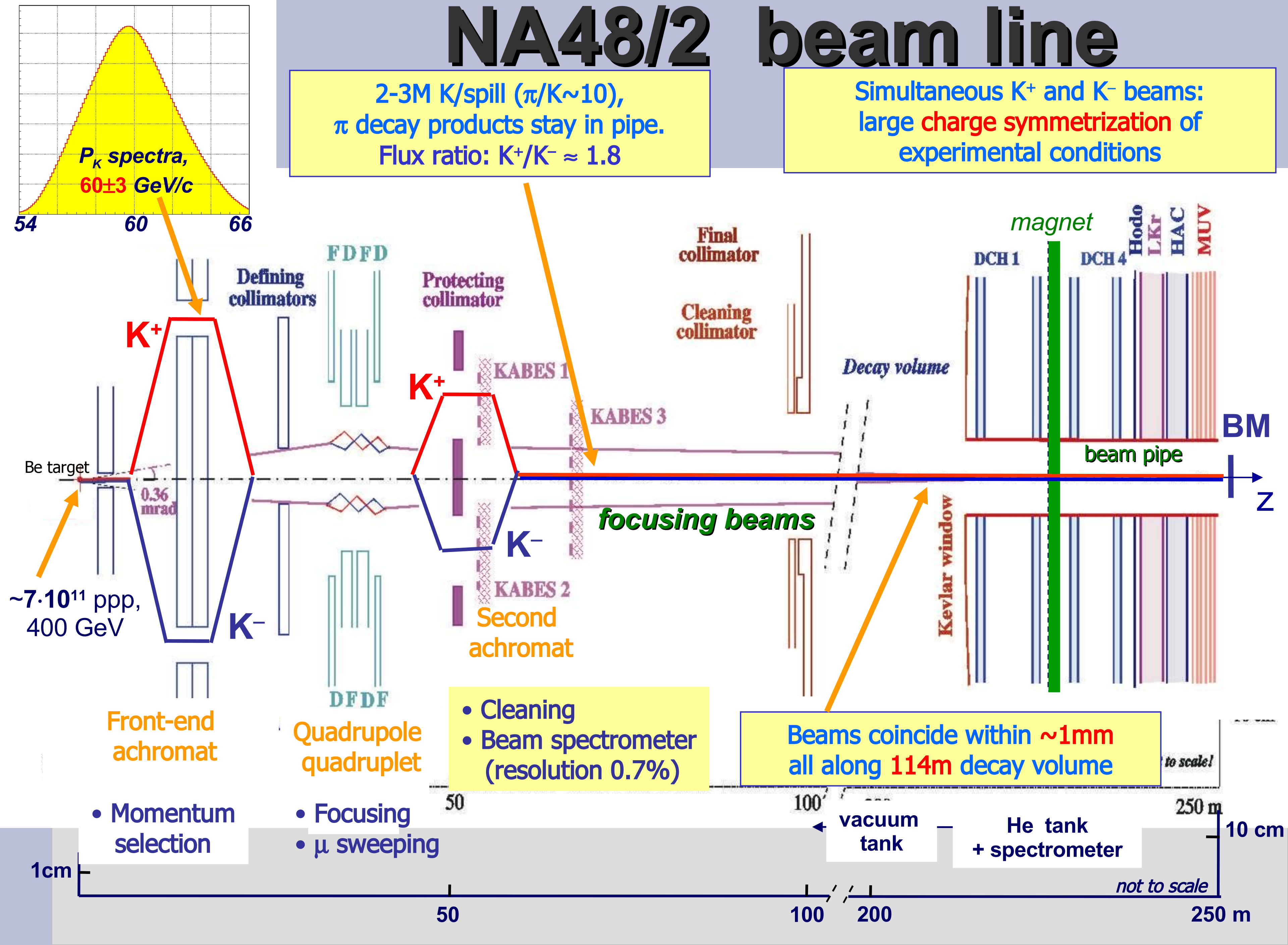
# Precision Measurement of $\pi\pi$ Scattering Lengths in Ke4 Decays

*Dmitry Madigozhin*
*(JINR, Dubna)*

on behalf of the NA48/2 Collaboration:

**Cambridge, CERN, Chicago, Dubna, Edinburgh, Ferrara, Firenze, Mainz, Northwestern, Perugia, Pisa, Saclay, Siegen, Torino, Vienna**

