A decay of the ultra-high-energy neutrino $\nu_e \to e^- W^+$ in a magnetic field and its influence on the shape of the neutrino spectrum

Alexander Kuznetsov

Yaroslavl State (P.G. Demidov) University, Division of Theoretical Physics

June 11, 2010

16th International Seminar “Quarks-2010”, Kolomna, Moscow Region

In collaboration with N. Mikheev and A. Serghienko

Based on the paper to appear in Physics Letters B

arXiv:1002.3804[hep-ph]

Outline

1. The neutrino self-energy operator in magnetic field
2. The neutrino decay $\nu_e \to e^- W^+$ in an external field
3. The neutrino energy cutoff in a strong magnetic field

The neutrino self-energy operator in magnetic field

The most important achievement of the present-day neutrino physics: solving the solar-neutrino puzzle.

A problem of studying possible effects of an active environment on the neutrino dispersion properties becomes quite important.

A kind of external active medium: the strong magnetic field. The natural scale for the field strength exists: the critical value $B_e = m_e^2/e \simeq 4.41 \times 10^{13}\,\mathrm{G}$.

The neutrino self-energy operator in magnetic field

The neutrino self-energy operator $\Sigma(p)$ is defined in terms of the invariant amplitude for the transition $\nu_e \to \nu_e$:

$$\mathcal{M}(\nu_e \to \nu_e) = - \left[\bar{\nu}_e(p) \, \Sigma(p) \, \nu_e(p)\right].$$

The operator $\Sigma(p)$ defines the neutrino dispersion relation.

The additional neutrino energy in an external magnetic field is:

$$\Delta E = - \frac{1}{2\,E} \, \mathcal{M}(\nu_e \to \nu_e) \, .$$

The neutrino self-energy operator in magnetic field

Calculations of the neutrino dispersion relation in external magnetic fields have a long history.

- G. McKeon, 1981
- A. Borisov, V. Zhukovskiĭ, A. Kurilin and A. Ternov, 1985
- A. Erdas and G. Feldman, 1990
- A. Erdas and M. Lissia, 2003
- A. K., N. Mikheev, G. Raffelt and L. Vassilevskaya, 2006
- A. K. and N. Mikheev, 2007
- K. Bhattacharya and S. Sahu, 2009
- A. Erdas, 2009

The neutrino self-energy operator in magnetic field

Different regions of the parameter values were considered:

- a weak field case ($eB \ll m_e^2$);
- a moderately strong field case ($m_e^2 \ll eB \ll m_W^2$);
- the neutrino transverse (to $\mathbf{B}$) momentum $p_\perp$ is rather high, e.g. $p_\perp \gtrsim m_W$ or $p_\perp \gg m_W$, while the field strength is not too high, $eB \ll m_e^2$: the crossed-field approximation.

But the list is not comprehensive. And some results contradict to each other.

The neutrino self-energy operator in magnetic field

K. Bhattacharya and S. Sahu, 2009, made an attempt of reinvestigation the width of the process $\nu \to e^- W^+$, which is defined by $\mathrm{Im}\,\Delta E$, in the crossed field approximation, and obtained the result different from *A. Erdas and M. Lissia, 2003*.

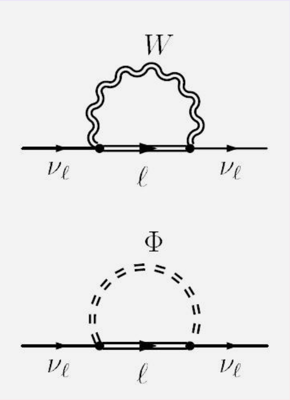
The neutrino self-energy operator in magnetic field

Another region of the physical parameter values: high neutrino transverse momenta, and high magnetic field strength. The crossed-field approximation is not valid.

A possibility of detecting cosmic neutrinos of ultrahigh energy, $E \gtrsim 10^{15}$ eV is discussed *(B. Zhang e.a., 2003; Q. Luo, 2005; K. Ioka e.a., 2005)*, originated from magnetars, the pulsars with superstrong magnetic fields ($B \sim 10^{15}$ G).

The emission of neutrinos having such energies cannot be adequately described without taking account of their interaction with the strong magnetic field of a magnetar.

