Troubles of describing multiple pion production in chiral dynamics.

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Outline

Quantum Chromodynamics.

Quantum chromodynamics QCD Lagrangian

$$\mathcal{L}_{\rm QCD} = -\frac{1}{4}G_{a\mu\nu}G_{a\mu\nu} + \sum_{q=u,d,s,c,b,t} \bar{q}_A \times \left[\gamma_\mu\left(i\partial_\mu - g\frac{\lambda^a}{2}G_{a\mu}\right) - m_q\right]_{AB} q_B,$$

[$a = 1\cdots 8$, $A, B = 1\cdots 3$]. Calculational tools based on perturbation theory work beautifully at high energy due to the property of asymptotic freedom: effective coupling strength of interaction of quarks and gluons tends to zero.

• At low energy ≤ 1 GeV coupling is not small, and perturbation theory fails in predicting hadronic properties.

Chiral symmetry and its spontaneous breaking

The theory aimed at describing low energy hadron processes should be formulated in terms of effective colorless degrees of freedom introduced on the basis of spontaneously broken chiral symmetry $SU(3)_L \times SU(3)_R$ which is the symmetry of QCD Lagrangian relative independent rotations of right and left fields of approximately massless u, d, s quarks:

$$q_L \equiv \frac{1+\gamma_5}{2} q \quad \to \quad g_L q_L,$$
$$q_R \equiv \frac{1-\gamma_5}{2} q \quad \to \quad g_R q_R,$$

$[g_{L,R} \in SU(3)_{L,R}]$

The pattern of the spontaneous breaking is

$$SU(3)_L \times SU(3)_R \Rightarrow SU(3)_{L+R}.$$

Goldstone Theorem

- The Goldstone theorem: Spontaneous breaking of global symmetry results in appearance of massless fields (Goldstone bosons).
  - Idea of proof: $\hat{H}$-Hamiltonian, $\hat{Q}$-symmetry generator, $[\hat{Q},\hat{H}] = 0$, $|0\rangle$-zero energy state (vacuum), $\hat{H}|0\rangle = 0$. If $\hat{Q}|0\rangle = |\psi\rangle \neq 0$ (spontaneous breaking) then $[\hat{Q},\hat{H}]|0\rangle = -\hat{H}|\psi\rangle = 0 \Rightarrow |\psi\rangle$ is zero energy state (Goldstone bosons).

In case of spontaneous breaking of $SU(3)_L \times SU(3)_R \Rightarrow SU(3)_{L+R}$ they are light $J^P = 0^-$ mesons π^+, π^-, π^0, K^+, K^0, K^-, $\bar{K}^0$, η.

Linear Realization

- Mesons as condensates of quark-antiquark pairs: $M \sim \langle \bar{q}_L q_R \rangle$.
- Transformation Law: $q_{L,R} \to q_{L,R} g^\dagger_{L,R} \Longrightarrow M \to g_L M g^\dagger_R$
- In case of $SU_R(2 \times SU_L(2)$ meson fields are $\boldsymbol{\pi} = (\pi^+, \pi^- \pi^0)$ (pseudoscalar pions) and scalar σ: $M = \frac{1}{2}(\sigma + \boldsymbol{\tau} \cdot \boldsymbol{\pi})$.
- Effective Lagrangian: $\mathcal{L} = \frac{1}{2}\mathrm{Tr}(\partial_\mu M \partial_\mu M^\dagger) - V(M)$.

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Nonlinear Realization

Fields are transformed nonlinearly under action of g: $\pi^a \to \varphi^a(g,\pi)$. The transformation law $U \to g_L U g_R^\dagger$, $U = \exp\left(i\boldsymbol{\Phi}\sqrt{2}/f_\pi\right)$,

$$\boldsymbol{\Phi} = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta}{\sqrt{6}} \end{pmatrix}.$$

Effective Lagrangian in Nonlinear Realization

Transformation law fixes the Lagrangian of interacting Goldstone mesons:

$$\mathcal{L}_{\mathrm{GB}} = \frac{f_\pi^2}{4}\mathrm{Sp}\left(\partial_\mu U \partial_\mu U^\dagger\right) + m_\pi^2 \mathrm{Sp}(U + U^\dagger) + \cdots .$$

The last term explicitly breaks chiral symmetry and makes Goldstone bosons massive.