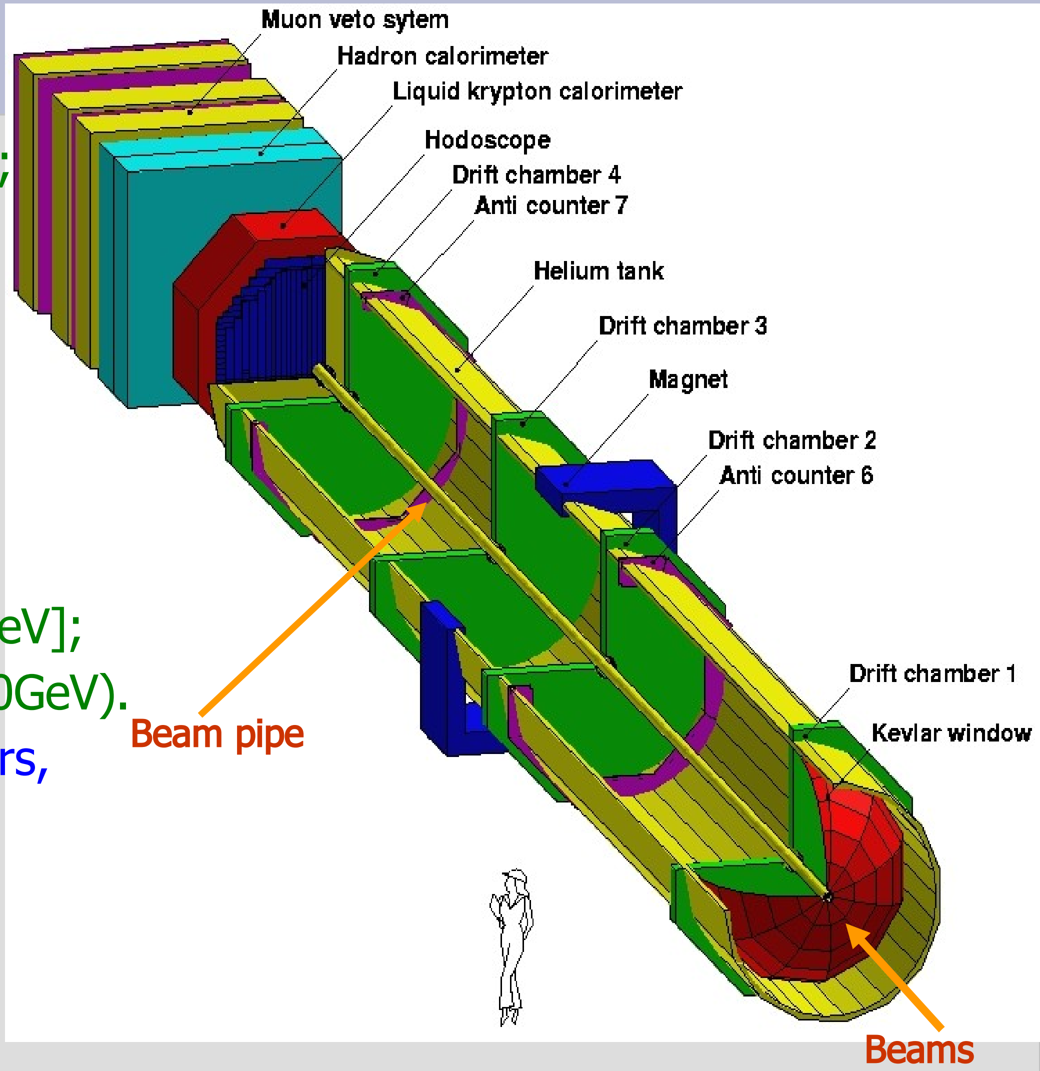
# NA48/2 beam line



# The NA48 detector

**Main detector components:**

- Magnetic spectrometer (4 DCHs):
  4 views/DCH: redundancy ⇒efficiency;
  used in trigger logic;
  $\Delta p/p = 1.0\% + 0.044\% * p$ [GeV/c].
- Hodoscope
  fast trigger;
  precise time measurement (150ps).
- Liquid Krypton EM calorimeter (LKr)
  High granularity, quasi-homogenious;
  $\sigma_E/E = 3.2\%/E^{1/2} + 9\%/E + 0.42\%$ [GeV];
  $\sigma_x=\sigma_y=0.42/E^{1/2} + 0.6$mm (1.5mm@10GeV).
- Hadron calorimeter, muon veto counters,
  photon vetoes.



# NA48/2 data:

**2003 run: ~ 50 days**

**2004 run: ~ 60 days**

A view of the NA48/2 beam line

Total statistics in 2 years:

**$K^{\pm} \to \pi^{-}\pi^{+}\pi^{\pm}$: $\sim 4 \cdot 10^{9}$**

**$K^{\pm} \to \pi^{0}\pi^{0}\pi^{\pm}$: $\sim 1 \cdot 10^{8}$**

Rare $K^{\pm}$ decays:
BR's down to $10^{-9}$
can be measured

**>200 TB of data recorded**

# Pion scattering lengths

The important free parameter of ChPT is the quark condensate <qq>, it determines the relative size of mass and momentum terms in the power expansion.

$a_0$ and $a_2$ are S-wave $\pi\pi$ scattering lengths in isospin states I=0 and I=2, correspondingly. They enter into all $\pi\pi$ scattering amplitudes.

The relation between <qq> and the scattering lengths $a_0$ and $a_2$ is known from this theory, so the experimental measurement of $a_0$ and $a_2$ provides an important constraints for ChPT Lagrangian parameters.

Pion scattering lengths are measured also in the

**study of the cusp-effect in $K^{\pm}\rightarrow\pi^{\pm}\pi^{0}\pi^{0}$ decays**

[NA48/2 final result: EPJ C64(2009)589]

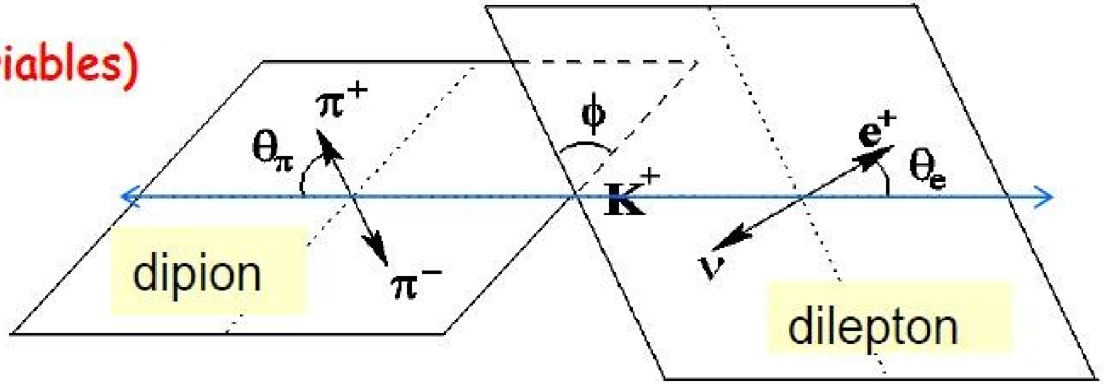
# Ke4 decays : formalism

**Five kinematic variables (Ca.Ma. variables)**

(Cabibbo-Maksymowicz 1965)

$S_\pi$ ($M^2_{\pi\pi}$), $S_e$ ($M^2_{e\nu}$),

$\cos\theta_\pi$, $\cos\theta_e$ and $\phi$.