The neutrino self-energy operator in magnetic field



Feynman diagrams representing the magnetic-field-induced contribution to the neutrino self-energy operator in the Feynman gauge. Double lines correspond to the exact propagators for the charged lepton, the W boson, and the nonphysical scalar charged Φ boson in an external magnetic field.

The neutrino decay $\nu_e \to e^- W^+$ in magnetic field

The neutrino decay width:

$$w(\nu_e \to e^- W^+) = -2\,\mathrm{Im}\,\Delta E = \frac{1}{E}\,\mathrm{Im}\,\mathcal{M}(\nu_e \to \nu_e)\,.$$

In the crossed field approximation, the width is expressed in terms of the dynamical field parameter χ and the lepton mass parameter λ:

$$\chi = \frac{eB\;p_\perp}{m_W^3}\,, \qquad \lambda = \frac{m_e^2}{m_W^2}\,.$$

The neutrino decay $\nu_e \to e^- W^+$ in magnetic field

A general expression for the decay width in the crossed field approximation *(A. K. and N. Mikheev, 2007)*

$$w(\nu_e \to e^- W^+) = \frac{\sqrt{2}\, G_\mathrm{F}\, m_W^4}{12\sqrt{3}\,\pi^2\, E} \int\limits_0^1 \frac{\mathrm{d}z}{z(1-z)^2}\, K_{2/3}(U)$$

$$\times \left[z + \lambda\,(1-z)\right]\left[2(1+z)(2+z) + \lambda\,(1-z)(2-z)\right],$$

where $K_{2/3}(U)$ is the modified Bessel function,

$$U = \frac{2}{3\,\chi}\, \frac{\left[z + \lambda\,(1-z)\right]^{3/2}}{z(1-z)}\,.$$

The neutrino decay $\nu_e \to e^- W^+$ in magnetic field

In the limit $\chi, \lambda \ll 1$, the result can be expressed in terms of the modified dynamical field parameter only:

$$\xi = \frac{\chi}{\sqrt{\lambda}} = \frac{eB\ p_\perp}{m_e\, m_W^2}\,.$$

The decay width, in agreement with *A. Erdas and M. Lissia, 2003*:

$$w(\nu \to e^- W^+) = \frac{\sqrt{2}\, G_{\mathrm{F}}}{3\pi} \frac{(eB\ p_\perp)^2}{m_W^2\, E} \left(1 + \frac{\sqrt{3}}{\xi}\right) \exp\left(-\frac{\sqrt{3}}{\xi}\right)$$

The neutrino decay $\nu_e \longrightarrow e^- W^+$ in magnetic field

The range for the $\xi = eB\ p_\perp/(m_e\, m_W^2)$ parameter appears to be rather large, $0 < \xi \ll m_W/m_e \simeq 1.6 \times 10^5$.

In the limit $\xi \ll 1$ the result by *A. Borisov e.a., 1985*, is reproduced.

The result by *K. Bhattacharya and S. Sahu, 2009,* is incorrect.

The most possible reason: they used the W boson propagator expanded over the field tensor $F^{\mu\nu}$ to the linear terms, while the quadratic terms are also essential.

The neutrino decay $\nu_e \to e^- W^+$ in magnetic field

In the field of the magnetar scale, $\sim 10^{14} - 10^{15}$ G, the crossed-field approximation is inapplicable. We use the hierarchy $p_\perp^2 \gg m_W^2 \gg eB \gg m_e^2$, neglecting the electron mass as the smallest parameter. For the process width we obtain:

$$w(\nu \to e^- W^+) = \frac{G_{\mathrm{F}} \, (eB)^{3/2} \; p_\perp}{\pi \sqrt{2\pi} \, E} \, \Phi(\eta) \, ,$$

where $\Phi(\eta)$ is the function depending on the parameter η only:

$$\eta = \frac{4 \; eBp_\perp^2}{m_W^4} \, .$$

The neutrino decay $\nu_e \to e^- W^+$ in magnetic field

The function $\Phi(\eta)$ is rather cumbersome in the general case. It can be essentially simplified at large and small values of the argument. In the limit $\eta \gg 1$ one obtains:

$$\Phi(\eta \gg 1) \simeq \frac{1}{3} \sqrt{\pi(\eta - 0.3)} \, ,$$

and the error is less than 1 % for $\eta > 10$.

In the other limit $\eta \ll 1$ one obtains

$$\Phi(\eta \ll 1) \simeq \exp\left(-\frac{1}{\eta}\right) \left(1 - \frac{1}{2}\,\eta + \frac{3}{4}\,\eta^2\right) .$$

and the error is less than 1 % for $\eta < 0.5$.

