Gauge invariant approach to anomalous conformal currents and shadow fields

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Plan

1) **Introduction**

2) **Modified (Lorentz) de Donder gauge and computation of two-point functions from AdS**

3) **Gauge invariant approach to anomalous conformal currents and shadow fields**

$\mathbf{S_{AdS}(\Phi)}$

$\Phi = \phi$ scalar

ϕ^A vector

ϕ^{AB} tensor

$\phi^{A_1 \ldots A_s}$ arbitrary spin

fields in AdS space

AdS_{d+1}

$$ds^2 = \frac{R^2}{z^2}(dx^a dx^a + dz dz)$$

$\mathbf{x^a}$ boundary flat coordinates

$\mathbf{z}$ radial coordinate

$R = 1$

bulk $\mathbf{so(d,1)}$ $\longrightarrow$ boundary $\mathbf{so(d-1,1)}$

$$\frac{\delta S_{AdS}}{\delta \Phi} = 0$$

$$\Phi(x,z) \sim z^{\Delta} \Phi_{\text{cur}}(x)$$

$$\Phi(x,z) \sim z^{-\Delta} \Phi_{\text{sh}}(x)$$

$$\Delta = \frac{d}{2} + \sqrt{m^2 + (s + \frac{d-4}{2})^2}$$

$$\Delta_{\Phi_{\text{sh}}} = d - \Delta$$

$m \neq 0$

$\boldsymbol{\Phi}_{\mathsf{cur}}(\mathbf{x})$ anomalous currents

$\boldsymbol{\Phi}_{\mathsf{sh}}(\mathbf{x})$ anomalous shadow fields

$$S_{AdS}(\Phi) \equiv S_{eff}(\Phi_{\mathsf{sh}})$$

$$\langle \Phi_{\mathsf{cur}}(x_1) \ldots \Phi_{\mathsf{cur}}(x_n) \rangle$$

$$= \frac{\delta^n S_{eff}}{\delta \Phi_{\mathsf{sh}}(x_1) \ldots \delta \Phi_{\mathsf{sh}}(x_n)}$$

CFT

$\phi_{\mathbf{SYM}}$ fields of boundary conformal theory, e.g. SYM

$$S(\phi_{\mathbf{SYM}})$$

$$\Phi_{\mathsf{cur}} = \Phi_{\mathsf{cur}}(\phi_{\mathbf{SYM}})$$

$$\mathbf{V} = \int d^d x \; \mathbf{\Phi}_{\mathsf{sh}}(\mathbf{x}) \mathbf{\Phi}_{\mathsf{cur}}(\mathbf{x})$$

$$e^{-\mathbf{S_{cft}}} = \int D\phi_{\mathbf{SYM}} e^{-S(\phi_{\mathbf{SYM}})+\mathbf{V}}$$

AdS/CFT

$$S_{eff}(\Phi_{sh}) \stackrel{?}{=} S_{cft}(\Phi_{sh})$$

Goal

Find

$S_{\text{eff}}(\Phi_{\text{sh}})$

for massive arbitrary spin fields

by using AdS

scalar

$$S = \int d^d x dz \mathcal{L}$$

$$\mathcal{L} = \frac{1}{2}\sqrt{g}(g^{\mu\nu}\partial_\mu \Phi \partial_\nu \Phi + m^2 \Phi^2)$$

$$\Phi = z^{\frac{d-1}{2}} \phi$$

scalar

$$\mathcal{L} = \frac{1}{2}|\partial^{\mathbf{a}}\phi|^{\mathbf{2}} + \frac{1}{2}|\mathcal{T}_\nu\phi|^{\mathbf{2}}$$

$$\mathcal{T}_\nu \equiv \partial_z + \frac{\nu}{z}$$

$$\nu = \sqrt{m^2 + \frac{d^2}{4}}$$

scalar

Solution to Dirichlet problem

$$\left(\Box + \partial_{\mathbf{z}}^{\mathbf{2}} - \frac{\nu^2}{z^2}\right)\phi = \mathbf{0}$$

$$\phi(\mathbf{x},\mathbf{z}) \quad \xrightarrow{z\longrightarrow 0} \quad \mathbf{z}^{-\nu+\frac{1}{2}}\ \phi_{\mathsf{sh}}(\mathbf{x})$$