**Partial Wave expansion of the amplitude** into s and p waves (Pais-Treiman 1968) + Watson theorem (T-invariance) for $\delta_\ell^I$

$$\delta_0^0 \equiv \delta_s \text{ and } \delta_1^1 \equiv \delta_p$$

**F, G = 2 Axial Form Factors**

$F = F_s\, e^{i\delta s} + F_p\, e^{i\delta p} \cos\theta_\pi$

$G = G_p\, e^{i\delta g}$

**H = 1 Vector Form Factor**

$H = H_p\, e^{i\delta h}$

F, G, H are complex and dimensionless

**Map the** distributions of the Ca.Ma. variables in the **five-dimensional space** with 4 Form factors and one phase shift , assuming identical phases for the p-wave Form Factors $F_p$, $G_p$, $H_p$:

The fit parameters are :

$F_s$ $F_p$ $G_p$ $H_p$ and $\delta = \delta_s - \delta_p$

($F_s$ , $F_p$ , $G_p$ , $H_p$ are real )

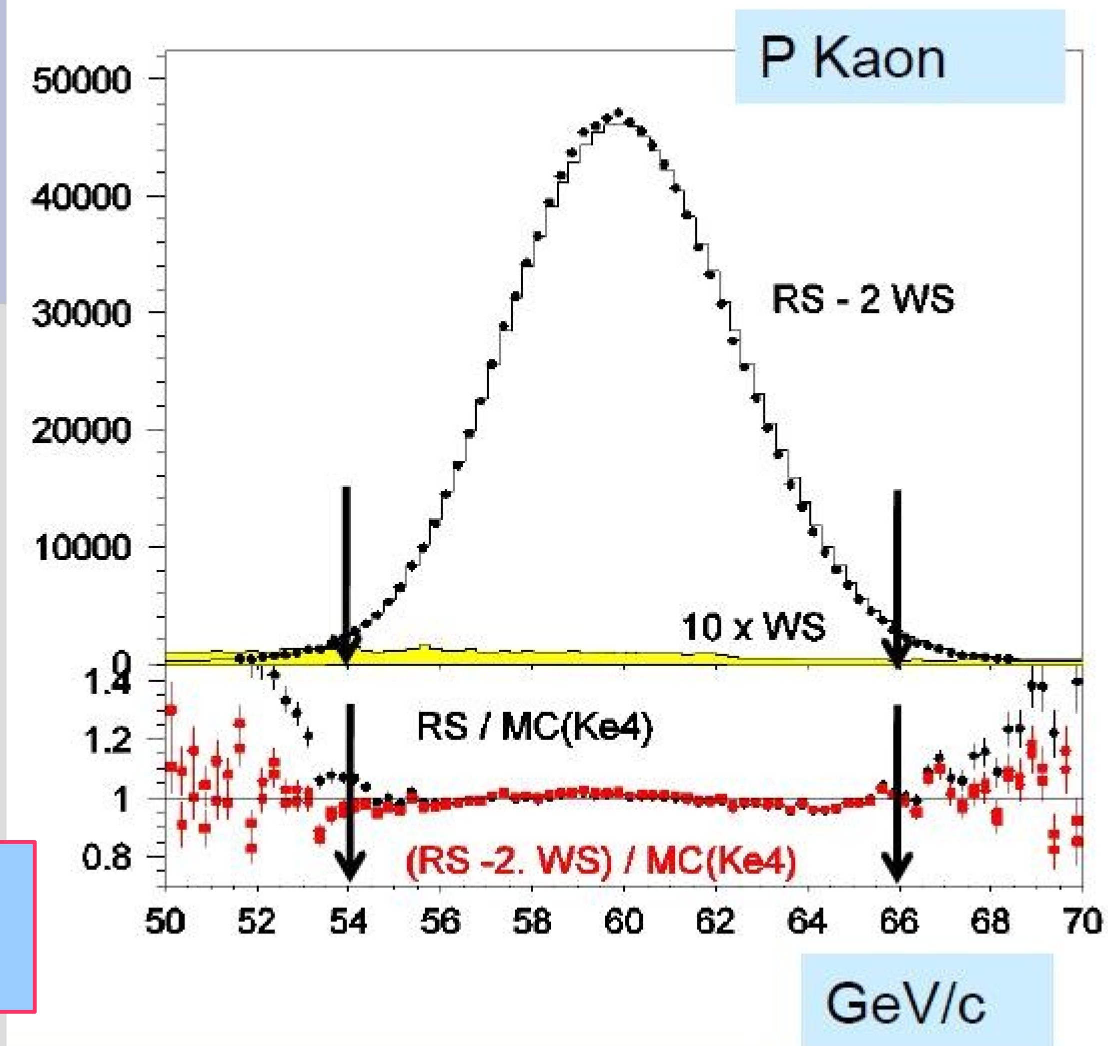
Signal ($\pi$+ $\pi$- e+/- $\nu$) topology:

- 3 charged tracks, good vertex
- Opposite sign 2 pions
- 1 electron (Elkr /P ~ 1)

Main background sources:

- a) $K^+ \to [\pi^- \to e^- \nu]\pi^+\pi^+$
- b) $K^+ \to [\pi^0 \to e^+e^- (\gamma)] (\pi^0) \pi^+$

Background has a «wrong sign» (WS) component (same sign pion charges): 1/3 for a) and 1/2 for b).




Total background is at the level of 2 x 0.3 % (estimated from WS events and checked by MC)

# Ke4 decays : fitting procedure

Total (2003+2004) 1.13 Million Ke4 decays

Using iso-populated boxes in the 5-dimension space of the Ca.Ma. variables, ($M_{\pi\pi}$, $M_{e\nu}$, $\cos\theta_\pi$, $\cos\theta_e$ and $\phi$) one defines a grid of

10x5x5x5x12=15000 variable size boxes.

