Strong versus Weak Coupling Confinement in Supersymmetric QCD

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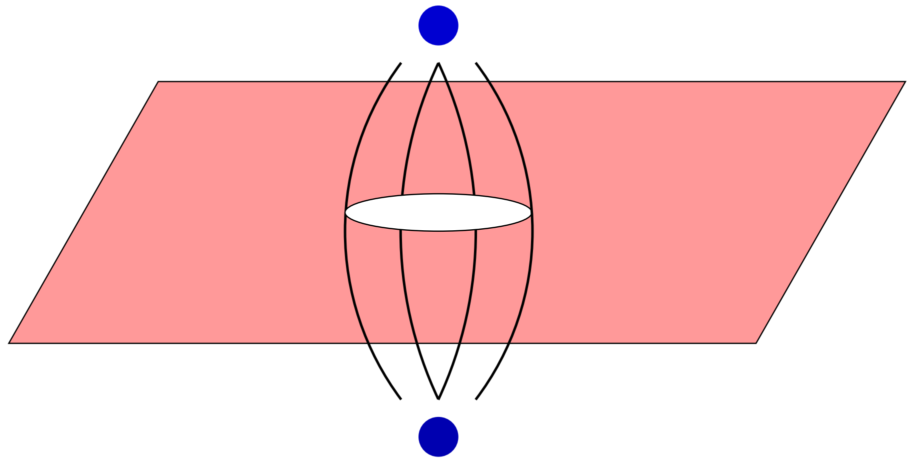
based on

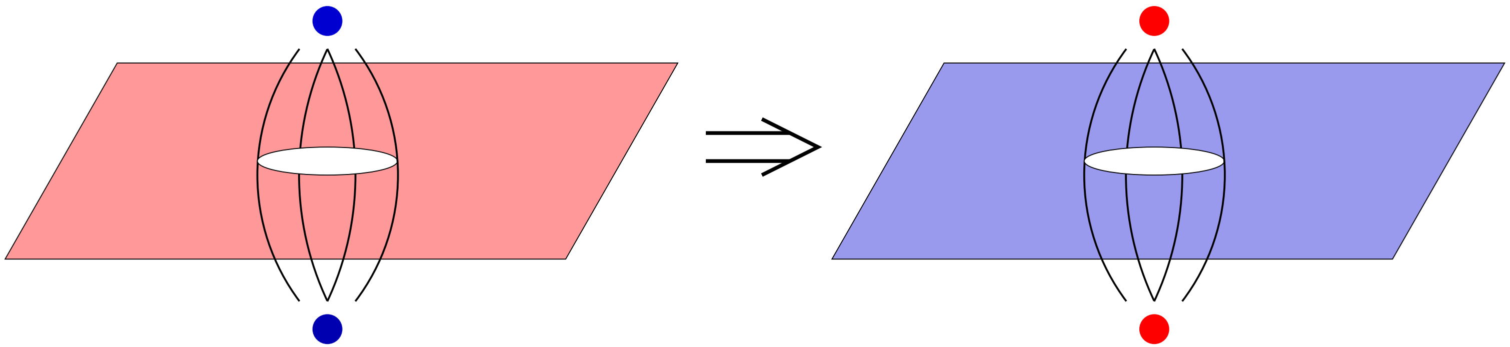
A.M. & Alexei Yung, Nucl. Phys. **B647** (2002) 3;
A.M. & Alexei Yung, Nucl. Phys. **B831** (2010) 72;
A.M., *Period Integrals, Quantum Numbers and Confinement in SUSY QCD*, arXiv:1003.2089

A scenario of confinement:
Polyakov, Mandelstam, 't Hooft, . . . Meissner mechanism in superconductor: condensation of electric charge (red) kills magnetic field except for a tube, ensuring confinement of magnetic monopoles (blue).



(Relativistic version of) Landau-Ginzburg theory: the Abelian Higgs model - a part of generic Yang-Mills-Higgs system

To turn into problem of particle QFT one needs duality between electric and magnetic charges.



Does it give confinement of quarks?
Can it give confinement with non-Abelian features (doublets, generally multiplets, of the confined objects)?

Dual picture: condensation of monopoles !?