$$\phi(\mathbf{x},\mathbf{z}) \quad \xrightarrow{z\longrightarrow\infty} \quad 0$$

scalar

Solution to Dirichlet problem

$$\phi(\mathbf{x}, \mathbf{z}) = \int d^d y \, \mathbf{G}_\nu(\mathbf{x} - \mathbf{y}, \mathbf{z}) \phi_{\mathsf{sh}}(\mathbf{y})$$

$$\mathbf{G}_\nu(\mathbf{x}, \mathbf{z}) = \frac{z^{\nu + \frac{1}{2}}}{(z^2 + |x|^2)^{\nu + \frac{d}{2}}}$$

scalar

$$S_{\mathsf{eff}} = \int d^d x \, \mathcal{L}_{\mathsf{eff}}|_{z \to 0}$$

$$\mathcal{L}_{\mathsf{eff}} = \phi \mathcal{T}_\nu \phi$$

$$\mathcal{T}_\nu \equiv \partial_z + \frac{\nu}{z}$$

scalar

Effective action

$$S_{\text{eff}} = \int d^d x_1 d^d x_2 \, \boldsymbol{\Gamma_{12}}$$

$$\boldsymbol{\Gamma_{12}} = \frac{\phi_{\text{sh}}(x_1)\phi_{\text{sh}}(x_2)}{|x_{12}|^{2\nu+d}}$$

$$|x_{12}|^2 = (x_1 - x_2)^a (x_1 - x_2)^a$$

massive spin-1

Field content: $\phi^{\mathbf{A}}, \quad \phi$

$$\mathcal{L} = -\frac{1}{4} F^{AB} F^{AB}$$

$$+ \frac{1}{2}(m\phi^A + \partial^A \phi)^2$$

$$F^{AB} = D^A \phi^B - D^B \phi^A$$

$$\delta\phi^A = \partial^A \xi \,, \qquad \delta\phi = -m\xi$$

massive spin-1

bulk		boundary
$\mathbf{so(d,1)}$	$\longrightarrow$	$\mathbf{so(d-1,1)}$
$\phi^{\mathbf{A}}$	$\longrightarrow$	$\phi^{\mathbf{a}} \oplus \phi^{\mathbf{z}}$
ϕ	$\longrightarrow$	ϕ

tower of so(d-1,1) fields

$$\phi^{\mathbf{a}}$$

$$\phi^{\mathbf{z}} \qquad\qquad \phi$$

massive spin-1

standard Lorentz gauge

$$D^A \phi^A + m\phi = 0$$

leads to coupled equations

$$(\Box + \partial_z^2 - \frac{m_1^2}{z^2})\phi^{\mathbf{a}} + \partial^a \phi^{\mathbf{z}} = 0$$

$$(\Box + \partial_z^2 - \frac{m_2^2}{z^2})\phi^{\mathbf{z}} + m\phi = 0$$

$$(\Box + \partial_z^2 - \frac{m_3^2}{z^2})\phi + \phi^{\mathbf{z}} = 0$$

massive spin-1

Modified Lorentz gauge

$$\mathbf{D^A}\phi^{\mathbf{A}} + \mathbf{m}\phi + \frac{2}{R}\phi^{\mathbf{z}} = 0$$

RRM, 2009

gives

Decoupled equations

massive spin-1

$$\phi_{\mathbf{1}} = \mathrm{v}_{11}\phi^{\mathbf{z}} + \mathrm{v}_{12}\phi$$

$$\phi_{-\mathbf{1}} = \mathrm{v}_{21}\phi^{\mathbf{z}} + \mathrm{v}_{22}\phi$$

Orthogonal transformation

massive spin-1
Decoupled equations

$$(\Box + \partial_z^2 - \frac{{\nu_0}^2}{z^2})\phi^a = 0$$

$$(\Box + \partial_z^2 - \frac{{\nu_1}^2}{z^2})\phi_1 = 0$$

$$(\Box + \partial_z^2 - \frac{{\nu_{-1}}^2}{z^2})\phi_{-1} = 0$$

$$\nu_0 = \sqrt{m^2 + \frac{(d-2)^2}{4}}$$

$$\nu_1 = \nu_0 + 1\,, \qquad \nu_{-1} = \nu_0 - 1$$

massive spin-1

Solution to Dirichlet problem

$$\phi^a(x,z) = \int d^d y \mathbf{G}_{\nu_{\mathbf{0}}}(\mathbf{x}-\mathbf{y},\mathbf{z})\ \phi^a_{\mathsf{sh}}(y)$$

