Perturbative calculations in holographic QCD: Magnetic susceptibility of the vector current.

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In this talk we study the formulation of the perturbation theory technique in holographic QCD (AdS/QCD), known as Witten diagrams. This method is extremely useful in computing various correlation functions in holographic models. It turns out that not in all AdS/QCD models a consistent perturbation theory exists.

As an example, we compute the correlator of two electromagnetic quark currents of QCD in external magnetic field, which is related to the four-point correlation function of vector currents. This value is named the magnetic susceptibility of electromagnetic current and can be observed in heavy ion collisions experiments (RHIC).

Space geometry in AdS/QCD

AdS/CFT	$\longrightarrow$	**AdS/QCD**
N=4 SYM	$\longrightarrow$	**QCD**
R-symmetry breaking:		
S_5	$\longrightarrow$	$\varnothing$
absense of conformality :		
AdS_5	$\longrightarrow$	AdS_5, deformed at $z = z_{IR}$

Types of deformation at IR

Hard wall: $ds^2 = \frac{1}{z^2}(dz^2 + dx_\mu dx^\mu)$	cut at $z = z_{IR}$ $z_{IR} \sim \Lambda_{QCD}^{-1}$
Soft wall (type +/-): $ds^2 = \frac{e^{\pm\gamma^2 z^2}}{z^2}(dz^2 + dx_\mu dx^\mu)$ or AdS_5 with $\phi(z) = \mp\gamma^2 z^2$	exponential cut $\gamma \sim \Lambda_{QCD}$
Thermal black hole: $ds^2 = \frac{1}{z^2}(f_{BH}(z) dz^2 - \frac{dt^2}{f_{BH}(z)} + dx_i dx^i)$ $f_{BH}(z) = 1 - \frac{z}{z_H}$	horizon at z_H $z_H \sim \frac{1}{T}$

Calculation of the correlator

The AdS/CFT correspondence states that

$$Z[J_i(x_\mu)] = iS_{str}[\Phi_{i\ cl}(z, x_\mu)]|_{\Phi_{i\ cl}(0)=J_i}.$$

Namely,

$$\langle O_1 \dots O_n \rangle_{connected} = \frac{\delta^n}{\delta\Phi_1(0)\dots\delta\Phi_n(0)} S_5(\Phi_i(z)_{classic})\Big|_{\Phi_i(0)=0}$$

This means that for a calculation of the n-point function it's enough to find the action up to the n-th order in boundary values of the corresponding field.

Perturbation theory in boundary values: 1st order

Consider the equation of motion for the field $\Phi^a(x,z)$ expanding it in the b.v. of fields $(\tilde{\Phi}^a(x))$:

$$K[\Phi^a(x,z)] = 0 + O(\tilde{\Phi}^a(x))$$

Here K is the kinetic kernel of the corresponding action:

$$S(\Phi^a) = \int \Phi^a(x,z) K[\Phi^a(x,z)] + \text{interactions}$$

Perturbation theory in boundary values: 1st order

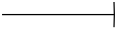
The first order solution with boundary condition $\Phi^a(x,z)|_{z=0} = \tilde{\Phi}^a(x)$ is

$$\Phi^{(1)a}(x,z) = \int d^4x \; \phi(z,x)\tilde{\Phi}^a(x),$$

where

$$\phi(z,x)|_{z=0} = 1, \qquad K[\phi(x,z)] = 0.$$

The function $\phi(z,x)$ is known as the bulk-to-boundary propagator and will be depicted as


$\tilde{\Phi}^a$

Perturbation theory in boundary values: 2-nd order

The in the second order, one gets the equation, which includes nonlinear terms with the first order solution:

$$K[\Phi^{(2)}(x,z)] = \mathbb{V}^3[\Phi^{(1)}(x,z)^2] + \mathbb{V}^4[\Phi^{(1)}(x,z)^3] + \dots$$

Where ($\mathbb{V}^3$, $\mathbb{V}^4$, ...) are interaction vertices in the action:

$$S(\Phi^a) = \int \Phi K[\Phi] + \Phi\mathbb{V}^3[\Phi^2] + \Phi\mathbb{V}^4[\Phi^3] + \dots$$

Perturbation theory in boundary values: 2-nd order

The solution for $\Phi^{(2)}(x,z)$ is obtained via the Green function $G_\Phi(x_1,x_2,z_1,z_2)$:

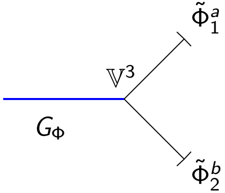
$$\Phi^{(2)}(x_1,z_1) = \int G_\Phi(x_1,z_1,x_2,z_2)(\mathbb{V}^3[\Phi^{(1)}(x_2,z_2)^2])\ d^4x_2 dz$$

which is the inverse of the kinetic kernel with zero boundary conditions

$$G = K^{-1}, \qquad G_\Phi(x_1,z_1,x_2,z_2)|_{z_1=0} = 0$$

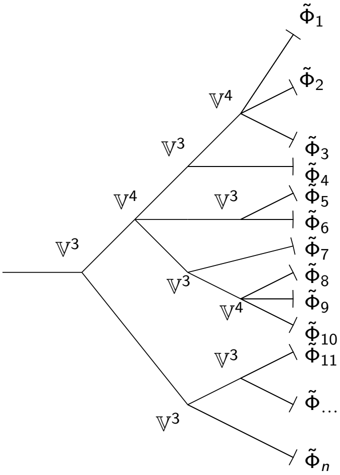
This Green function is named the bulk-to-bulk propagator.

And the 2-nd order solution is depicted as:



Perturbation theory in boundary values: n-th order

The general n-th order solution can look like

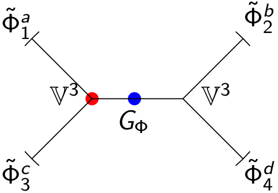


The action

Now we substitute the solution into the action. For example for the action of the 4-th order:

$$S(\Phi^{(1)} + \Phi^{(2)} + \dots)^{(4)} = \int \Phi^{(1)} K[\Phi^{(3)}] + \Phi^{(2)} K[\Phi^{(2)}] + \\ + \Phi^{(2)} \mathbb{V}^3[(\Phi^{(1)})^2] + \dots$$

We get the diagram, where all bulk-to-bulk and bulk-to-boundary propagators are glued by kinetic and vertex operators of the action.



Problems with bulk-to-bulk propagator

Any diagram with bulk-to-bulk propagator (starting from the 4-th order in sources) contains the integral:

$$\int_{\epsilon}^{z_{IR}} dz_2 \; G_{\Phi}(z_1, z_2) f(z_2)$$

In the model like type "-" soft-wall, which has the action

$$S = \int \frac{e^{-\gamma^2 z^2}}{z^2} (\Phi \partial_z^2 \Phi) + \dots$$

and has no cut off ($z_{IR} = \infty$) the Green function behaves as:

$$G_{\Phi}(z1, z2) \sim K^{-1} \sim e^{+\gamma^2 z^2}$$

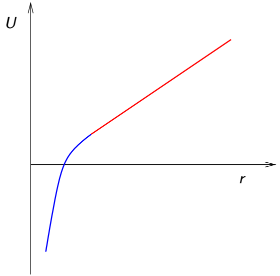
and the perturbation series diverges exponentially.

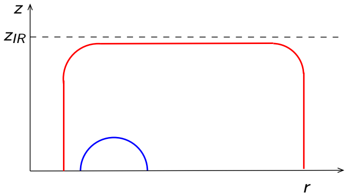
Connection with confinement

Confinement of quarks is manifested by the linear quark interaction potential at large distances.

In the AdS/CFT the interaction potential is related to the area of the minimal surface in the bulk.

If the surface is saturated at some coordinate z_{IR} its area behaves as r, so the linear potential is arises.





Connection with confinement

The the type “-” soft-wall model does not demonstrate the confinement.
For the linear potential the growing warp factor in the metric is needed, for example that of the soft-wall type “+”

$$ds^2 = \frac{e^{+\gamma^2 z^2}}{z}(dz^2 + dx_\mu dx^\mu)$$

in order for the minimal surface to have a z_{IR} bound

Example:

The 4-point correlation function of vector currents in AdS/QCD

Field content of the AdS/QCD model

$$\left(\frac{2}{z}\right) X^{\alpha\beta} \leftrightarrow \bar{q}_R^{\alpha} q_L^{\beta} \qquad \text{scalar quark current}$$

$$\phi \leftrightarrow tr\, G_{\mu\nu}^2 \qquad \text{gluon operator}$$

$$A_{L\mu}^a \leftrightarrow \bar{q}_L \gamma^{\mu} t^a q_L \qquad \text{left quark current}$$

$$A_{R\mu}^a \leftrightarrow \bar{q}_R \gamma^{\mu} t^a q_R \qquad \text{right quark current}$$

$$\Downarrow$$

$$V_{\mu}^a \leftrightarrow \bar{q} \gamma^{\mu} t^a q \qquad \text{vector current}$$

$$A_{\mu}^a \leftrightarrow \bar{q} \gamma^{\mu} \gamma^5 t^a q \qquad \text{axial-vector current}$$

