On the Chiral Magnetic Effect in Soft-Wall AdS/QCD

A. S. Gorsky[1], P. N. Kopnin[1,2], A. V. Zayakin[1,3]

[1]ITEP, Moscow, Russia
[2]MIPT, Moscow, Russia
[3]Ludwig-Maximillian University, Munich, Germany

Quarks-2010, Kolomna, Russia,

June 7^{th}, 2010.

arXiv: 1003.2293[hep-ph]

Outline:

1. Essence of the Chiral Magnetic Effect (CME)
2. CME - weak-coupling results & the gauge/gravity framework
3. Experimental status of CME
4. "Bottom-Up" AdS/QCD Models, gauge sector of the soft-wall AdS/QCD
5. Classical solutions, boundary conditions, on-shell action and currents
6. Anomalies and the Bardeen counterterm & related discussion
7. Pseudoscalar contribution to the effect
8. Summary & Conclusions

Essence of the Chiral Magnetic Effect (CME)

Generation of an electric current parallel to a magnetic field in a topologically nontrivial background

[D. E. Kharzeev, L. D. McLerran and H. J. Warringa, 2008;

K. Fukushima, D. E. Kharzeev, and H. J. Warringa, 2008].

- Chirally symmetric phase of QCD with massless quarks q_L, q_R of a unit electromagnetic charge

- $q_L \rightarrow (q^+_{-1/2}, q^-_{+1/2})$ and $q_R \rightarrow (q^+_{+1/2}, q^-_{-1/2})$ (charge and helicity), magnetic moment $\propto$ charge $\times$ spin

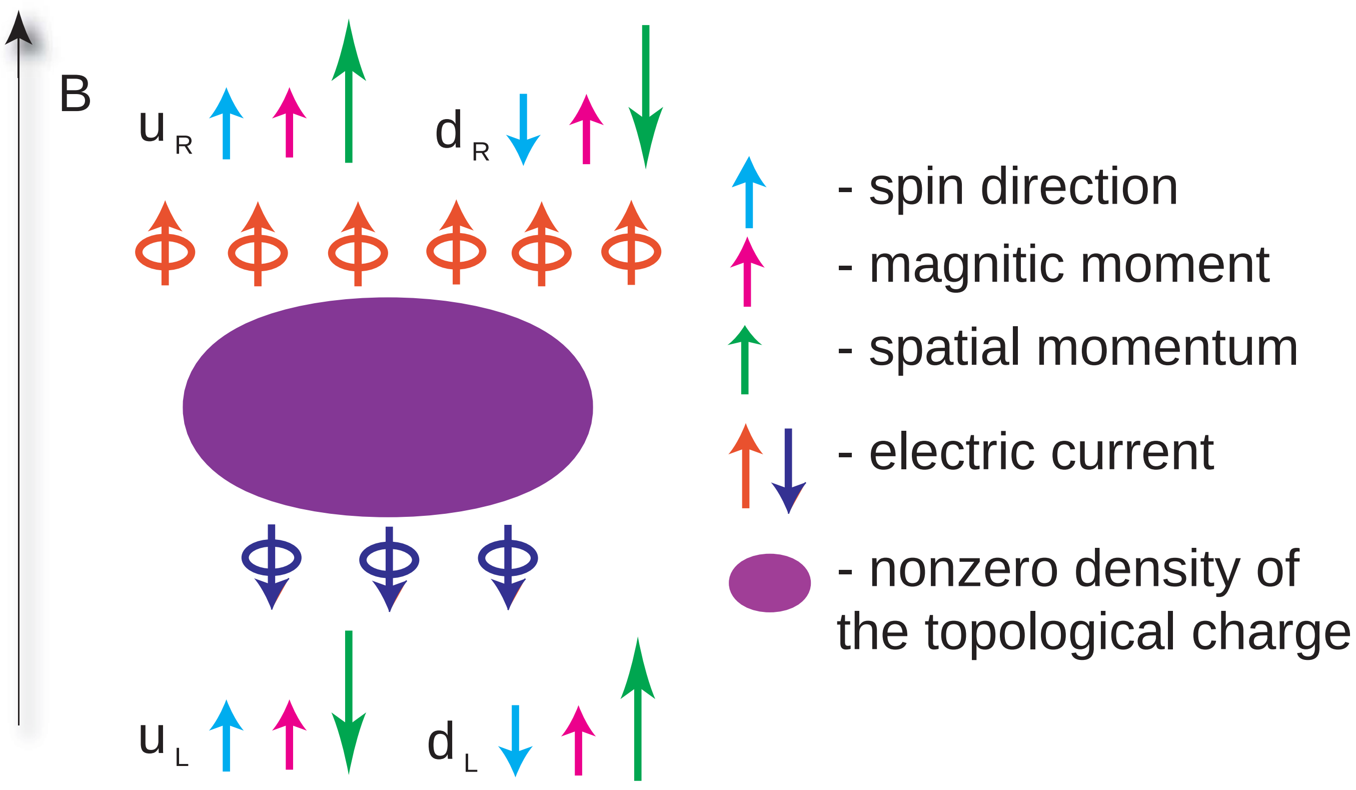
- External magnetic field aligns magnetic moments $\Rightarrow$ spins and momenta are correlated with the magnetic field