$$\phi_1(x,z) = \int d^d y \mathbf{G}_{\nu_{\mathbf{1}}}(\mathbf{x}-\mathbf{y},\mathbf{z})\ \phi_{\mathsf{sh},-1}(y)$$

$$\phi_{-1}(x,z) = \int d^d y \mathbf{G}_{\nu_{-\mathbf{1}}}(\mathbf{x}-\mathbf{y},\mathbf{z})\ \phi_{\mathsf{sh},1}(y)$$

$$\mathbf{G}_\nu(\mathbf{x},\mathbf{z}) \equiv \frac{z^{\nu+1/2}}{(z^2+|x|^2)^{\nu+\frac{d}{2}}}$$

massive spin-1

Conformal dimensions

$$\Delta(\phi^a_{\mathsf{sh}}) = \frac{d}{2} - \nu_0$$

$$\Delta(\phi_{\mathsf{sh},1}) = \frac{d}{2} - \nu_0 + 1$$

$$\Delta(\phi_{\mathsf{sh},-1}) = \frac{d}{2} - \nu_0 - 1$$

massive spin-1

$$S_{\text{eff}} = \int d^d x \, \mathcal{L}_{\text{eff}}|_{z\to 0}$$

$$\mathcal{L}_{\text{eff}} = \phi^a \mathcal{T}_{\nu_{\mathbf{0}}} \phi^a + \phi_1 \mathcal{T}_{\nu_{\mathbf{1}}} \phi_1 + \phi_{-1} \mathcal{T}_{\nu_{-\mathbf{1}}} \phi_{-1}$$

$$\mathcal{T}_\nu = \partial_z + \frac{\nu}{z}$$

massive spin-1

Effective action

$$S_{\text{eff}} = \int d^d x_1 d^d x_2 \mathbf{\Gamma_{12}}$$

$$\Gamma_{12} = \frac{\phi^{\mathbf{a}}_{\mathsf{sh}}(\mathbf{x_1})\phi^{\mathbf{a}}_{\mathsf{sh}}(\mathbf{x_2})}{|x_{12}|^{2\nu_{\mathbf{0}}+d}}$$

$$+ \frac{\phi_{\mathsf{sh},\mathbf{1}}(\mathbf{x_1})\phi_{\mathsf{sh},\mathbf{1}}(\mathbf{x_2})}{|x_{12}|^{2\nu_{-\mathbf{1}}+d}}$$

$$+ \frac{\phi_{\mathsf{sh},-\mathbf{1}}(\mathbf{x_1})\phi_{\mathsf{sh},-\mathbf{1}}(\mathbf{x_2})}{|x_{12}|^{2\nu_{\mathbf{1}}+d}}$$

$$\nu_{\mathbf{0}} = \sqrt{m^2 + \tfrac{(d-2)^2}{4}}$$

$$\nu_{\mathbf{1}} = \nu_0 + 1\,, \qquad \nu_{-\mathbf{1}} = \nu_0 - 1$$

Gauge symmetries

of

effective action

and differential constraints

for

shadow field

Modified Lorentz gauge for bulk AdS fields

$$\mathbf{D^A\phi^A + m\phi + 2\phi^z = 0}$$

leads to

differential constraint for shadow fields

$$\partial^{\mathbf{a}}\phi^{\mathbf{a}}_{\mathsf{sh}} + \phi_{\mathsf{sh},1} + \Box\phi_{\mathsf{sh},-1} = 0$$