- Monopoles do not exist (or at least *classical* monopoles are heavy $m_M \sim \frac{m_W}{g^2}$ in the weakly-coupled world?);

- Classical monopole solutions exist in the non-Abelian theories with extra *scalar* fields;

- Who are the quantum monopoles? Can they be light? Can they condense?

Partial answer in SUSY gauge theories . . .

Realization of scenario:
Supersymmetric QCD ($\mathcal{N} = 2$ softly broken to $\mathcal{N} = 1$).

- Adjoint vector multiplet: $A = A_\mu dx^\mu$, Φ, ...
  Gauge field necessarily requires adjoint scalar!

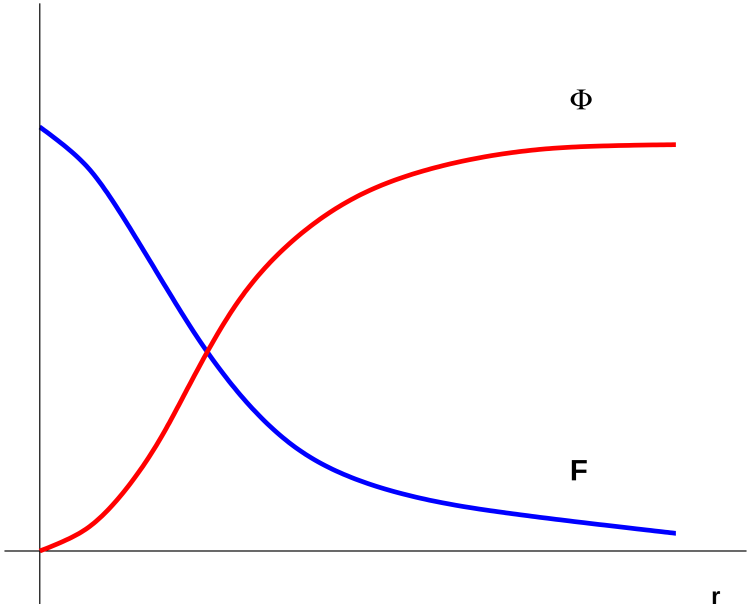
- Fundamental matter (scalar quarks): Q^B, $\tilde{Q}_B$, ... with masses m_B; $B = 1, \ldots, N_f = \#$ of flavors

Scalars can easily condense:
If $\langle \Phi \rangle \neq 0$: the Coulomb phase of Abelian gauge theory;
If $\langle \tilde{Q} Q \rangle \neq 0$: gauge group is (totally) “Higgsed”, as in superconductor.

String solutions: vortices in Landau-Ginzburg theory

$$\mathcal{L} \sim |\nabla Q|^2 + V(Q) + \ldots, \quad \nabla Q = (\partial - iA)\, Q \tag{1}$$

String or flux tube in $(x, y) = (r, \theta)$ plane:



profile for the condensate $\Phi \to Q$ and magnetic field F.

Exactly the picture for Abelian theory, coming from $SU(2)$

$$\Phi = a \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = a \cdot \sigma_3$$

and the gauge field on the Coulomb branch

$$\mathbf{A} = \begin{pmatrix} A & W^+ \\ W^- & -A \end{pmatrix} = A\sigma_3 \oplus W^{\pm}\sigma_{\pm}$$

breaks into massless photon A and massive W-bosons, due to

$$[\Phi, \mathbf{A}] \sim \pm a \cdot W^{\pm}\sigma_{\pm}$$

Generally $U(1)^{N_c-1}$ gauge group, by adjusting parameters to be enlarged up to $SU(n) \times U(1)^{N_c-n}$.

Next, $\mathcal{W} = \tilde{Q}\Phi Q + m\tilde{Q}Q$, so if $a \pm m \approx 0$, (some, if many) quarks Q can be light - and condense (r - number of condensed flavors, distinguishing different vacua).

The effective theory is

$$\mathcal{L}_{\text{eff}} = \text{Im}T(a)F_{\mu\nu}^2 + \ldots + |\nabla Q|^2 + \ldots$$

with renormalized coupling $\tau_0 = \frac{\theta_0}{\pi} + \frac{8\pi i}{g_0^2} \Rightarrow T(a) = \frac{\theta(a)}{\pi} + \frac{8\pi i}{g^2(a)}$

Classical supersymmetric theory - complicated, quantum - easy. $T(a)$ can be determined and computed *exactly*!