In each $M_{\pi\pi}$ "slice" (1500 boxes), a set of 4 fit parameters is found which minimizes the difference between the data and predicted populations

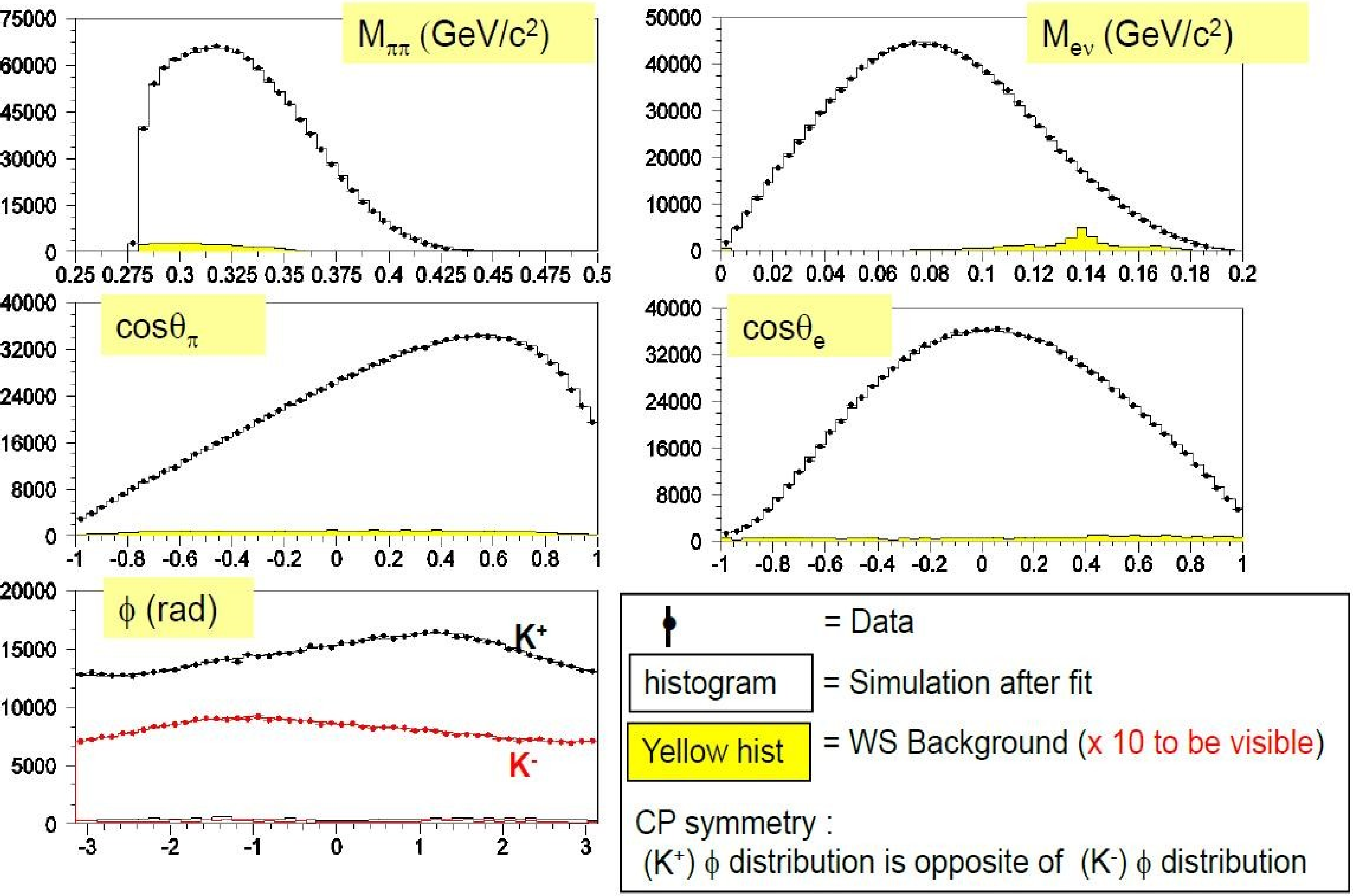
The normalisation $F_s^2$ is obtained in each bin/slice by the ratio $x_{slice} = \Sigma_{j \text{ in slice}} Nj/\Sigma_{j \text{ in slice}} MCj$

| Data sample | MC sample |
|---|---|
| $K^+$ sample (726 400 events) 48 events/box | $K^+$ MC (17.4 Million events) 1160 events/box |
| $K^-$ sample (404 400 events) 27 events/box | $K^-$ MC ( 9.7 Million events) 650 events/box |

$K^+$ and $K^-$ samples fitted separately in 10 independent $M_{\pi\pi}$ bins/slices, then combined in each slice according to their statistical error.

No assumption is made on the shape of the variation of the phase $\delta$ (and FF) from one $M_{\pi\pi}$ slice to the next ( i.e. "model independent" analysis)

# Ke4 decays : Data/MC comparison after fit



# Ke4 formfactors: fit results

Series expansion with:

- $q^2 = S\pi/4m_\pi^2 - 1$
- $Se/4m_\pi^2$

$F_s^2 = f_s^2(1+f'_s/f_s q^2+f''_s/f_s q^4+f'_e/f_s Se/4m_\pi^2)^2$

$G_p/f_s = g'_p/f_s + g'_p/f_s q^2$

Preliminary (2003+2004)

| | value | stat | syst |
|---|---|---|---|
| $f_s'/f_s$ | 0.152 | ± 0.007 | ± 0.005 |
| $f_s''/f_s$ | -0.073 | ± 0.007 | ± 0.006 |
| $f_e'/f_s$ | 0.068 | ± 0.006 | ± 0.007 |
| $f_p/f_s$ constant | -0.048 | ± 0.003 | ± 0.004 |
| $g_p/f_s$ | 0.868 | ± 0.010 | ± 0.010 |
| $g_p'/f_s$ | 0.089 | ± 0.017 | ± 0.013 |
| $h_p/f_s$ constant | -0.398 | ±0.015 | ± 0.008 |

# Comparison of Ke4 phase shift experimental measurements

Apply Isospin corrections (10-15 mrad) to all published points :