Gauge symmetry of differential constraint

$$\delta\phi^a_{\mathsf{sh}} = \partial^a \xi_{\mathsf{sh}}$$

$$\delta\phi_{\mathsf{sh},1} = \Box\xi_{\mathsf{sh}}$$

$$\delta\phi_{\mathsf{sh},-1} = \xi_{\mathsf{sh}}$$

$\phi_{\mathsf{sh},-1}$ Goldstone field

Modified Lorentz gauge for bulk AdS fields

$$\mathbf{D^A \phi^A + m\phi + 2\phi^z = 0}$$

has left-over gauge symmetry

$$\delta\phi^A = \partial^A \xi\,, \qquad \delta\phi = -m\xi$$

$$(\Box + \partial_z^2 - \frac{\nu_{\mathbf{0}}{}^2}{z^2})\xi = 0$$

$$\xi(\mathbf{x}, \mathbf{z}) = \int d^d y G_{\nu_{\mathbf{0}}}(x - y, z)\ \xi^{\mathbf{a}}_{\mathsf{sh}}(\mathbf{y})$$

Use of gauge conditions

for shadow fields

may be useful for various

applications

Light-cone gauge may be useful

in AdS superstring theory

Light-cone frame

$$x^a = x^+, x^-, x^i\,, \qquad i = 1, \ldots d-2$$

$$x^\pm = x^{d-1} \pm x^0$$

$$\phi^a = \phi^+\,, \phi^-\,, \phi^i$$

$$\phi^+_{\mathsf{sh}} = 0 \qquad \text{light-cone gauge}$$

Solution to differential constraint

$$\phi^-_{\mathsf{sh}} = -\frac{\partial^j}{\partial_-}\phi^j_{\mathsf{sh}} - \frac{1}{\partial_-}\phi_{\mathsf{sh},1} - \frac{\Box}{\partial_-}\phi_{\mathsf{sh},-1}$$

Light-cone gauge fixed S_{eff}

$$\mathbf{S}_{\text{eff}}^{\text{light-cone}} = \int d^d x_1 d^d x_2 \; \mathbf{\Gamma}_{\mathbf{12}}^{\text{light-cone}}$$

$$\Gamma_{12}^{\text{light-cone}} = \frac{\phi^i_{\text{sh}}(x_1)\phi^i_{\text{sh}}(x_2)}{|\mathbf{x_{12}}|^{\mathbf{2\nu_0+d}}}$$

$$+ \frac{\phi_{\text{sh},1}(x_1)\phi_{\text{sh},1}(x_2)}{|\mathbf{x_{12}}|^{\mathbf{2\nu_{-1}+d}}}$$

$$+ \frac{\phi_{\text{sh},-1}(x_1)\phi_{\text{sh},-1}(x_2)}{|\mathbf{x_{12}}|^{\mathbf{2\nu_1+d}}}$$

ϕ^i_{sh} , $\phi_{\text{sh},1}$, $\phi_{\text{sh},-1}$ unconstrained fields

massive spin-2

Field content

so(d,1) fields

$$\mathbf{\Phi^{AB}}, \quad \mathbf{\Phi^{A}}, \quad \mathbf{\Phi}$$

Use **gauge invariant approach**

$$\mathcal{L} = \mathbf{\Phi^{AB}} \Box_{AdS} \mathbf{\Phi^{AB}} + \mathbf{\Phi^{A}} \Box_{AdS} \mathbf{\Phi^{A}} + \mathbf{\Phi} \Box_{AdS} \mathbf{\Phi} + \ldots$$

$$\delta \mathbf{\Phi^{AB}} = D^A \xi^{\mathbf{B}} + D^B \xi^{\mathbf{A}} + \ldots$$

$$\delta \mathbf{\Phi^{A}} = D^A \xi + \ldots$$

$$\delta \mathbf{\Phi} = \ldots$$

massive spin-2

modified de Donder gauge

$$\mathbf{D^B\Phi^{AB} - \frac{1}{2}D^A\Phi^{CC} + m\Phi^A}$$

$$\mathbf{+2\Phi^{zA} - \eta^{zA}\Phi^{CC}} = 0$$

$$\mathbf{D^A\Phi^A + m\Phi + \Phi^{AA}}$$

$$\mathbf{+2\Phi^z} = 0$$

leads to **decoupled** equations

$$\mathbf{so(d,1)} \qquad\qquad \mathbf{so(d-1,1)}$$

$$\mathbf{\Phi^{AB}} \quad = \quad \mathbf{\Phi^{ab}} \oplus \mathbf{\Phi^{za}} \oplus \mathbf{\Phi^{zz}}$$