- Expansion parameter: $\frac{|\boldsymbol{q}_\pi|}{f_\pi} \ll 1 \Rightarrow$ theory should work for pions with low momenta (soft pions).

Problems

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Hidden Local Symmetry

- "Hidden" means that if $U = \xi_L^\dagger \xi_R$ then $\xi_{L,R} \to h\xi_{L,R} g_{L,R}^\dagger$ implies $U \to g_L U g_R^\dagger$ where h transform vector meson fields as $V_\mu \to hV_\mu h^\dagger - i\partial_\mu h h^\dagger$.
- "Generalized hidden" means that axial vector mesons are included.
- Numerous chiral invariant models with vector and axial vector mesons. So Why Hidden Local Symmetry (HLS)?
- HLS has many appealing features. One of them is that Electroweak sector is introduced in a way that low energy relations among strong coupling constants are not violated by electroweak gauge bosons.

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GHLS lagrangian (strong interactions)

$$
\begin{aligned}
\mathcal{L} \;=\;& a_0 f_\pi^{(0)2}\mathrm{Tr}\left(\frac{\partial_\mu\xi^\dagger\xi+\partial_\mu\xi\xi^\dagger}{2i}-gV_\mu\right)^2+\\
& b_0 f_\pi^{(0)2}\mathrm{Tr}\left(\frac{\partial_\mu\xi^\dagger\xi-\partial_\mu\xi\xi^\dagger}{2i}+gA_\mu\right)^2+\\
& c_0 f_\pi^{(0)2}g^2\mathrm{Tr}A_\mu^2+d_0 f_\pi^{(0)2}\mathrm{Tr}\left(\frac{\partial_\mu\xi^\dagger\xi-\partial_\mu\xi\xi^\dagger}{2i}\right)^2-\\
& \frac{1}{2}\mathrm{Tr}\left(F_{\mu\nu}^{(V)2}+F_{\mu\nu}^{(A)2}\right)-i\alpha_4 g\mathrm{Tr}[A_\mu,A_\nu]F_{\mu\nu}^{(V)}+\\
& 2i\alpha_5 g\mathrm{Tr}\left(\left[\frac{\partial_\mu\xi^\dagger\xi-\partial\xi\xi^\dagger}{2ig},A_\nu\right]+[A_\mu,A_\nu]\right)F_{\mu\nu}^{(V)}.
\end{aligned}
$$

Notations

- Vector and axial vector field strengths:

$$F^{(V)}_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu - ig[V_\mu, V_\nu] - ig[A_\mu, A_\nu],$$
$$F^{(A)}_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[V_\mu, A_\nu] - ig[A_\mu, V_\nu],$$

- Fields $V_\mu = \left(\frac{\tau}{2} \cdot \rho_\mu\right)$, $A_\mu = \left(\frac{\tau}{2} \cdot \mathbf{A}_\mu\right)$, $\xi = \exp i \frac{\tau \cdot \pi}{2f^{(0)}_\pi}$,
- Exclude axial vector-pseudoscalar mixing by means of introduction of physical $a_1(1260)$ meson field a_μ

$$A_\mu = a_\mu - \frac{b_0}{g(b_0 + c_0)} \frac{\partial_\mu \xi^\dagger \xi - \partial_\mu \xi \xi^\dagger}{2i}$$

- Renormalize according to $f_\pi \to Z^{-1/2} f_\pi$, $\pi \to Z^{-1/2}\pi$, $(a_0, b_0, c_0, d_0) = Z(a, b, c, d)$,

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Lagrangian of simple HLS without axial vectors