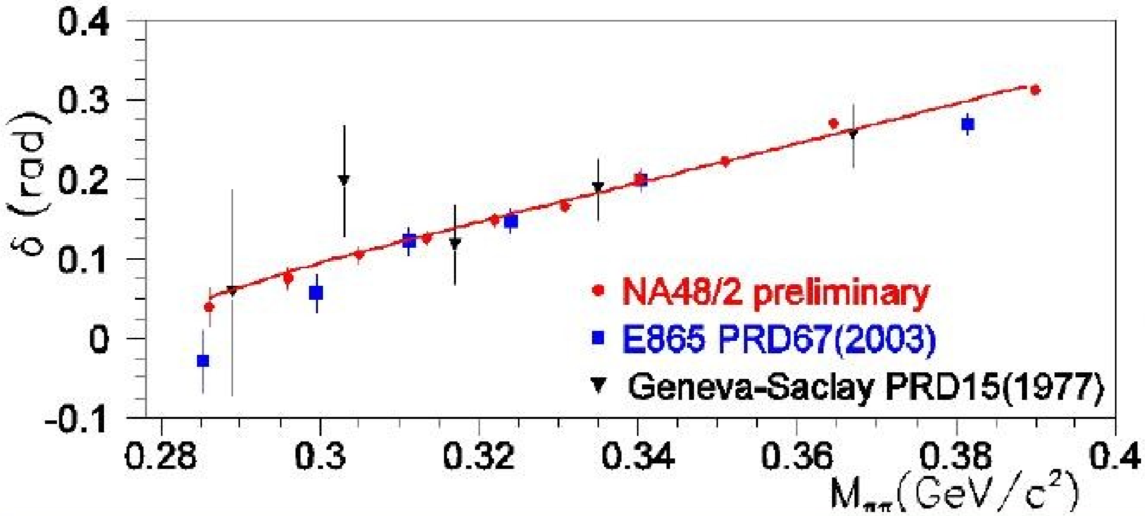
**S118 (Geneva-Saclay)**: typical error 40-50 mrad

**E865**: typical error 15-20 mrad

Correction small wrt experimental error but coherent shift downwards for all data points

NA48/2 typical error 7-8 mrad

improved precision due to both
- -larger statistics ~3 x E865
- -larger acceptance at high $\pi\pi$ mass




- All Phase points corrected for isospin mass effects
- Independent experiments
- Errors = stat + syst

Line from a 2p fit to NA48 data alone

Systematic errors are mailny bin to bin uncorrelated:
- Binning choise
- Trigger efficiency
- Acceptance control
- Background level (the only correlated source)
- Background shape
- Electron identification
- Radiative correction

# From phase shifts to pion-pion scattering lengths

$\pi\pi$ phases at threshold can be predicted from data above 0.8 GeV using Roy equations (unitarity, analyticity and crossing symmetries) and 2 subtraction constants $a_0$ and $a_2$

Numerical solutions have been developed (ACGL, DFGS) valid in the Isospin symmetry limit **(Universal Band in the $[a_2, a_0]$ plane)**, but broken in the experimental world.

factorization of electromagnetic and mass effects :

Gamow factor x PHOTOS generator × Isospin corrections

**Radiative effects** (except mass effects) included in the simulation,

Gamow factor : "classical" Coulomb attraction between the 2 charged pions

PHOTOS generator: real photon(s) are emitted and tracked in the simulation

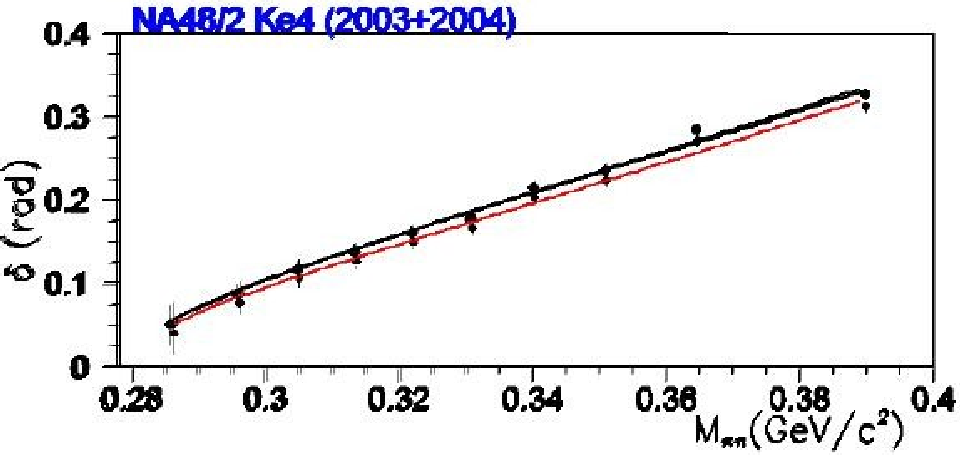
(-> effect on event selection + possible bias on reconstructed quantities)

**Mass effects**:

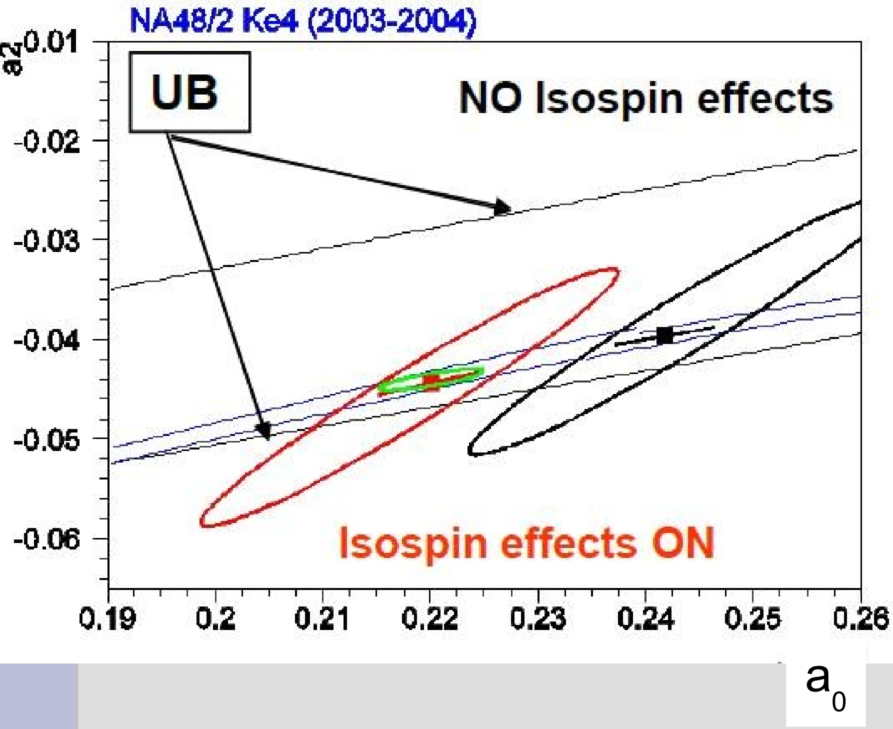
- recently computed as a correction to the measurements
- even larger than current experimental precision !