- Components $(q^+_{-1/2}, q^-_{+1/2})$ move in the opposite direction to the field, $(q^+_{+1/2}, q^-_{-1/2})$ – along the field

- A $U(1)_A$ chemical potential μ_5 induced by a sphaleron produces a state with a positive helicity:

$$N(q^-_{+1/2}) + N(q^+_{+1/2}) > N(q^+_{-1/2}) + N(q^-_{-1/2})$$

An electromagnetic current $J \propto$ l.h.s. - r.h.s.



The weak coupling results

In the weak-coupling limit [K. Fukushima, D. E. Kharzeev, and H. J. Warringa, 2008] the resulting current is

$$J_3^V = \frac{\mu_5 B}{2\pi^2} \equiv \mathcal{J}_{FKW}$$

For nonzero frequencies of the external field there is a drop to $1/3 \times \mathcal{J}_{FKW}$. A temperature-dependent expression for the susceptibility $\chi \propto T$ has also been obtained [K. Fukushima, D. E. Kharzeev, and H. J. Warringa, 2009].

CME in the gauge/gravity framework

The CME has been studied in the dual models to shed light into its properties in the strong-coupling limit:

- [H.-U. Yee, 2009] – in a model of Einstein gravity with a $U(1)_L \times U(1)_R$ Maxwell theory in the AdS_5 space and in the Sakai-Sugimoto model with a non-zero frequency. Results agree with those in the weak-coupling limit, except for the drop at $\omega = 0$.

- [A. Rebhan, A. Schmitt and S. A. Stricker, 2010] – in the Sakai-Sugimoto model at zero frequency. Result is 2/3 of the weak-coupling result in the absence of the Bardeen counterterm and is zero at zero frequency with the counterterm.

Experimental status of CME

Experimental status is discussed in [**STAR** collaboration and S. A. Voloshin, 2009], overview was given by Prof. Andianov.

Lattice calculations by the ITEP Lattice group [P. V. Buividovich, M. N. Chernodub, E. V. Luschevskaya and M. I. Polikarpov, 2009] – overview given by Pavel Buividovich.

Approaches to holographic QCD – "bottom-up"

Operators under consideration in "bottom-up" AdS/QCD:

- Symmetry currents $J^{a}_{L,R\ \mu}$ where a is an adjoint $SU(N_f)_{L,R}$ index;
- Chiral symmetry violation order parameter $\Sigma^{\alpha\beta} = \left\langle \bar{q}^{\alpha} q^{\beta} \right\rangle$ where $\alpha, \beta = 1...N_f$ are fundamental flavor indices.

Radial coordinate of the AdS is interpreted as the energy scale with UV region near the boundary. QCD is asymptotically conformal in the UV $\Rightarrow$ our 5D space is asymptotically AdS near the boundary. We have to modify the geometry in the IR region to reflect the confinement.

In order to sharpen the model we have to test its consistency – one has to calculate quantities known in QCD from the AdS point of view.