Chiral symmetry breaking

The chiral symmetry breaking order parameter $\sigma \sim \langle \bar{q} q \rangle$ is encoded in the normalizeble mode of the vacuum profile of the field X

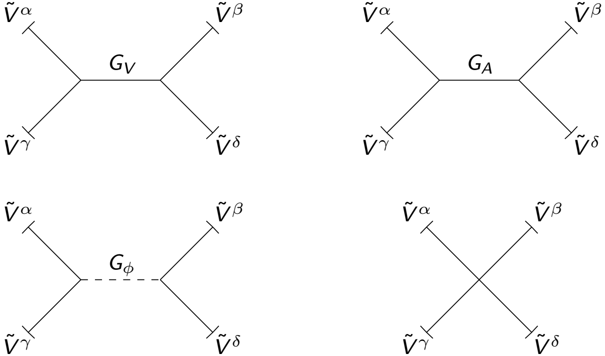
$$X^a(z) = e^{i\pi^a t^a} \chi(z) = e^{i\pi^a t^a}(mz + \sigma z^3)$$

In the model only axial-vector field A^a_μ interacts with X^a, so the $L \leftrightarrow R$ symmetry is broken.

$$S \sim \int \sqrt{g} \langle F_V^2 + F_A^2 + 6\chi(z)^2 (A - \partial\pi)^2 + \ldots \rangle$$

the 4-point correlator of vector current

The action of the 4-th order in vector sources is depicted as.



The dilaton propagator is proportional to $\frac{1}{N_c^2}$, so its diagram is negligeble.

The two currents correlator in external magnetic field

We are interested in the correlator

$$\langle J_\alpha(Q) J_\beta(-Q) \rangle_B$$

In the second order in the field the correlator can be expressed via the 4-point function:

$$\langle J_\alpha(Q) J_\beta(-Q) \rangle_{B^2} = \langle J_\alpha(Q) J_\beta(-Q) J_\gamma(k_1) J_\delta(k_2) \rangle e^1_\gamma e^2_\delta |_{k_1, k_2 \to 0}$$

Result: hard wall

$$\langle J^V(Q)J^V(Q)\rangle_B\Big|_{HW} = B^2 \times \left(-\frac{3}{2}\right)\frac{N_c}{\pi^2}Q^4(-1.22\,Gev^{-6}).$$

- The coefficient changes weakly if $\sigma \to 0$, as the chiral symmetry is restored.
- It diverges if $z_{IR} \sim \Lambda_{QCD}^{-1} \to \infty$, as the conformal symmetry is restored.

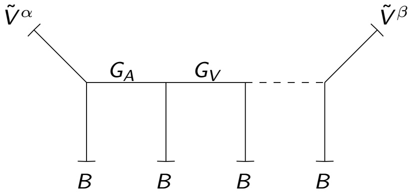
Result: thermal black hole

In the thermal black hole background the temperature T is nonzero, and the model is in the deconfined phase $\langle \bar{q}q \rangle = 0$. In this setting the result is nonzero at small momenta:

$$\langle J^V(Q) J^V(Q) \rangle_B \Big|_{BH, Q \to 0} = \left(-\frac{3}{2} \right) \frac{N_c}{\pi^2} (0.19) \frac{B^2}{T^2}.$$

Result: thermal black hole

One can evaluate the whole series of diagrams of the type:



and get the result as series in $\frac{B^2}{T^4}$

$$\langle J^V(Q) J^V(Q) \rangle_B \sim T^2 \left(-0.19 \frac{B^2}{T^4} + 0.018 \frac{B^4}{T^8} - 0.00176 \frac{B^6}{T^{12}} + \dots \right)$$

$$? \sim T^2 \; F\left(\frac{B^2}{T^4} \right)$$

Conclusion

- The diagram technique for computation of correlators is only applicable for models with confinement.
- The 2-point function of vector currents in external field is governed not by the chiral symmetry breaking, but by the confinement properties. In absence of chiral chemical potential and at $T = 0$ it behaves as Q^4 at small Q.
- At finite temperature the correlator is finite and the summation of all orders of perturbation theory in $\frac{B^2}{T^4}$ is possible.

Thank you for your attention!

Let's eat!