(CGR EPJ C59 (2009) 777,
DK preliminary June 2008 in progress )

# Ke4 decays: from phase shifts to scattering lengths ($a_0$,$a_2$)




a tiny effect from theory…. a big change in now precise experimental measurement !




This induces a large change on ($a_0$,$a_2$) values

| from a 2p fit | | | from a 1p fit | |
|---|---|---|---|---|
| $\Delta a_0$ = -0.025, $\Delta a_2$ = -0.007 | | | $\Delta a_0$ = -0.022 | |
| error | stat | syst | stat | syst |
| $\sigma(a_0)$: | ± 0.0128 | ± 0.0050 | ± 0.005 | ± 0.002 |
| $\sigma(a_2)$: | ± 0.0084 | ± 0.0034 | | |

Ellipses are 68% CL contours in 2p fits

# Ke4 decays: comparison with theoretical predictions

Preliminary (2003+2004)

**THEORY prediction**

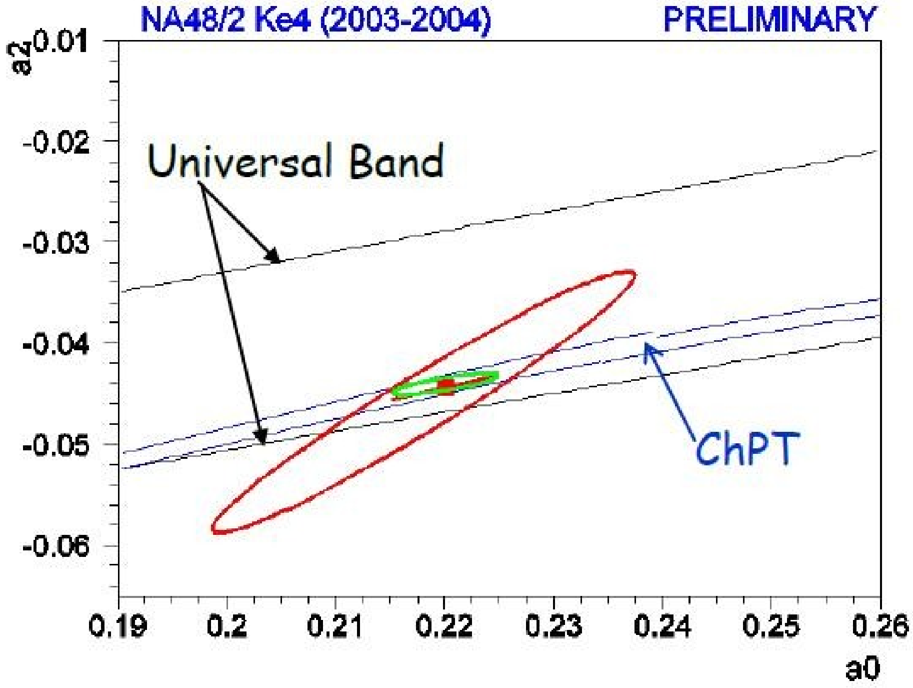
Using more inputs from ChPT and low energy constants, the prediction is better constrained (CGL NPB603(2001),PRL86(2001)):

$a_0$ = 0.220 ± 0.005

$a_2$ = -0.0444 ± 0.0008

**Experimental measurement**

| | |
|---|---|
| $a_0$ ChPT 1p fit | 0.2206 ± 0.0049 stat ± 0.0018 syst ± 0.0064 theo * |
| $a_0$ free | 0.2220 ± 0.0128 stat ± 0.0050 syst ± 0.0037 theo* |
| $a_2$ free 2p fit | -0.0432 ± 0.0086 stat ± 0.0034 syst ± 0.0028 theo* |
| | Correlation 96.7% |



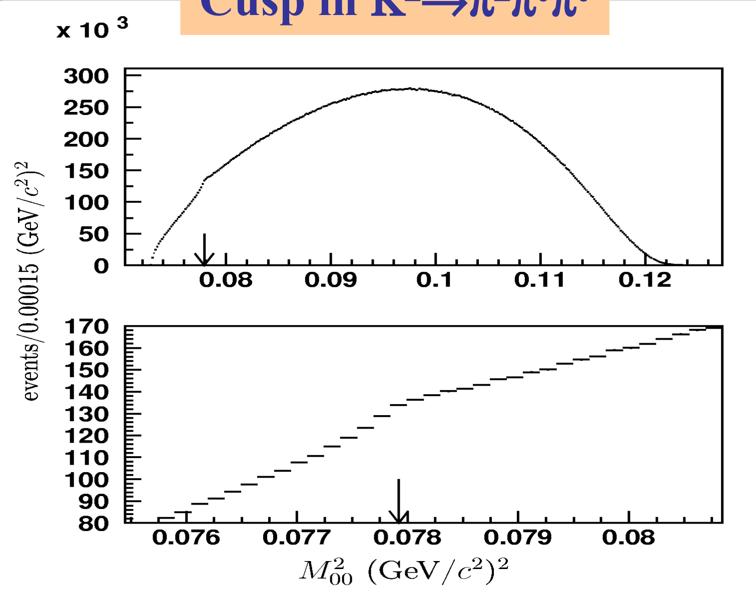

*Theory error evaluated from control of the isospin corrections & inputs to Roy equation numerical solutions (CGR EPJ C59 (2009)777)