Supersymmetry and complex numbers: in $\mathcal{N} = 2$ theory scalars are complex!

Light states - the BPS mass formula

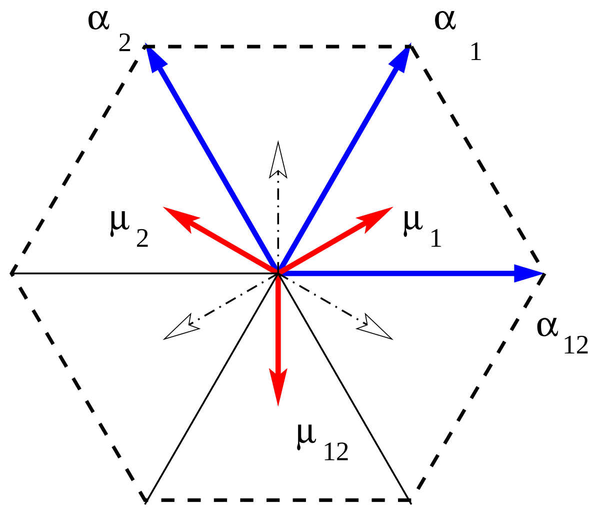
$$M \sim |n_e \cdot a + n_m \cdot a_D + n_B \cdot m| \qquad (2)$$

where the condensates a (and a_D) and the bare masses m are *all complex*!

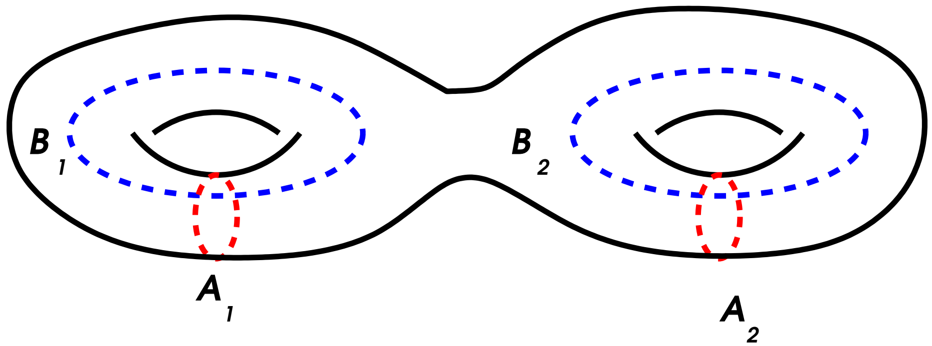
Holomorphic properties: in the vicinity, say, of $a_D = 0$ formula (2) leads to changing of the quantum numbers $(n_e, n_m) \to (n_e \pm n_m, n_m)$.

Quantum numbers in non-Abelian theories

Quark color charges (weights) and monopole charges (roots) of the $SU(3)$ gauge group