- The lagrangian of the hidden local symmetry at lowest order in derivatives:

$$
\begin{aligned}
\mathcal{L}^{\mathrm{HLS}} &= -\frac{1}{4}\boldsymbol{\rho}^2_{\mu\nu} + \frac{1}{2}ag^2f^2_\pi\boldsymbol{\rho}^2_\mu + \\
&\frac{1}{2}\left(\partial_\mu\boldsymbol{\pi}\right)^2 - \frac{1}{2}m^2_\pi\boldsymbol{\pi}^2 + \frac{m^2_\pi}{24f^2_\pi}\boldsymbol{\pi}^4 + \\
&\frac{1}{2f^2_\pi}\left(\frac{a}{4} - \frac{1}{3}\right)\left[\boldsymbol{\pi} \times \partial_\mu\boldsymbol{\pi}\right]^2 + \\
&\frac{1}{2}ag\left(1 - \frac{\boldsymbol{\pi}^2}{12f^2_\pi}\right)\left(\boldsymbol{\rho}_\mu \cdot \left[\boldsymbol{\pi} \times \partial_\mu\boldsymbol{\pi}\right]\right)
\end{aligned}
$$

Adding axial vector meson a_1

GHLS lagrangian generates additional terms. • The lagrangian of the $a_1\rho\pi$ at lowest order in derivatives:

$$\begin{aligned}\mathcal{L}_{a_1} &= -\frac{1}{4}\boldsymbol{a}_{\mu\nu}^2 + \frac{1}{2}(b+c)g^2f_\pi^2\boldsymbol{a}_\mu^2 - \frac{\alpha_5}{f_\pi}\boldsymbol{\rho}_{\mu\nu}\cdot[\boldsymbol{a}_\mu \times \partial_\nu\boldsymbol{\pi}] - \\ &\frac{r}{f_\pi}\boldsymbol{a}_{\mu\nu}\cdot[\boldsymbol{\rho}_\mu \times \partial_\nu\boldsymbol{\pi}] - \frac{r^2}{gf_\pi^3}[\boldsymbol{a}_\mu \times \partial_\nu\boldsymbol{\pi}]\cdot[\partial_\mu\boldsymbol{\pi} \times \partial_\nu\boldsymbol{\pi}] - \\ &\frac{r}{2gf_\pi^3}\partial_\mu\boldsymbol{a}_\nu\cdot[\boldsymbol{\pi} \times [\partial_\mu\boldsymbol{\pi} \times \partial_\nu\boldsymbol{\pi}]].\end{aligned}$$

$\boldsymbol{\rho}_{\mu\nu} = \partial_\mu\boldsymbol{\rho}_\nu - \partial_\nu\boldsymbol{\rho}_\mu$, $\boldsymbol{a}_{\mu\nu} = \partial_\mu\boldsymbol{a}_\nu - \partial_\nu\boldsymbol{a}_\mu$, $r = \frac{b}{b+c}$.

Outline

GHLS (charged electroweak sector)

- Small momenta expansion of electroweak piece of GHLS lagrangian

$$\Delta\mathcal{L}_{\rm EW} = \frac{1}{2}g_2 V_{ud}\boldsymbol{W}_\perp\left(-f_\pi\partial_\mu\boldsymbol{\pi}_\perp + \frac{1}{3f_\pi}[\boldsymbol{\pi}\times[\boldsymbol{\pi}\times\partial_\mu\boldsymbol{\pi}]]_\perp + bgf_\pi^2\boldsymbol{a}_{\mu\perp} + agf_\pi[\boldsymbol{\pi}\times\boldsymbol{\rho}_\mu]\right),$$

$$M_{a_1 3\pi} \equiv M[a_1^-(q) \to \pi^+(q_1)\pi^-(q_2)\pi^-(q_3)],$$

$$iM_{a_1 3\pi} = \frac{agr}{2f_\pi}\epsilon_\mu \left(A_1 q_{1\mu} + A_2 q_{2\mu} + A_3 q_{3\mu}\right),$$

where ϵ_μ is the polarization four-vector of a_1 meson, and

$$A_1 = (1+\hat{P}_{23})\left\{\frac{\beta[(q_3,q_1-q_2) - qq_3 + m_\pi^2] - qq_3}{D_\rho(q_1+q_2)} + \frac{4r^2(\beta-1)q_2q_3 + q^2 - qq_1}{2m_\rho^2}\right\},$$

$$A_2 = \frac{\beta[(q_3,q_1-q_2) + qq_3 - m_\pi^2] + qq_3}{D_\rho(q_1+q_2)} + \frac{(q_2,q_1-q_3)}{D_\rho(q_1+q_3)} - \frac{2r^2(\beta-1)q_1q_3 + qq_1}{m_\rho^2}.$$

