ &- \frac{m_\nu}{2} \, (C_L - C_R + 4K_2) \, [(\vec{\xi} \cdot \vec{B}_t) + \frac{m_\nu}{E} \, (\vec{\xi} \cdot \vec{B}_l)], \end{aligned}$$

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The contribution to neutrino energy caused by μ_ν

$$\Delta E_\nu^{(\mu)} = -\mu_\nu \, [(\vec{\xi} \cdot \vec{B}_t) + \frac{m_\nu}{E} \, (\vec{\xi} \cdot \vec{B}_l)].$$

The neutrino additional energy

$$\begin{aligned} \Delta E_\nu \;\; &= \;\; \frac{m_\nu^2}{2E_\nu} \, (A_L + A_R + 2K_1) + B_L \, \frac{1 - (\vec{\xi} \cdot \vec{v})}{2} + B_R \, \frac{1 + (\vec{\xi} \cdot \vec{v})}{2} \\ &- \;\; \frac{m_\nu}{2} \, (C_L - C_R + 4K_2) \, [(\vec{\xi} \cdot \vec{B}_t) + \frac{m_\nu}{E} \, (\vec{\xi} \cdot \vec{B}_l)], \end{aligned}$$

The magnetic moment of neutrino in a magnetized plasma

$$\mu_\nu = \frac{m_\nu}{2} \, (C_L - C_R + 4K_2)$$

Magnetic moment of neutrino in a magnetized plasma

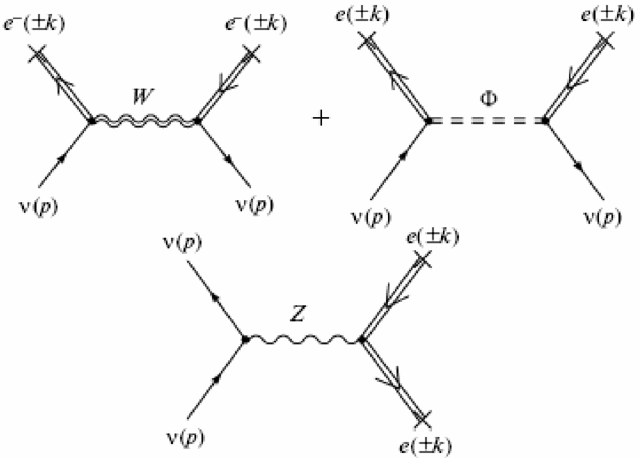
In a magnetized plasma

$$\mu_\nu = \mu_\nu^{field} + \mu_\nu^{plasma}$$

The expression for μ_ν^{field} in a broad range of neutrino energy and magnetic field

$$m_l^2/m_W^2 \ll (eB)^2 p_\perp^2/m_W^6 \ll 1$$

A.V.Kuznetsov, N.V.Mikheev, Phys. At. Nucl. 70(7),1258 (2007)



Feynman diagrams determining the contribution of a magnetized plasma to the amplitude of neutrino forward scattering. Double lines correspond to charged particles.

So,

$$\Sigma(p) = \Sigma^W(p) + \Sigma^Z(p) + \Sigma^\Phi(p)$$

We consider the physical conditions

$$m_e^2, \mu^2, T^2, eB \ll m_W^2$$

The magnetic moment of neutrino in magnetized plasma

$$\mu_\nu = \frac{m_\nu}{2} \left(C_L - C_R + 4K_2 \right)$$

The contribution from W-boson

$$C_L^W = -\frac{e\,G_F}{\sqrt{2}\,\pi^2\,E}\int\limits_0^{+\infty} dk\,(f(\omega_0) - \tilde{f}(\omega_0)),\; C_R^W = K_2^W = 0,$$

The contribution from Φ-boson

$$C_L^\Phi = -\frac{m_e^2}{2m_W^2}\,C_L^W,\;\; C_R^\Phi = -\frac{m_\nu^2}{2m_W^2}\,C_L^W,$$

$$K_2^\Phi = -\frac{e\,G_F}{4\sqrt{2}\pi^2}\,\frac{m_e^2}{m_W^2}\int\limits_0^{+\infty}\frac{dk}{\omega_0}\,(f(\omega_0) - \tilde{f}(\omega_0)).$$

The contribution from Z-boson

$$C_L^Z = -\frac{e\,G_F}{2\sqrt{2}\,\pi^2\,E}\int\limits_0^{+\infty} dk\,(f(\omega_0) - \tilde{f}(\omega_0)),\; C_R^Z = K_2^Z = 0.$$

Finally, for the neutrino magnetic moment one have obtained

$$\mu_\nu \simeq \frac{m_\nu}{2}\, C_L + \mu_\nu^{field} = \frac{m_\nu}{2}\left(C_L^W + C_L^Z\right) + \mu_\nu^{field}$$

$$\mu_\nu \simeq \frac{3eG_F m_\nu}{8\sqrt{2}\,\pi^2}\left(1 \mp \frac{2}{3\,E}\int\limits_0^{+\infty} dk\,(f(\omega_0) - \tilde{f}(\omega_0))\right)$$

Where „-“ corresponds to the ν_e, „+“ corresponds to the ν_μ, ν_τ.

In the ultrarelativistic plasma

$$\mu_\nu = \frac{C_L\, m_\nu}{2} \simeq \frac{3e\, G_F\, m_\nu}{8\sqrt{2}\,\pi^2}\left(1 \mp \frac{2}{3}\,\frac{\mu}{E}\right)$$

In the charge symmetric plasma

$$\mu_{\nu_e} \simeq \frac{3e\, G_F\, m_\nu}{8\sqrt{2}\,\pi^2}\left(1 + \frac{4\pi^2}{9}\,\frac{T^2}{m_W^2}\right)$$

Conclusions

- The influence of a magnetized plasma on the neutrino self-energy operator was studied.
- It was shown that only a part of the additional neutrino energy depending on its spin and magnetic field, relates to neutrino magnetic moment.
- It was found that presence of a magnetized plasma does not lead to enhancement of the neutrino magnetic moment in contrast to results in previous literature. Moreover, the magnetic moment of the neutrino is suppressed by its mass, while in a charge symmetric plasma the magnetic moment it is additionally suppressed by a factor of $T/m_W \ll 1$.