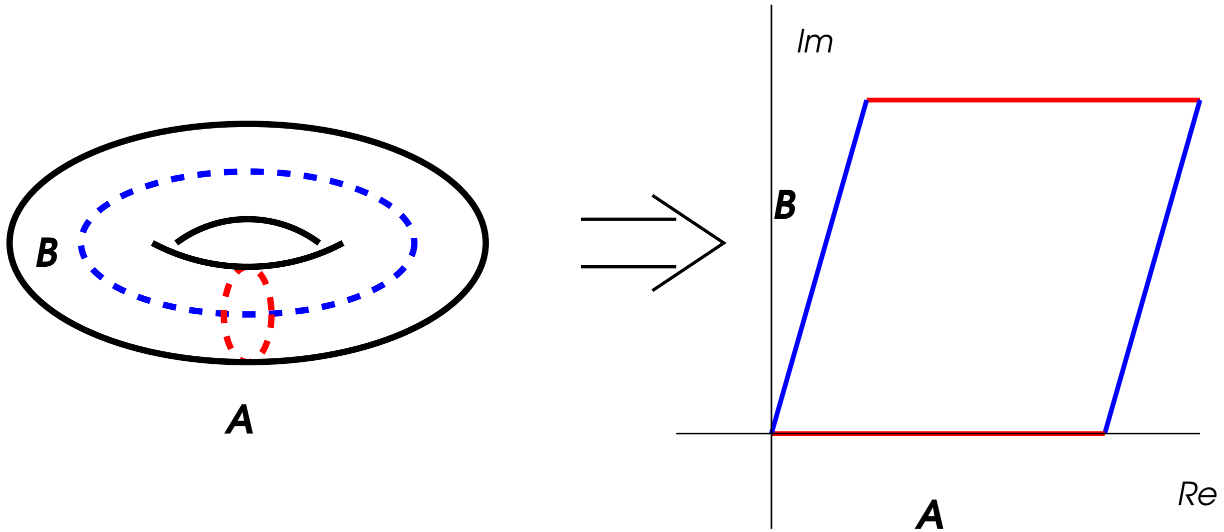


Dirac quantization condition $\boldsymbol{\mu}_i \cdot \boldsymbol{\alpha}_j = \delta_{ij} \Rightarrow A_i \circ B_j = \delta_{ij}$ turns in $\mathcal{N} = 2$ SUSY Yang-Mills into the intersection form of the cycles.



Riemann surface (complex curve) of genus $g = 2$, corresponding to the $SU(3)$ gauge theory. Elementary quark's charges $\boldsymbol{\mu}_{1,2} \leftrightarrow A_{1,2}$ correspond to the A-cycles, while the monopole's ones are $\boldsymbol{\alpha}_{1,2} \leftrightarrow B_{1,2}$ - to the B-cycles; (in "homological normalization" charges are always *integer*).

Duality $A \leftrightarrow B$ (different choice of the cycles corresponds to the same complex structure) is equivalent to electric-magnetic duality.

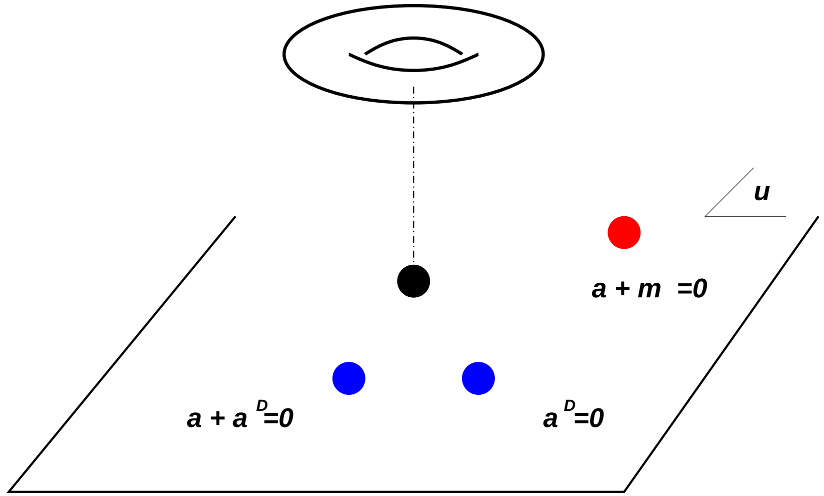


Duality $A \leftrightarrow B$ corresponds to $T \leftrightarrow -\frac{1}{T}$ or $g^2 \leftrightarrow \frac{1}{g^2}$, since (for rectangle $T \sim \frac{i}{g^2} \sim i\frac{R_2}{R_1}$)

$$T = \frac{\partial a^D}{\partial a}, \quad \Leftarrow \quad \frac{1}{g^2} = \frac{M_{\mathrm{mon}}}{M_W} \tag{3}$$

Periods $a = \oint_A dS$, $a_D = \oint_B dS$ are never simultaneously real, except for singular degenerate cases: extra massless states!

Supersymmetric QCD with matter of mass m



Quark (red), monopole and dyon (blue) singularities (=vacua).
At blue points the adjoint fields with the *root* charges condence: an Abelian confinement by the Seiberg-Witten scenario.
In order to get non-Abelian structures, one needs to turn the quark vacuum (red point) into the strong coupling by $m \to 0$.