$$\mathbf{\Phi^{A}} \quad = \quad \mathbf{\Phi^{a}} \oplus \mathbf{\Phi^{z}}$$

$$\mathbf{\Phi} \quad = \quad \mathbf{\Phi}$$

tower of $so(d-1,1)$ tensor,vector, scalar fields

$$\mathbf{\Phi^{ab}}$$

$$\mathbf{\Phi^{za}} \qquad \mathbf{\Phi^{a}}$$

$$\mathbf{\Phi^{zz}} \qquad \mathbf{\Phi^{z}} \qquad \mathbf{\Phi}$$

$$\phi^{\mathbf{ab}} \equiv \mathbf{\Phi}^{\mathbf{ab}} + \eta^{ab}(\mathbf{\Phi}^{\mathbf{zz}} + \mathbf{\Phi}^{\mathbf{z}} + \mathbf{\Phi})\,,$$

$$\begin{pmatrix} \phi_{\mathbf{1}}^{\mathbf{a}} \\ \\ \phi_{\mathbf{-1}}^{\mathbf{a}} \end{pmatrix} = V_{2\times 2} \begin{pmatrix} \mathbf{\Phi}^{\mathbf{za}} \\ \\ \mathbf{\Phi}^{\mathbf{a}} \end{pmatrix}$$

$$\begin{pmatrix} \phi_{\mathbf{2}} \\ \\ \phi_{\mathbf{0}} \\ \\ \phi_{\mathbf{-2}} \end{pmatrix} = V_{3\times 3} \begin{pmatrix} \mathbf{\Phi}^{\mathbf{zz}} \\ \\ \mathbf{\Phi}^{\mathbf{z}} \\ \\ \mathbf{\Phi} \end{pmatrix}$$

Orthogonal transformations

$$\phi^{\mathbf{ab}}$$

$$\phi^{\mathbf{a}}_{\mathbf{-1}} \qquad \phi^{\mathbf{a}}_{\mathbf{1}}$$

$$\phi_{\mathbf{-2}} \qquad \phi_{\mathbf{0}} \qquad \phi_{\mathbf{2}}$$

Decoupled equations

$$(\Box + \partial_z^2 - \frac{\nu_\lambda^{\mathbf{2}}}{z^2})\phi_\lambda^{\mathbf{ab}} = 0\,, \qquad \lambda = 0$$

$$(\Box + \partial_z^2 - \frac{\nu_\lambda^{\mathbf{2}}}{z^2})\phi_\lambda^{\mathbf{a}} = 0\,, \qquad \lambda = \pm 1$$

$$(\Box + \partial_z^2 - \frac{\nu_\lambda^{\mathbf{2}}}{z^2})\phi_\lambda = 0\,, \qquad \lambda = 0, \pm 2$$

$$\nu_\lambda = \sqrt{m^2 + \frac{d^2}{4} + \lambda}$$

$$\mathbf{S}_{\text{eff}} = \int d^d x \, \mathcal{L}_{\text{eff}}|_{\mathbf{z}\to\mathbf{0}}$$

$$\begin{aligned}
\mathcal{L}_{\text{eff}} &= \phi^{ab}\mathcal{T}_{\nu_{\mathbf{0}}}\phi^{ab} \\
&+ \phi^a_{-1}\mathcal{T}_{\nu_{-\mathbf{1}}}\phi^a_{-1} + \phi^a_1\mathcal{T}_{\nu_{\mathbf{1}}}\phi^a_1 \\
&+ \phi_{-2}\mathcal{T}_{\nu_{-\mathbf{2}}}\phi + \phi_0\mathcal{T}_{\nu_{\mathbf{0}}}\phi_0 + \phi_2\mathcal{T}_{\nu_{\mathbf{2}}}\phi_2
\end{aligned}$$

$$\mathcal{T}_\nu = \partial_z + \frac{\nu}{z}$$

$$\nu_\lambda = \sqrt{m^2 + \frac{d^2}{4} + \lambda}$$

massive spin-2

Solution to Dirichlet problem

$$\phi^{ab}(x,z) = \int d^d y \mathbf{G}_{\nu_{\mathbf{0}}}(\mathbf{x}-\mathbf{y},\mathbf{z})\ \phi^{ab}_{\mathsf{sh}}(y)$$