Couplings and masses through free parameters

- Couplings and masses through free parameters

$$g_{\rho\pi\pi} = \frac{ag}{2},\; m_\rho^2 = ag^2 f_\pi^2,\; m_{a_1}^2 = (b+c)g^2 f_\pi^2,$$

$f_\pi = 92.4$ MeV is the pion decay constant.

- Parameters r and β are the combinations of the GHLS parameters:

$$r = \frac{b}{b+c},\; \beta = \frac{\alpha_5}{r}.$$

- Cancelation of momentum-dependent $\rho\pi\pi$ vertex

$$\frac{a}{2} = d + \frac{bc}{b+c}$$

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Feynman diagrams for required amplitudes

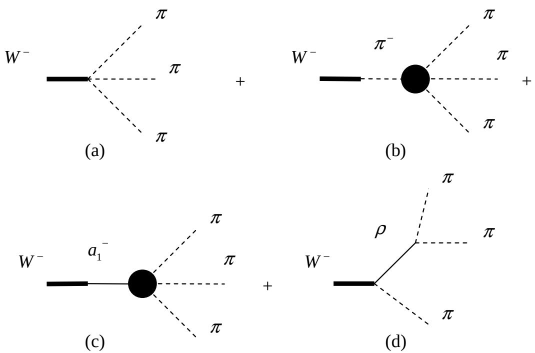

Figure: The diagrams due to HLS lagrangian

$W^- \to \pi^+\pi^-\pi^-$ decay amplitude

- $W^- \to \pi^+\pi^-\pi^-$ decay amplitude $iM = \frac{g_2 V_{ud}}{2f_\pi}\epsilon^{(W)}_\mu J_\mu$,

$$J_\mu = -q_{1\mu} + \frac{q_\mu}{D_\pi(q)}\left[m_\pi^2 - qq_1 + \frac{am_\rho^2}{2}(1+\hat{P}_{23})\frac{(q_2, q_1 - q_3)}{D_\rho(q_1+q_3)}\right] - \frac{ar^2 m_{a_1}^2}{2D_{a_1}(q)}\left\{A_1 q_{1\mu} + A_2 q_{2\mu} + A_3 q_{3\mu} - \frac{2q_\mu}{m_{a_1}^2}\times (1+\hat{P}_{23})\left[(m_\pi^2 + q_1 q_2)(q_3, q_1 - q_2)\times \left(\frac{\beta}{D_\rho(q_1+q_2)} - \frac{r^2(\beta-1)}{m_\rho^2}\right)\right]\right\} + \frac{am_\rho^2}{2}(1+\hat{P}_{23})\frac{(q_1-q_3)_\mu}{D_\rho(q_1+q_3)}.$$

Divergence of axial current

- Divergence of axial vector current in $\tau \to 3\pi\nu_\tau$

$$
\begin{aligned}
q_\mu J_\mu &= \frac{m_\pi^2}{D_\pi(q)}\left[q^2 - qq_1 + \frac{a}{2}m_\rho^2(1+\hat{P}_{23})\times\right.\\
&\left.\frac{(q_2, q_1 - q_3)}{D_\rho(q_1+q_3)}\right] - ar^2\frac{m_{a_1}^2 - q^2}{D_{a_1}(q)}\times\\
&(1+\hat{P}_{23})(m_\pi^2 + q_1q_2)(q_3, q_1 - q_2)\times\\
&\left[\frac{\beta}{D_\rho(q_1+q_2)} - \frac{r^2(\beta-1)}{m_\rho^2}\right].
\end{aligned}
$$