Five-Dimensional Effective Action

$$S_{5D} = \int d^5x\sqrt{g}e^{-\Phi}\text{tr}\left\{\Lambda^2\left(|DX|^2 + \frac{3}{R^2}|X|^2 - V(X)\right) - \frac{1}{4g_5^2}(F_L^2 + F_R^2)\right\}$$

with a metric $ds^2 = \dfrac{R^2}{z^2}(-dz^2 + dx_\mu dx^\mu)$ and dilaton Φ.

- "Hard-wall": $\Phi(z) \equiv 0,\ V(X) \equiv 0,\ 0 \leqslant z \leqslant z_m,$
- "Soft-wall": $\Phi(z) \sim \lambda z^2 (z \to \infty),\ V(X) \neq 0,\ 0 \leqslant z < \infty.$

$$\begin{aligned} L_\mu^a(x, z=0) &= \text{source of } \bar{q}_L(x)\gamma_\mu t^a q_L(x), \\ R_\mu^a(x, z=0) &= \text{source of } \bar{q}_R(x)\gamma_\mu t^a q_R(x), \\ \lim_{z\to 0}\frac{2}{z}X^{\alpha\beta}(x,z) &= \text{source of } \bar{q}_L^\alpha(x) q_R^\beta(x). \end{aligned}$$

If we KK-decompose all the fields and integrate out the dynamics along the z-axis, we get an effective action for mesons - a chiral Lagrangian.

Gauge sector of the soft-wall AdS/QCD

In the chirally symmetric phase we only need to consider the gauge sector, $|X| = 0$ (its phase – the 5D pion – is a more involved issue).

$$\begin{aligned} S &= S_{YM}[L] + S_{YM}[R] + S_{CS}[L] - S_{CS}[R] & (1) \\ S_{YM}[A] &= -\frac{1}{8g_5^2}\int e^{-\phi} F \wedge * F = -\frac{1}{8g_5^2}\int dz\ d^4x\ e^{-\phi}\sqrt{g} F_{MN}F^{MN} & (2) \\ S_{CS}[A] &= -\frac{N_c}{24\pi^2}\int A \wedge F \wedge F - \frac{1}{2} A \wedge A \wedge A \wedge F + \frac{1}{10} A \wedge A \wedge A \wedge A \wedge A \\ &= -\frac{N_c}{24\pi^2}\int dz\ d^4x\ \epsilon^{MNPQR} A_M F_{NP} F_{QR} & (3) \end{aligned}$$

with a metric tensor

$$ds^2 = g_{MN} dX^M dX^N = \frac{R^2}{z^2}\eta_{MN} dX^M dX^N = \frac{R^2}{z^2}(-dz^2 + dx_\mu dx^\mu) \quad (4)$$

.

Classical equations and the boundary conditions

Taking into account $\frac{R}{g_5^2} = \frac{N_c}{12\pi^2}$ and imposing a gauge $L_z = R_z = 0$ we obtain the following E.o.M.'s for the gauge fields:

$$\partial_z \left(\frac{e^{-\phi(z)}}{z} \partial_z L^\mu \right) - 24\epsilon^{\mu\nu\rho\sigma} \; \partial_z L_\nu \partial_\rho L_\sigma = 0 \tag{5}$$

$$\partial_z \left(\frac{e^{-\phi(z)}}{z} \partial_z R^\mu \right) + 24\epsilon^{\mu\nu\rho\sigma} \; \partial_z R_\nu \partial_\rho R_\sigma = 0 \tag{6}$$

with the following boundary conditions:

$$L_0(0) = \mu_L, \qquad R_0(0) = \mu_R, \tag{7}$$

$$L_3(0) = j_L, \qquad R_3(0) = j_R, \tag{8}$$

$$L_1(0, x_2) = -\frac{1}{2} x_2 B, \qquad R_1(0, x_2) = -\frac{1}{2} x_2 B, \tag{9}$$

$$L_\mu(\infty) = R_\mu(\infty), \qquad \partial_z L_\mu(\infty) = -\partial_z R_\mu(\infty). \tag{10}$$

- Here $\mu = \frac{1}{2}(\mu_L + \mu_R), \mu_5 = \frac{1}{2}(\mu_L - \mu_R)$ are the chemical potentials.