$$\phi^{a}_{\lambda}(x,z) = \int d^d y \mathbf{G}_{\nu_{\lambda}}(\mathbf{x}-\mathbf{y},\mathbf{z})\ \phi^{a}_{\mathsf{sh},-\lambda}(y)$$

$$\phi_{\lambda}(x,z) = \int d^d y \mathbf{G}_{\nu_{\lambda}}(\mathbf{x}-\mathbf{y},\mathbf{z})\ \phi_{\mathsf{sh},-\lambda}(y)$$

$$\mathbf{G}_{\nu}(\mathbf{x},\mathbf{z}) \equiv \frac{z^{\nu+1/2}}{(z^2+|x|^2)^{\nu+\frac{d}{2}}}$$

massive spin-2

Effective action

$$S_{\text{eff}} = \int d^d x_1 d^d x_2 \; \mathbf{\Gamma_{12}}$$

$$\Gamma_{12} = \frac{\phi^{ab}_{\text{sh}}(x_1)\phi^{ab}_{\text{sh}}(x_2)}{|\mathbf{x_{12}}|^{\mathbf{2\nu_0+d}}}$$

$$+ \sum_{\lambda=\pm 1} \frac{\phi^{a}_{\text{sh},\lambda}(x_1)\phi^{a}_{\text{sh},\lambda}(x_2)}{|\mathbf{x_{12}}|^{\mathbf{2\nu_{-\lambda}+d}}}$$

$$+ \sum_{\lambda=0,\pm 2} \frac{\phi_{\text{sh},\lambda}(x_1)\phi_{\text{sh},\lambda}(x_2)}{|\mathbf{x_{12}}|^{\mathbf{2\nu_{-\lambda}+d}}}$$

1)

Modified de Donder gauge for bulk AdS fields

leads

differential constraints for shadow fields

2)

On-shell left-over gauge symmetries

of bulk AdS fields

lead

to gauge symmetries of shadow fields

Arbitrary spin-s AdS field

Field content : $so(d,1)$ **tensor fields**

$$\Phi^{A_1\ldots A_s}$$

$$\Phi^{A_1\ldots A_{s-1}}$$

$$\Phi^{A_1\ldots A_{s-2}}$$

$$\cdots\cdots$$

$$\cdots\cdots$$

$$\Phi^{A_1 A_2}$$

$$\Phi^{A_1}$$

$$\Phi$$

Zinoviev 2001

Impose modified de Donder gauge

$$\mathbf{D^A\Phi^{AA_2\ldots A_s} - \frac{1}{2}D^{A_2}\Phi^{AAA_3\ldots A_s}}$$

$$\mathbf{+\, m\Phi^{A_2\ldots A_{s-1}}}$$

$$\mathbf{+\, 2\Phi^{zA_2\ldots A_s} - \eta^{zA_2}\Phi^{AAA_3\ldots A_s} = 0}$$

Decompose $\quad \mathbf{so(d,1) \longrightarrow so(d-1,1)}$

$$\mathbf{\Phi^{A_1\ldots A_s} = \Phi^{a_1\ldots a_s}}$$

$$\mathbf{\Phi^{a_1\ldots a_{s-1}}}$$

$$\cdots\cdots$$

$$\mathbf{\Phi^{a_1a_2}}$$

$$\mathbf{\Phi^{a_1}}$$

$$\mathbf{\Phi}$$

$$\phi_{\lambda}^{a_1 \ldots a_{\mathbf{s}'}}$$

$$\mathbf{s}' = \mathbf{0}, \mathbf{1}, \mathbf{2}, \ldots \mathbf{s} - \mathbf{1}, \mathbf{s}$$

$$\lambda = -\mathbf{s}', -\mathbf{s}' + \mathbf{2}, -\mathbf{s}' + \mathbf{4}, \ldots, \mathbf{s}' - \mathbf{4}, \mathbf{s}' - \mathbf{2}, \mathbf{s}'$$