vanishes at a_1 mass shell in the limit $m_\pi \to 0$

Spectrum of 3π in $\tau \to 3\pi\nu_\tau$ decay

- Spectrum of $\pi^+\pi^-\pi^-$ in $\tau^- \to \pi^-\pi^-\pi^+\nu_\tau$ normalized to its branching fraction:

$$\frac{dB}{ds} = \frac{(G_F V_{ud})^2 (m_\tau^2 - s)^2}{2\pi (2m_\tau)^3 \Gamma_\tau} \left[(m_\tau^2 + 2s)\rho_t(s) + m_\tau^2 \rho_l(s) \right],$$

$s = q^2$.

- Spectral functions

$$\rho_t(s) = \frac{1}{3\pi s f_\pi^2} \int d\Phi_{3\pi} \left(\frac{|qJ|^2}{s} - |J|^2 \right),$$

$$\rho_l(s) = \frac{1}{\pi s^2 f_\pi^2} \int d\Phi_{3\pi} |qJ|^2,$$

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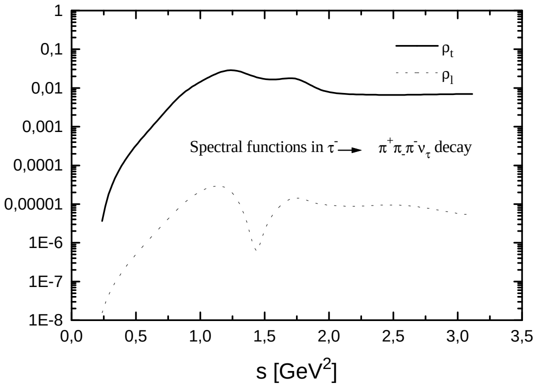

Figure: Spectral functions for $\tau^- \to \pi^+\pi^-\pi^-\nu_\tau$ in GHLS.

Outline

1. Introduction
2. GHLS lagrangian
3. Confronting GHLS with ALEPH data on $\tau^- \to \pi^+\pi^-\pi^-\nu_\tau$.
   - Decay amplitudes
   - Results
4. GHLS and the reaction $e^+e^- \to \pi^+\pi^-\pi^+\pi^-$ at $\sqrt{s} \leq 1$ GeV.
   - Diagrams and decay width of $\rho \to 4\pi$.
   - Results
5. Conclusion

Fitted parameters

- Canonical choice $(a, b, c, d, \alpha_5) = (2, 2, 2, 0, 1)$ does not reproduce data. Fitting them results in $\chi^2 = 690/112 d.o.f$.
- Set of fitted parameters is first taken to be

$$(m_{a_1}, a, r, \beta, m_{a_1'}, a', r', \beta', w', m_{a_1''}, a'', r'', \beta'', w''),$$

- w' parameterizes the coupling $a_1'\rho\pi$ as $g_{\rho\pi\pi}w'r'/f_\pi$. Analogously for w''.
- Fit chooses $w' \approx 1$, $\chi^2 = 122/102 d.o.f$ weakly depends on $w' \Longrightarrow$ instead of w' free is $\psi'-$ phase of a_1' contribution. Imitates a' mixing.

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$$(m_{a_1}, a, r, \beta, m_{a_1'}, a', r', \beta', w', m_{a_1''}, a'', r'', \beta'', w''),$$

- w' parameterizes the coupling $a_1'\rho\pi$ as $g_{\rho\pi\pi} w' r'/f_\pi$. Analogously for w''.
- Fit chooses $w' \approx 1$, $\chi^2 = 122/102 d.o.f$ weakly depends on $w' \Longrightarrow$ instead of w' free is $\psi'-$ phase of a_1' contribution. Imitates a' mixing.