- $j_{L,R}$ are the gauge field boundary values, a variation with respect to which gives the currents

$$\frac{\delta S[L,R]}{\delta L_3(z=0)} = \frac{1}{V_{4D}} \frac{\partial S[L,R]}{\partial j_L} = \mathcal{J}_L, \tag{11}$$

$$\frac{\delta S[L,R]}{\delta R_3(z=0)} = \frac{1}{V_{4D}} \frac{\partial S[L,R]}{\partial j_R} = \mathcal{J}_R. \tag{12}$$

- Being source of the Abelian vector currents in the theory on the boundary, the Abelian magnetic field manifests itself as a boundary value for the 5D vector field strength.

- Boundary condition at $z = \infty$ is an adaptation of an analytical continuation of an analogous condition in the chirally broken Sakai-Sugimoto model.

On-shell action and currents

Action, estimated on-shell for the solutions of (5,6,10):

$$L_0(z) = \mu_L + \left(\mu_5 - \frac{1}{2}j_5\right)\left(e^{-|\beta|w(z)} - 1\right), \quad L_3(z) = j_L - \left(\mu_5 - \frac{1}{2}j_5\right)\left(e^{-|\beta|w(z)} - 1\right),$$
$$R_0(z) = \mu_R - \left(\mu_5 + \frac{1}{2}j_5\right)\left(e^{-|\beta|w(z)} - 1\right), \quad R_3(z) = j_R - \left(\mu_5 + \frac{1}{2}j_5\right)\left(e^{-|\beta|w(z)} - 1\right),$$
$$R_1(z, x_2) = -\frac{1}{2}x_2 B, \qquad L_1(z, x_2) = -\frac{1}{2}x_2 B, \tag{13}$$

here $j = j_L + j_R, j_5 = j_L - j_R$, and $w(z) = \int\limits_0^z du\ u\ e^{\phi(u)}$, $\dfrac{e^{-\phi(z)}}{z}\ w'(z) = 1$,

yields:

$$\mathcal{J} = \frac{2}{V_{4D}}\frac{\partial S}{\partial j} = \frac{N_c}{3\pi^2}B\mu_5, \tag{14}$$
$$\mathcal{J}_5 = \frac{2}{V_{4D}}\frac{\partial S}{\partial j_5} = -\frac{N_c}{3\pi^2}B(\mu + j_5). \tag{15}$$

The AdS/CFT prescription implies that we set $j_5 = 0$.

Anomalies and the Bardeen counterterm

In our setup there are two external gauge fields on the boundary – $V_\mu(z=0)$ and $A_\mu(z=0)$. $V_\mu(z=0)$ corresponds to $e\times$ an external electromagnetic field and provides μ, while a nonzero $A_\mu(z=0)$ accounts for μ_5. It has been pointed out in [A. Rebhan, A. Schmitt and S. A. Stricker, 2010] that the divergence of the vector current

$$\partial_\mu \mathcal{J}^\mu = -\frac{N_c}{24\pi^2} F^V_{\mu\nu} \tilde{F}^{A\ \mu\nu}$$

has to be compensated for by a local counterterm

$$S_{Bardeen} = c \int d^4x \epsilon^{\mu\nu\rho\sigma} L_\mu R_\nu (F^L_{\rho\sigma} + F^R_{\rho\sigma}), \qquad (16)$$

with an appropriate choice of the constant c.

$S_{Bardeen}$ may be considered as a product of holographic renormalization. Whether it needs to be taken into account remains unclear.

In our model $c = -\dfrac{N_c}{12\pi^2}$ and

$$\mathcal{J}_{subtracted} = \mathcal{J} + \mathcal{J}_{Bardeen} = \frac{N_c}{3\pi^2} B\mu_5 + \left(-\frac{N_c}{12\pi^2}\right) \times 4B\mu_5 = 0. \qquad (17)$$

Discussion of the relevance of the Bardeen counterterm

It was suggested by V. Rubakov [arXiv: 1005.1888[hep-ph]] that the counterterm has to be excluded from the calculation, since it fixes the anomaly of the vector current only in the presence of a real dynamical axial gauge field, while in our case we are dealing with a constant axial chemical potential, which is different from a constant temporal component of an axial gauge field.