# Final results of pion scattering lengths measurement from the Ke4 and the cusp results

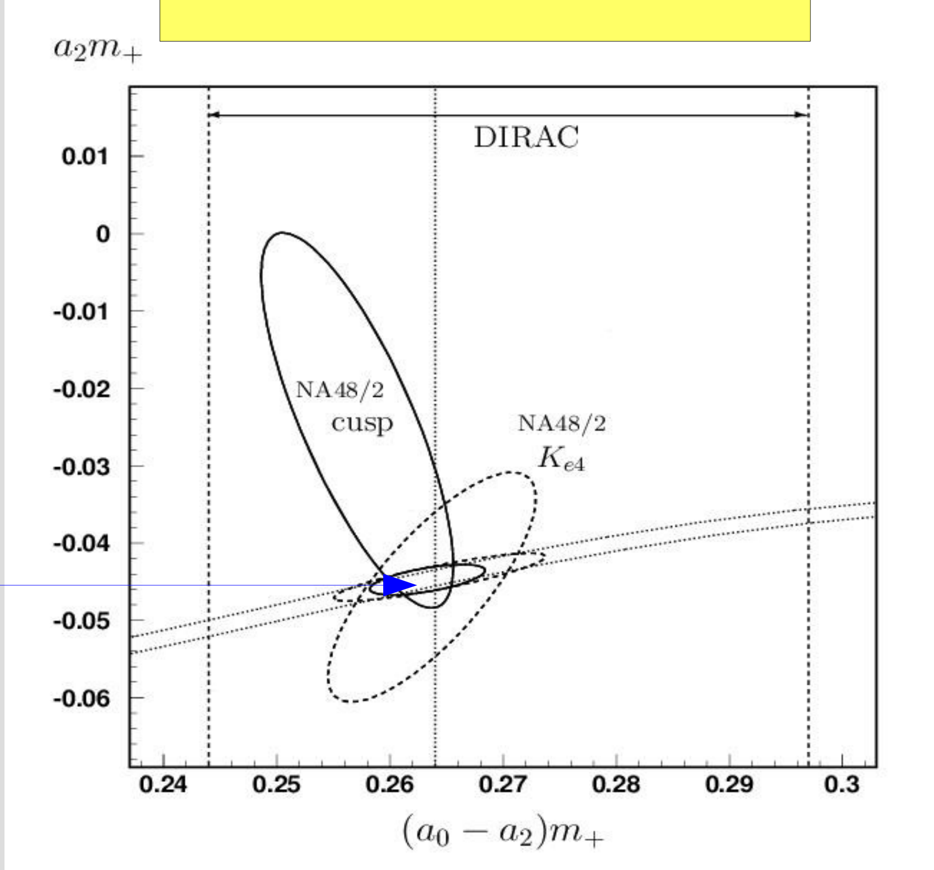
68% - probability ellipses, full uncertainties:

combined
- Statistical,
- Systematical,
- External.

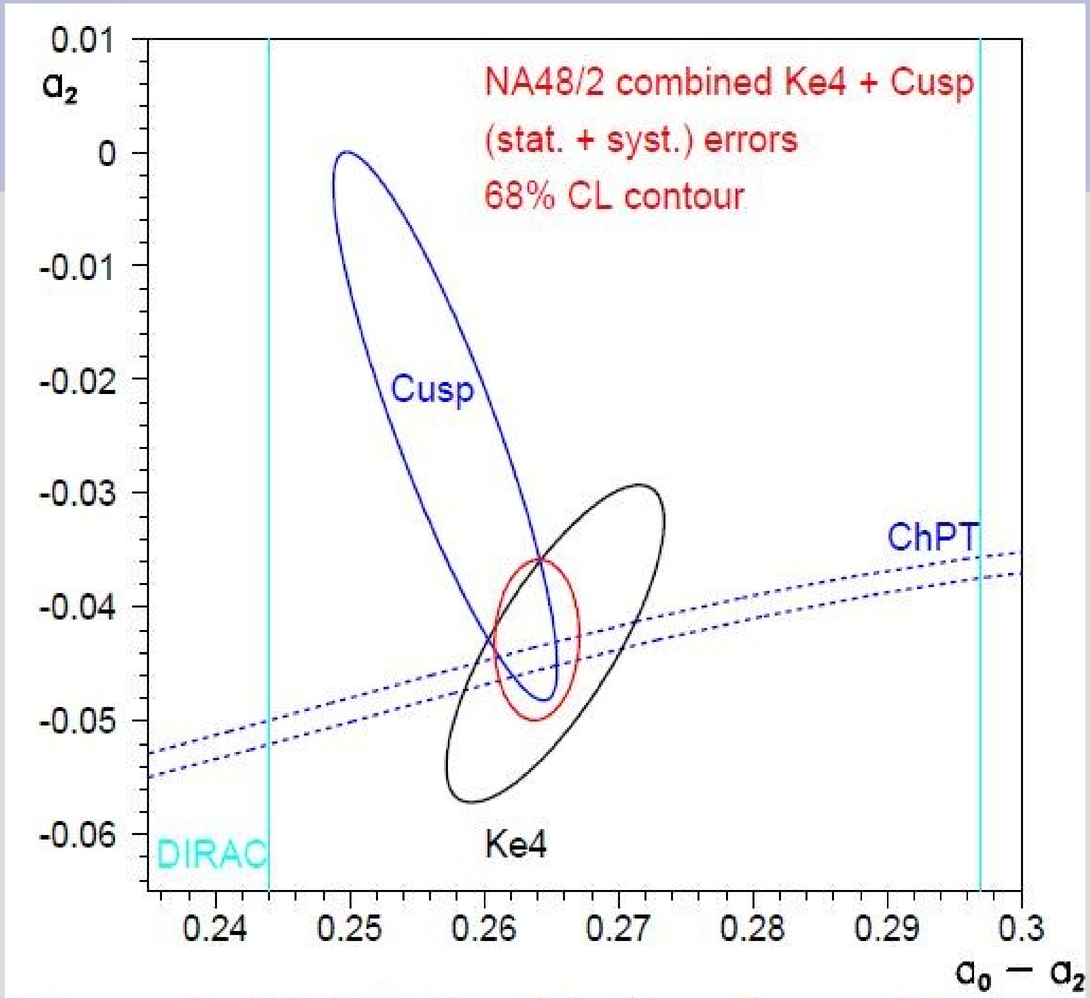
**Cusp in $K^{\pm} \to \pi^{\pm}\pi^{0}\pi^{0}$**