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Best fit

- Fitted parameters:

$$\begin{aligned} m_{a_1} &= 1.332 \pm 0.015\ \text{GeV},\ a = 1.665 \pm 0.011, \\ & r = 0.332 \pm 0.007,\ \beta = 8.5 \pm 0.3, \\ & m_{a_1'} = 1.59 \pm 0.01\ \text{GeV},\ a' = 0.99 \pm 0.01, \\ & r' = 0.96 \pm 0.01,\ \beta' = 0.07 \pm 0.02, \\ & \psi' = 28^\circ \pm 1^\circ, \\ & m_{a_1''} = 1.88 \pm 0.02\ \text{GeV},\ a'' = 0.46 \pm 0.01, \\ & r'' = 1.45 \pm 0.02,\ \beta'' = 0.91 \pm 0.05, \\ & w'' = 1.14 \pm 0.01, \end{aligned}$$

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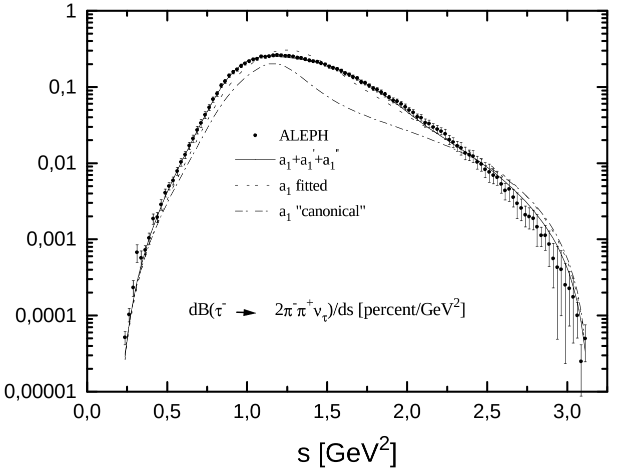

Figure: Spectrum normalized to $B(\tau^- \to \pi^+\pi^-\pi^-\nu_\tau)$

Comparison of canonical and fitted parameters

- Couplings and masses through free parameters:

$$g_{\rho\pi\pi} = \frac{ag}{2},\, m_\rho^2 = ag^2 f_\pi^2,\, m_{a_1}^2 = (b+c)g^2 f_\pi^2,$$

- Condition of cancelation of point-like $\gamma\pi^+\pi^-$, $W^-\pi^-\pi^0$ vertices in GHLS:

$$\frac{a}{2} = d + \frac{bc}{b+c} \Longrightarrow$$

- $(a, b, c, d, \alpha_5) =$
$(1.665 \pm 0.011, 1.5 \pm 0.1, 3.0 \pm 0.1, -0.16 \pm 0.04, 2.8 \pm 0.1)$

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Contributions to spectrum

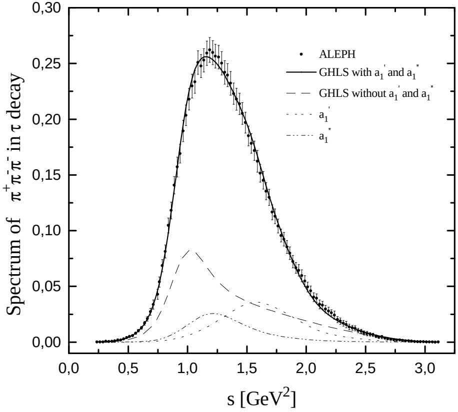

Figure: Contributions to spectrum of $\pi^-\pi^-\pi^+$ in τ decay.

- Contribution of diagrams (a), (b), (c), and (d) $B(\tau^- \to \pi^+\pi^-\pi^-\nu_\tau) \approx 2.65\%$.
- Contribution of (c) $B(\tau^- \to \pi^+\pi^-\pi^-\nu_\tau) \approx 0.33\%$
- Net contribution of (a), (b), and (d) (without a_1) is $B(\tau^- \to \pi^+\pi^-\pi^-\nu_\tau) \approx 0.47\%$.
- These should be compared to (c) in which a_1 is replaced by a_1' and a_1'': $B(\tau^- \to \pi^+\pi^-\pi^-\nu_\tau) \approx 1.15\%$ and 0.67%, respectively.