$so(d-1,1)$ **tensor fields**

$$\phi_{(\mathbf{s}',\lambda)} \sim \phi_\lambda{}^{\mathbf{a_1\ldots a_{s'}}}$$

$$\phi_{(\mathbf{s},\mathbf{0})}$$

$$\phi_{(\mathbf{s-1},\mathbf{-1})} \qquad \phi_{(\mathbf{s-1},\mathbf{1})}$$

$$\cdots \qquad \cdots \qquad \cdots$$

$$\phi_{(\mathbf{1},\mathbf{1-s})} \qquad \phi_{(\mathbf{1},\mathbf{3-s})} \qquad \cdots \qquad \phi_{(\mathbf{1},\mathbf{s-3})} \qquad \phi_{(\mathbf{1},\mathbf{s-1})}$$

$$\phi_{(\mathbf{0},\mathbf{-s})} \qquad \phi_{(\mathbf{0},\mathbf{2-s})} \qquad \cdots \qquad \phi_{(\mathbf{0},\mathbf{s-2})} \qquad \phi_{(\mathbf{0},\mathbf{s})}$$

arbitrary spin-s

Decoupled equations

$$(\Box + \partial_z^2 - \frac{\nu_\lambda^2}{z^2})\phi_\lambda^{a_1\ldots a_{s'}} = 0$$

$$\nu_\lambda = \sqrt{m^2 + (s + \frac{d-4}{2})^2 + \lambda}$$

$$\phi_\lambda^{a_1\ldots a_{s'}}(x,z) = \int d^d y \, G_{\nu_\lambda}(x-y,z)\phi_{sh,-\lambda}^{a_1\ldots a_{s'}}(y)$$

$$S_{\mathsf{eff}} = \int dx_1^d dx_2^d \, \mathbf{\Gamma_{12}}$$

$$\mathbf{\Gamma_{12}} = \sum_{s',\lambda} \mathbf{\Gamma_{12}^{(s',\lambda)}}$$

$$\mathbf{\Gamma_{12}^{(s',\lambda)}} = \frac{\phi_{\mathsf{sh},\lambda}^{\mathbf{a_1 \ldots a_{s'}}} \phi_{\mathsf{sh},\lambda}^{\mathbf{a_1 \ldots a_{s'}}}}{|x_{12}|^{2\nu_{-\lambda}+d}}$$

Light-cone gauge

$$S_{\text{eff}}^{\mathbf{LC}} = \int dx_1^d dx_2^d \, \mathbf{\Gamma}_{\mathbf{12}}^{\mathbf{LC}}$$

$$\mathbf{\Gamma}_{\mathbf{12}}^{\mathbf{LC}} = \sum_{s',\lambda} \mathbf{\Gamma}_{\mathbf{12}}^{(\mathbf{s}',\lambda)\mathbf{LC}}$$

$$\mathbf{\Gamma}_{\mathbf{12}}^{(\mathbf{s}',\lambda)\mathbf{LC}} = \frac{\phi_{\text{sh},\lambda}^{\mathbf{i_1 \ldots i_{s'}}} \phi_{\text{sh},\lambda}^{\mathbf{i_1 \ldots i_{s'}}}}{|x_{12}|^{2\nu_{-\lambda}+d}}$$

applications to superstring theory in $AdS_5 \times S^5$???

“technical” problem with standard cov. gauges, Lorentz, de Donder

1) Coupled equations

2) For spin 2, 3, 4, ...

solutions are expressible

in terms of Heyn functions

Little is known about Heun functions

asymptotic behavior ???

recurrent relations ???

Spin-1 conformal current
Standard approach

$$T^a \quad - \quad conformal \quad current$$

$$\partial^a T^a = 0$$

Conformal dimension

$$\Delta = d - 1$$

Spin-1 conformal current. Gauge inv. approach

$$\phi^a_{cur} \qquad \phi_{cur}$$

$$\mathbf{T^a = \phi^a_{cur} + \partial^a \phi_{cur}}$$

$$\partial^a T^a = 0$$

$$\partial^{\mathbf{a}} \phi^{\mathbf{a}}_{\mathbf{cur}} + \Box \phi_{\mathbf{cur}} = \mathbf{0}$$