Cusp and Ke4 results using Chiral constraint (shown with a Gaussian error)



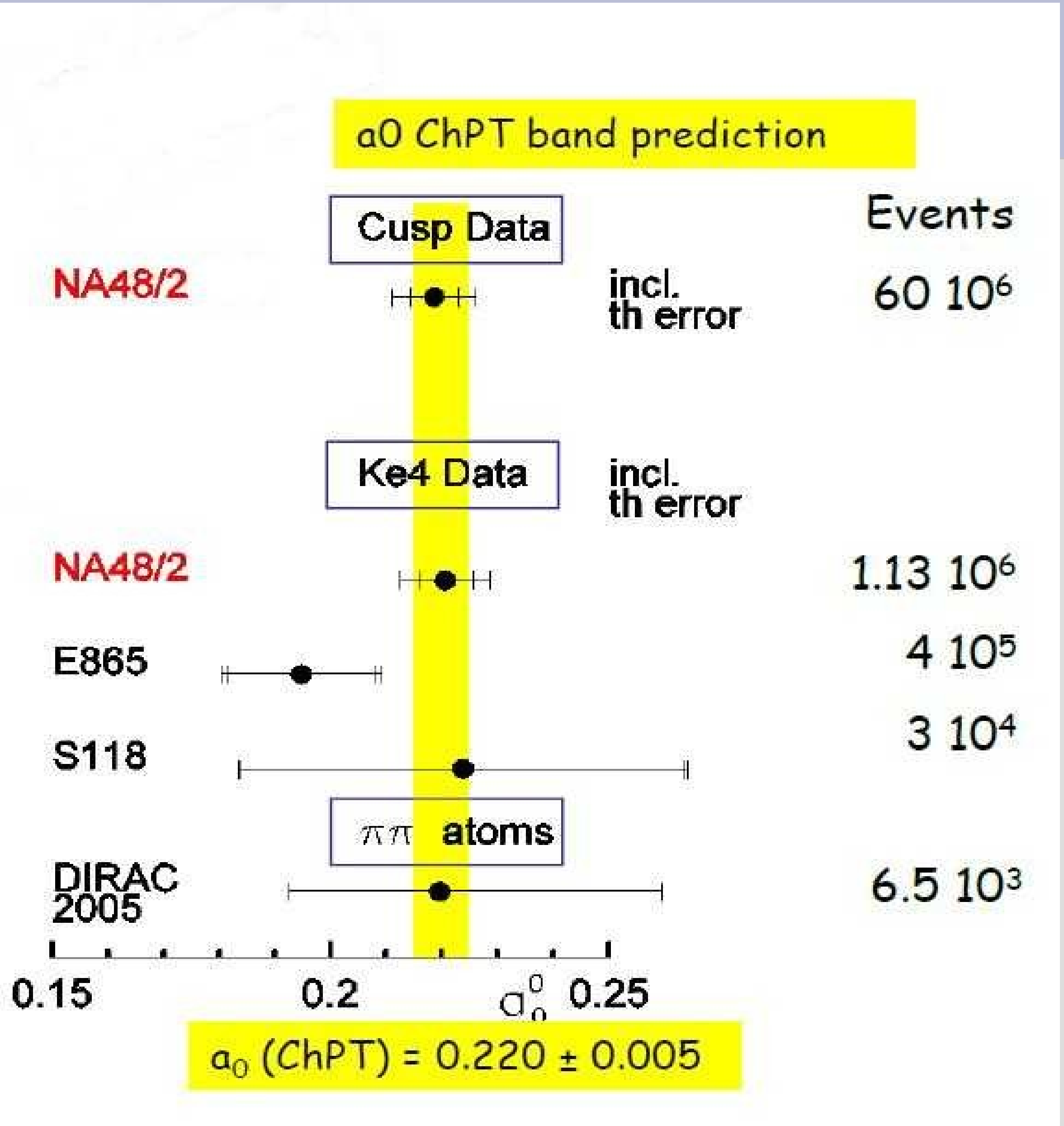
# NA48/2 Ke4 + cusp combined




$$a_0 = 0.2210 \pm 0.0047_{\rm stat} \pm 0.0015_{\rm syst}, \;\; a_2 = -0.0429 \pm 0.0044_{\rm stat} \pm 0.0016_{\rm syst}$$

$$a_0 - a_2 = 0.2639 \pm 0.0020_{\rm stat} \pm 0.0004_{\rm syst}.$$

# Comparison with another measurements



# Summary

- 1.13 millons of fully reconstructed $K^{\pm} \rightarrow \pi^{+}\pi^{-}\nu\ e^{\pm}$ (Ke4) decays (2003+2004 data).
- Ke4 Form Factors measured with an improved precision.
- Final result with ChPT link between $a_0$ and $a_2$ :

$$a_0 = 0.2206 \pm 0.0049(stat) \pm 0.0018(syst) \pm 0.0064(theor)$$

is in good agreement with the cusp measurement and with the prediction of ChPT.
- Experimental uncertainty is at the level of theoretical ChPT prediction $0.220 \pm 0.005$ (Colangelo, Gasser, Leutwyler 2000).