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Outline

1. Introduction
2. GHLS lagrangian
3. Confronting GHLS with ALEPH data on $\tau^- \to \pi^+\pi^-\pi^-\nu_\tau$.
   - Decay amplitudes
   - Results
4. GHLS and the reaction $e^+e^- \to \pi^+\pi^-\pi^+\pi^-$ at $\sqrt{s} \leq 1$ GeV.
   - Diagrams and decay width of $\rho \to 4\pi$.
   - Results
5. Conclusion

Effect of counter terms

- Generalized Hidden Local Symmetry model (GHLS) with particular choice of free parameters

$$(a, b, c, d, \alpha_5) = (2, 2, 2, 0, 1)$$

was applied to evaluate the four-pion process $\rho \to 4\pi$.

- The terms originating from counter terms and diagonalization of the $A_1 - \pi$ mixing:

$$\mathcal{L}^{(\rho\rho\pi\pi)} = -\frac{1}{16 f_\pi^2}\left([\boldsymbol{\rho}_\mu \times \partial_\nu \boldsymbol{\pi}] - [\boldsymbol{\rho}_\nu \times \partial_\mu \boldsymbol{\pi}]\right)^2 - \frac{1}{8 g f_\pi^4}[\boldsymbol{\rho}_\mu \times \partial_\nu \boldsymbol{\pi}] \cdot [\boldsymbol{\pi} \times [\partial_\mu \boldsymbol{\pi} \times \partial_\nu \boldsymbol{\pi}]]$$

Feynman diagrams for required amplitudes

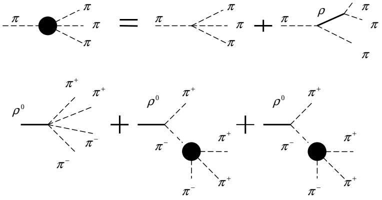

Figure: The diagrams due to HLS lagrangian

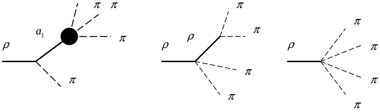

Figure: The diagrams due to $a_1\rho\pi$ and $\rho\rho\pi\pi$couplings (GHLS)

Decay width (proportional to probability)

- The reaction $\rho_q \to \pi_{q_1}\pi_{q_2}\pi_{q_3}\pi_{q_4}$. The $\rho \to 4\pi$ decay width

$$\Gamma_{\rho\to 4\pi}(s) = \frac{1}{3\pi^6 s^{3/2} 2^{12} N_s} \int_{s_{1-}}^{s_{1+}} ds_1 \int_{s_{2-}}^{s_{2+}} ds_2 \times \int_{u_{1-}}^{u_{1+}} \frac{du_1}{\lambda^{1/2}(s, s_2, s_2')} \int_{u_{2-}}^{u_{2+}} du_2 \int_{-1}^{1} \frac{d\zeta_2}{(1-\zeta_2^2)^{1/2}} \times |M_{\rho\to 4\pi}(s, s_1, s_2, u_1, u_2, t_2(\zeta_2))|^2,$$

$s = q^2$, $s_1 = (q - q_1)^2$, $s_2 = (q_3 + q_4)^2$, $u_1 = (q - q_2)^2$,
$u_2 = (q - q_3)^2$, $t_2 = (q_1 + q_4)^2$, $s_2' = (q_1 + q_2)^2$.

- $N_s = 4$ for $\pi^+\pi^-\pi^+\pi^-$ decay mode.

Point-like contribution

$$\mathcal{L}_{\text{photon}} = -e\mathcal{A}_\mu \left(2gf_\pi^2\rho^0_\mu - \frac{\pi^+\pi^-}{2f_\pi^2}[\boldsymbol{\pi} \times \partial_\mu \boldsymbol{\pi}]_3 - 2g\rho^0_\mu\pi^+\pi^- + 2gf_\pi[\boldsymbol{\pi} \times \boldsymbol{a}_\mu]_3\right).$$