In the absence of this counterterm the CME current in the strong coupling regime agrees exactly with the weak coupling limit (as it will be demonstrated below).

How to formally distinguish between the two aforementioned cases in holography is a problem still open to discussions.

Pseudoscalar contribution to the effect

The 5D scalar field $X = |X|e^{i\pi/f_\pi}$ interacts with the gauge fields via the covariant derivatives, thus inducing an interaction between π and the gauge field A_M.

- Usually the 4D pion is associated with a holonomy $\int A_z dz$ and the $\pi \to \gamma\gamma$ decay is determined by a part of the CS action $\int dzd^4x\ A_z F^V_{\mu\nu} F^{V\,\mu\nu}$. In the $A_z = 0$ gauge we have to reintroduce pion into the CS term.

- CS action is gauge invariant up to a surface term which is nonzero in our setup. In order to make it explicitly invariant we introduce 2 scalars:

$$S_{CS} = \frac{N_c}{24\pi^2}\left(\int L \wedge dL \wedge dL - \int R \wedge dR \wedge dR\right) \tag{18}$$

$$\to \frac{N_c}{24\pi^2}\left(\int (L + d\phi_L) \wedge dL \wedge dL - \int (R + d\phi_R) \wedge dR \wedge dR\right) \tag{19}$$

which under gauge transformations $L \to L + d\alpha_L$, $R \to R + d\alpha_R$ transform as $\phi_{L,R} \to \phi_{L,R} - \alpha_{L,R}$.

- $f_\pi(\phi_R - \phi_L)$ may be associated with the five-dimensional pion in the gauge in which A_z is set to zero.

- PCAC relation connecting the axial current and the pion field $\bar{\Psi}\gamma_\nu\gamma_5\Psi \Leftrightarrow f_\pi\partial_\nu\pi$ implies that we have to add the following term to the pion lagrangian $\mu_5 f_\pi\partial_0\pi$. Accordingly we have to modify the 5D action, obtaining an equation on the boundary: $\partial_0\pi + \mu_5 f_\pi = 0$. Thus, $\pi(z=0) = \frac{\mu_5}{f_\pi}t$.

  Additionally, in the D3/D7 models an R-symmetry chemical potential causes the D7 branes to rotate with an angular speed μ_R, so that the phase of the scalar field is $i\mu_R t$

- This yields another contribution to the CME:

$$\mathcal{J}_{\phi AA} = \frac{N_c}{6\pi^2}B\mu_5.$$

Summary table

Here is a summary of all the contributions to the CME:

Term in the action	Yang–Mills		Chern–Simons		Bardeen counterterm	Scalars in CS
	bulk	boundary	bulk	boundary		
Contribution to the current	$-\frac{1}{3}\frac{N_c}{2\pi^2}B\mu_5$	$\frac{1}{3}\frac{N_c}{2\pi^2}B\mu_5$	$\frac{1}{3}\frac{N_c}{2\pi^2}B\mu_5$	$\frac{1}{3}\frac{N_c}{2\pi^2}B\mu_5$	$-\frac{2}{3}\frac{N_c}{2\pi^2}B\mu_5$	$\frac{1}{3}\frac{N_c}{2\pi^2}B\mu_5$

Action taken into account	Total		Total without scalars	
	subtracted	nonsubtracted	subtracted	nonsubtracted
Resulting current, in terms of $\frac{N_c}{2\pi^2}B\mu_5$	$\frac{1}{3}$	1	0	$\frac{2}{3}$

Conclusions

- We have presented a calculation of the CME in soft-wall AdS/QCD.
- Model under consideration has the same basic features as the other holographic models where the CME has been studied, which has been confirmed independently. However, the results differ from those in the Sakai-Sugimoto model due to the presence of scalar fields.
- Those scalars act as 'catalysts' for the effect, only triggering it, while its magnitude is defined by the Chern-Simons action.
- The nature of the effect remains topological and it does not depend either on the dilaton or on the metric or on the details of the scalar Lagrangian.
- CME in soft-wall AdS/QCD amounts to 1/3 of the weak-coupling result if we take into account the Bardeen counterterm, and exactly agrees with the weak-coupling result without the counterterm.