Analytically,

$$(a, a^D) = \oint_{A,B} x\frac{dW}{W}, \quad m = \text{res } x\frac{dW}{W}, \quad W + \frac{1}{W} = \frac{P(x)}{\sqrt{Q(x)}}$$

$$T = \frac{\partial a^D}{\partial a}, \quad \Rightarrow \quad \exists \mathcal{F}: \; a^D = \frac{\partial \mathcal{F}}{\partial a} \tag{4}$$

with $P(x) = \langle \det(x - \Phi) \rangle$, $Q(x) = \Lambda^{2N_c - N_f} \prod_{B=1}^{N_f} (x + m_B)$.

In the simplest case $N_c = 2$: elliptic integrals. Generally - (at least) hyperelliptic curves of genus $g = N_c - 1$, where nothing can be computed explicitly ...

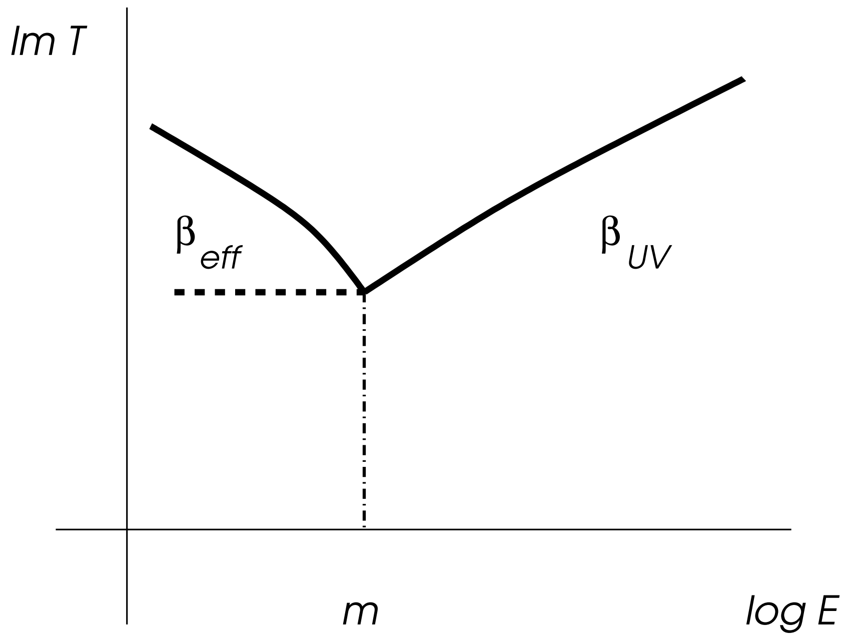
But at the singularities (vacua) the curve degenerates.

For our purposes elliptic case is not enough: $SU(2)$ breaks down to $U(1)$ - Abelian theory.

Non-Abelian: $SU(N_c)$ theory with $N_c < N_f < 2N_c$, broken down *minimally* to $\underbrace{SU(N_c - 1)}_{\text{neccesary factor}} \times U(1)$ (maximally to $U(1)^{N_c - 1}$).

E.g. if $SU(3) \to SU(2) \times U(1) \simeq U(2)$, one gets the $SU(2)$-*doublets* of confined objects in the spectrum.

Around each vacuum: different effective theories (light fields, Lagrangians,...) at different regions of moduli space: controlled only if effective theory is at weak coupling!



Regime of weak coupling in original theory:

$\beta_{\text{UV}} = 2N_c - N_f > 0$: asymptotic freedom in UV;

$\beta_{\text{eff}} = 2(N_c - 1) - N_f = \beta_{\text{UV}} - 2 \leq 0$: IR free or conformal theory: allows semiclassical analysis!

Hence, $2N_c - 2 \leq N_f < 2N_c$, e.g. $N_c = 3$, $N_f = 4,5$ with non Abelian confinement in IR: at least $SU(2) \times U(1)$ non Abelian gauge symmetry at the scales