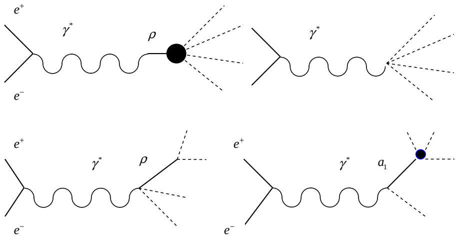

Figure: Diagrams describing process $e^+e^- \to \pi^+\pi^-\pi^+\pi^-$

Outline

Cross section

Cross section of $e^+e^- \to \pi^+\pi^-\pi^+\pi^-$:

$$\sigma_{e^+e^-\to 4\pi}(s) = \frac{12\pi m_\rho^3 \Gamma_{\rho e^+e^-}(m_\rho)\Gamma^{\text{eff}}_{\rho\to 4\pi}(s)}{s^{3/2}|D_\rho(q)|^2}$$

s is the total energy squared, $1/|D_\rho(q)|^2$ describes resonant production of pions, $\Gamma_{\rho\to f}$ proportional to probability of transition of ρ meson to final state f.

Results of evaluation of cross section in GHLS

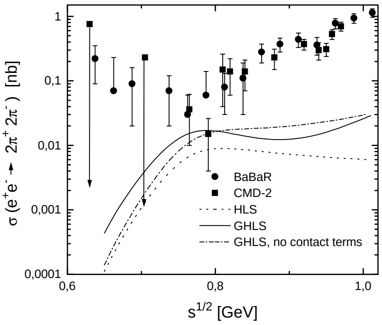

Figure: Results of evaluation of cross section in Generalized Hidden Local Symmetry Model

- Generalized hidden local symmetry model fails!

Adding heavier resonances

Taking into account ρ', ρ'' resonances: multiply cross section by

$$R(s) = \left|1 + \frac{D_\rho(q)}{1 + r(s)}\left[\frac{x_{\rho'}}{D_{\rho'}(q)} + \frac{x_{\rho''}}{D_{\rho''}(q)}\right]\right|^2,$$

$$r(s) = \left[\frac{\Gamma^{\mathrm{eff,no}a_1}_{\rho\to4\pi}}{\Gamma_{\rho\to a_1\pi\to4\pi}}\right]^{1/2} \exp(i\chi),$$

$$\chi = \cos^{-1}\frac{\Gamma^{\mathrm{eff}}_{\rho\to4\pi} - \Gamma^{\mathrm{eff,no}a_1}_{\rho\to4\pi} - \Gamma_{\rho\to a_1\pi\to4\pi}}{2\sqrt{\Gamma_{\rho\to a_1\pi\to4\pi}\Gamma^{\mathrm{eff,no}a_1}_{\rho\to4\pi}}}.$$

Results of fitting CMD-2 data

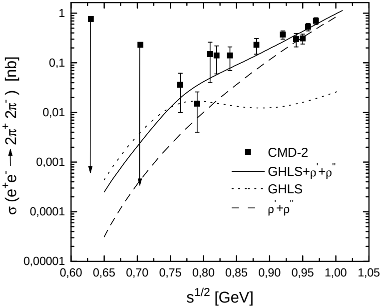

Figure: Fitting CMD-2 data

Results of fitting BaBaR data

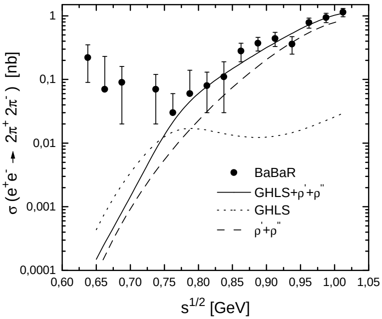

Figure: Fitting BaBaR data

Conclusion

- Simplest variant of GHLS model meets troubles when describing 3π in τ decay and 4π in e^+e^- annihilation.
- Chiral loops are insufficient in 4π (Ecker and Unterdorfer).
- Higher derivatives results in implosive growth of partial widths. Additional parameters stopping the growth are required. Alternatives:
- One should invoke the contributions of heavier axial vector mesons a_1', a_1'' in $\tau^- \to \pi^+\pi^-\pi^-\nu_\tau$.
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