The neutrino energy cutoff in a strong magnetic field

An upper limit exists on the energy spectrum of neutrinos propagating in a strong magnetic field.

If the neutrino mean free path $\lambda = 1/w$ is much less than the typical field size R (of the region with the strong magnetic field, $R \sim 10$ km), all the neutrinos are decaying in such the field.

For $\lambda = 1$ km $\ll R$, the cutoff energies E_c can be found for the neutrino spectrum, depending on the magnetic field strength.

Two limiting cases:
i) relatively weak field, $B \simeq 0.1 B_e$;
ii) relatively strong field, $B \simeq 10 B_e$.

The neutrino energy cutoff vs the magnetic field strength

i) In **the relatively weak field** limit, $B \simeq 0.1 B_e \simeq 4 \times 10^{12}$ G, the neutrino mean free path is:

$$\lambda \simeq \frac{4.9\,\mathrm{m}}{B_{0.1}\ \sin\theta}\ \exp\left(\frac{219}{B_{0.1}\, E_{15}\ \sin\theta}\right),$$

where $B_{0.1} = B/(0.1 B_e)$, $E_{15} = E/(10^{15}\mathrm{eV})$.

The cutoff energy corresponding to $\lambda = 1$ km, at $B_{0.1} = 1$, $\theta = \pi/2$, is

$$E_c \simeq 0.4 \times 10^{17}\mathrm{eV}\,.$$

The neutrino energy cutoff vs the magnetic field strength

ii) In **the relatively strong field** limit, $B \simeq 10 B_e \simeq 4 \times 10^{14}$ G, the neutrino mean free path is:

$$\lambda \simeq \frac{3.2\,\mathrm{cm}}{B_{10}^{3/2}\, \sin\theta}\, \exp\left(\frac{4.0}{B_{10}\, E_{15}^2\, \sin^2\theta} \right),$$

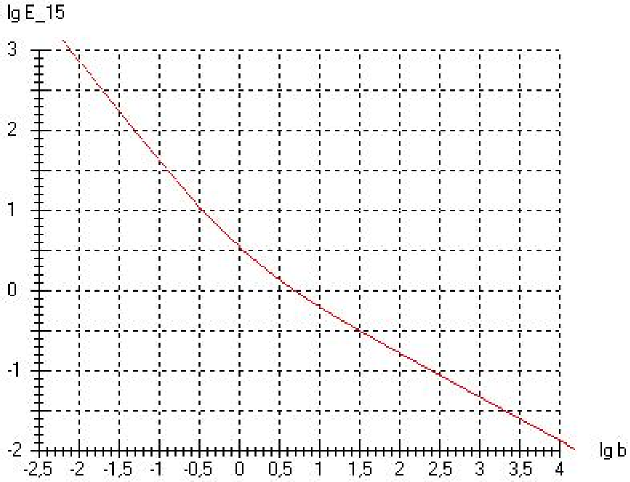
where $B_{10} = B/(10 B_e)$.

The cutoff energy corresponding to $\lambda = 1$ km, at $B_{10} = 1$, $\theta = \pi/2$, is

$$E_c \simeq 0.6 \times 10^{15} \mathrm{eV}\,.$$

The cutoff energy $E_c = 10^{15}\mathrm{eV}$ at $B \simeq 5 B_e \simeq 2.2 \times 10^{14}$ G.

The neutrino energy cutoff vs the magnetic field strength



Conclusions

- An influence of a strong external magnetic field on the neutrino self-energy operator is investigated.

Conclusions

- An influence of a strong external magnetic field on the neutrino self-energy operator is investigated.
- The width of the neutrino decay into the electron and W boson, and the mean free path of an ultra-high energy neutrino in a strong magnetic field are calculated.

Conclusions

- An influence of a strong external magnetic field on the neutrino self-energy operator is investigated.
- The width of the neutrino decay into the electron and W boson, and the mean free path of an ultra-high energy neutrino in a strong magnetic field are calculated.
- An energy cutoff for neutrinos propagating in a strong field is defined.
  The cutoff energy $E_c = 10^{15}\mathrm{eV}$ at $B \simeq 5 B_e \simeq 2.2 \times 10^{14}$ G.

Thank you for your attention!