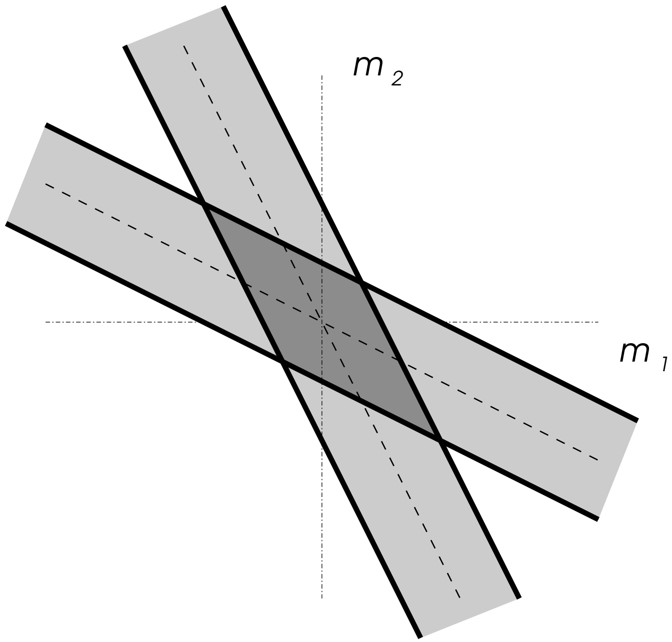
$$\sqrt{\mu m} < E < m$$

Quark $r = 2$ vacuum where at least two (light) flavors condense (up to $N_c - 1$ for $SU(N_c - 1)$).
Though for many flavors: partially coincident bare masses $|m_B - m_{B'}| \to 0$, Higgs branches, fat "semilocal" strings, etc.

Decreasing m: weakly coupled dual theory!

$$\tilde{\Lambda} = \max\left(\frac{m^2}{\Lambda}, \Lambda\right) \tag{5}$$

$N_c = 3$, $N_f = 4$ theory with pairwise coinciding masses $m_1 = m_3$, $m_2 = m_4$.



weakly-coupled (white), intermediate (grey) and strongly-coupled (dark) regions - the real slice of complex mass picture.
$N_f = 5$: classically the same, quantum theory - parabolas, "deformed rhombus" in the strong-coupled region.

Seiberg-Witten theory gives

$$a = \oint_A dS = -m, \quad |a + m| = 0 \tag{6}$$

or massless quark $(\frac{1}{2}, 0) = [1, 0]$ in the quark vacuum, and

$$a_D = \oint_B dS = -\frac{i}{\pi}\left(2\sqrt{m^2 - \Lambda^2} + m \log \frac{m - \sqrt{m^2 - \Lambda^2}}{m + \sqrt{m^2 - \Lambda^2}}\right) \tag{7}$$

the dual B-period on degenerate curve at quark vacuum.

At $m = \Lambda$ also $a^D = 0$: theory with (mutually nonlocal) light quark and monopole!
At $m = -\Lambda$, instead $\mathrm{Im}(a^D) = 0$, but $\mathrm{Re}(a^D) = 2m$, or $|a^D + 2a| = 0$, i.e. the dyon $(1, 1) = [2, 1]$ becomes massless.