$$\delta \phi^a_{cur} = \partial^a \xi$$

$$\delta \phi_{cur} = -\xi$$

Conformal dimensions

$$\Delta(T^a) = d - 1$$

$$T^a = \phi^a_{cur} + \partial^a \phi_{cur}$$

$$\mathbf{\Delta(\phi^a_{cur}) = d - 1}$$

$$\mathbf{\Delta(\phi_{cur}) = d - 2}$$

$$\phi^a(x,z) = \mathbf{U}_{\nu_1}\ \phi^a_{cur}(x)$$

$$\phi(x,z) = \mathbf{U}_{\nu_0}\ \phi_{cur}(x)$$

$$\mathbf{U}_\nu \equiv \sqrt{z}\mathbf{J}_\nu(\mathbf{z}\sqrt{\Box})$$

Bessel

modified Lorentz gauge

$$\mathbf{D^A \phi^A + 2\phi = 0}$$

leads to differential constraint for currents

$$\mathbf{D^A \phi^A + 2\phi}$$

$$= U_{\nu_1}(\partial^{\mathbf{a}} \phi^{\mathbf{a}}_{\mathbf{cur}} + \Box \phi_{\mathbf{cur}})$$

Spin-2 conformal current (energy-momentum tensor) Standard approach

$$\mathbf{T^{ab}} \qquad - \mathit{spin\ 2\ conformal\ \ current}$$

$$\partial^{\mathbf{a}}\mathbf{T^{ab}} = \mathbf{0}$$

$$\mathbf{T^{aa}} = \mathbf{0}$$

Conformal dimension

$$\mathbf{\Delta} = \mathbf{d}$$

Spin-2 current. Gauge inv. approach

Fields	$Conf.dim$
$\phi_{\mathbf{cur}}^{\mathbf{ab}}$	d
$\phi_{\mathbf{cur}}^{\mathbf{a}}$	$d-1$
$\phi_{\mathbf{cur}}$	$d-2$

Spin 2. Currents. Differential constraints

$$\partial^b \phi^{ab}_{cur} + \partial^a \phi^{bb}_{cur} + \Box \phi^a_{cur} = 0$$

$$\partial^a \phi^a_{cur} + \phi^{aa}_{cur} + \Box \phi_{cur} = 0$$

ϕ^a_{cur}, ϕ_{cur} **can be gauged away**

$$\partial^b \phi^{ab}_{cur} = 0$$

$$\phi^{aa}_{cur} = 0$$

Spin 2. Currents. Gauge transformations

$$\delta\phi^{ab}_{cur} = \partial^a\xi^b_{cur} + \partial^b\xi^a_{cur} + \eta^{ab}\Box\xi_{cur}$$

$$\delta\phi^{a}_{cur} = \partial^a\xi_{cur} + \xi^a_{cur}$$

$$\delta\phi_{cur} = \xi_{cur}$$

ϕ^a_{cur}, ϕ_{cur} **Stueckelberg fields**

summary of our study of AdS/CFT

1) Bulk fields are taken in modified de-Donder gauge

2) Modified de Donder gauge leads to decoupled equations of motion with on-shell leftover gauge symmetries

3) normalizable solutions → currents

non-normalizable solutions → shadows

Our currents and shadows

correspond to

bulk AdS fields taken

in modified de Donder gauge

4) leftover gauge symmetries

of bulk fields correspond to gauge

symmetries of boundary currents

and shadow fields,

5) Modified de Donder gauge for bulk fields

corresponds to differential constraints

for boundary conformal currents and shadows

5)

AdS field action

(Effective action)

evaluated on the solution to

Dirichlet problem

is equal to

two-point vertex of shadow field

Conclusions

1) Gauge invariant approach to
currents and shadows
give possibility to choice

various gauges which might
be helpful in applications

Standard currents are obtained
via Stueckelberg gauge fixing.

But others gauges
might also be interesting

2)

modified de Donder gauge leads to decoupled equations

and might be helpful for study

AdS/CFT

AdS/QCD

quantization of higher-spin AdS fields

3)

3-point 4-point gauge invariant

vertices of CFT from interacting

theory of massive higher-spin fields

string theory in $AdS_5 \times S^5$???