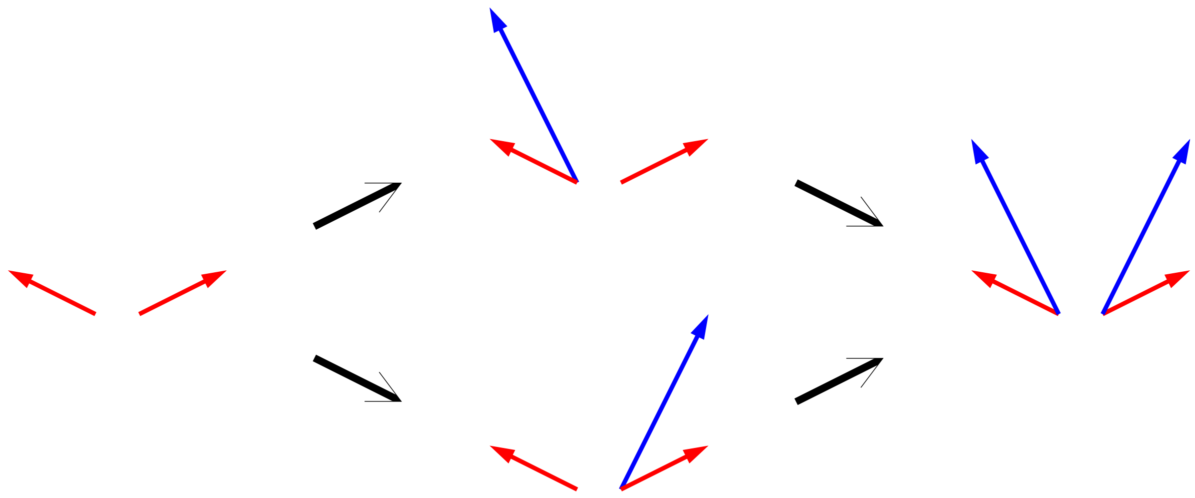
Moreover

- If $m > \Lambda$, a^D is imaginary, $|a + m + a^D| > |a + m|$, $(\frac{1}{2}, 1) = [1, 1]$ dyon is more heavy than quark;

- If $m < \Lambda$, a^D becomes real, $|a + m + a^D| < |a + m|$: i.e. at $m = \Lambda$, when $a^D = 0$, quark decouples into dyon, or acquires magnetic charge!

Exact computation of the periods for $SU(3)$ theory gives singularities on set of straight lines for $N_f = 4$ and on parabolas for $N_f = 5$ in mass moduli space.

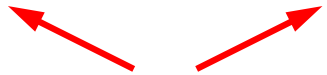
Change of the quantum numbers due to monodromies:

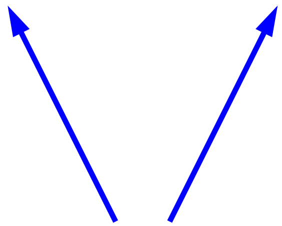


the doublet of quarks (weak coupling) turns into the doublet of dyons (at strong coupling).

In each region states are mutually local and can be described by an effective field theory!

Strings and confinement

Confinement in $r = 2$ vacuum: condensate $Q_{1,2}$ 

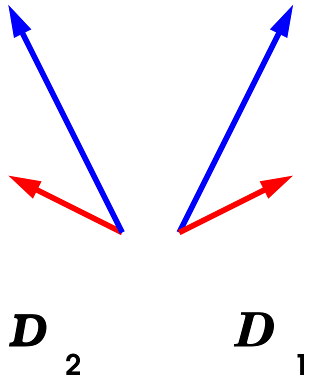
The doublet of magnetic strings $S_{1,2}$  which confine monopoles.

Generalization of common ANO strings from relativistic Landau-Ginzburg theory: the first order BPS vortex equations, etc.

The doublet of confined monopoles, w.r.t. $\boldsymbol{\alpha}_{12} = \boldsymbol{\alpha}_1 - \boldsymbol{\alpha}_2$ $SU(N_c - 1) = SU(2)$ subgroup.

Confinement at strong coupling:

Condensate formed by dyons:

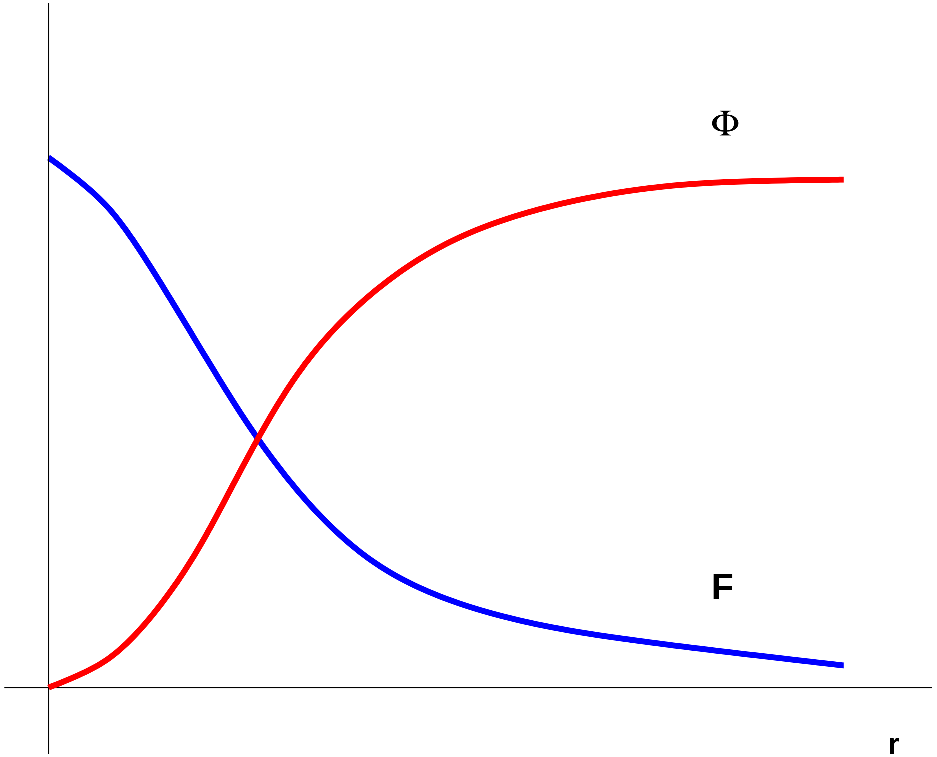


The fundamental strings: $\boldsymbol{\mu}_i \cdot \mathbf{A} + \boldsymbol{\alpha}_i \cdot \mathbf{A}^D \sim \delta_{iK} d\theta$, $K = 1, 2$ from the effective Lagranian for light dyons:

$$\mathcal{L} \sim \sum_{K=1,2} |\nabla \mathcal{D}_K|^2 + V(\mathcal{D}) + \ldots,$$
$$\nabla \mathcal{D}_K = \left(\partial - i(\boldsymbol{\mu}_K \cdot \mathbf{A} + \boldsymbol{\alpha}_K \cdot \mathbf{A}^D)\right) \mathcal{D}_K \tag{8}$$

and θ is angle in the (x, y)-plane, transversal to the direction of string.

String or flux tube in $(x, y) = (r, \theta)$ plane (back to the Meissner effect):



profile functions for the condensate $\Phi \rightarrow \mathcal{D}$ and gauge field $F \rightarrow F + F^D$.

Who is confined?

From vortex equations $\boldsymbol{\alpha}_{12} \cdot (\mathbf{A} + \mathbf{A}^D) \sim d\theta$, for the dual component $\boldsymbol{\alpha}_{12} \cdot (\mathbf{A} - \mathbf{A}^D) \sim 0$.

Dynamic in the only non-Abelian direction $\boldsymbol{\alpha}_{12}$ is determined by "difference dyon" $\mathcal{D}_1 - \mathcal{D}_2 \; =$ 
and "difference string" $S_1 - S_2 =$ .

Screening of the electric charge of "difference string" by the condensate of the "difference dyon"

$$S_1 - S_2 + \frac{1}{2}(\mathcal{D}_1 - \mathcal{D}_2) \; =$$

causes still the *confinement of monopoles at strong coupling*!.

Ambiguity

The monopole mass $a_D(m)$ is multivalued function: near $m \sim \Lambda$ it contains the square root $\sqrt{m^2 - \Lambda^2}$ under logarithm.

Dependently on the particular choice of trajectory $m \curvearrowright \Lambda$, quark picks up an *ambiguous* sign of magnetic charge:

$$[1, 0] \to [1, q], \qquad q = \pm 1$$

Changes some sings of charges in effective theory

but

the *same* magnetic monopoles are confined at strong coupling, i.e. the main physical conclusion does not depend on this ambiguity!

Conclusions

- Naive scenario for the dual Meissner effect: the condensation of monopoles should cause the confinement of quarks;

- This can be realized only of one takes the strongly coupled $r = 0$ vacuum with the massless monopoles and adds there external probe quarks “by hands”. Only Abelian confinement!

- Constructive way: transporting the picture from weak (where we have started from non-Abelian theory) into strong coupling domain;

- Quantum numbers change, and the condensate is formed by light *dyons* (the weights cannot disappear!)

- One does *not* get the confinement of quarks - the confined objects are still monopoles! (Apart of more “weak” technical problems with the Higgs branchs, fat strings etc . . .)

- Results: a lot of useful knowledge about the strongly-coupled phase of the gauge theory. The $\mathcal{N} = 2$ simplified partner allows in principle to imagine and study what could be in *quantum* theory!

  We have achieved *some* confinement with the realistic non-Abelian features and understanding of its mechanism.

  But

- Citing "The Moonstone" by Wilkie Collins: "*We have only partially reproduced the conditions, and the experiment has been only partially successful in consequence* . . ."