CRITICAL EXAMINATION OF THE "FIELD-THEORETICAL APPROACH" TO THE NEUTRON-ANTINEUTRON OSCILLATIONS IN NUCLEI

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The $n-\bar{n}$ transition is possible due to the $(\Delta B = 2)$ interaction predicted within some variants of GUT.
Experimental results are available,
in vacuum (reactor experiments M. Baldo-Ceolin et al, Z.Phys. C63,409 (1994) Yad.Fiz. 59, 1612 (1996)), $\tau_n > 8.6\,10^7 sec \simeq 2.7\,yr$,

in nucleus ^{16}O, $\tau(^{16}O) > 4.3\,10^{31}\,yr$, **(M. Takita et al. (KAMIOKANDE) Phys.Rev. D34, 902 (1986))**

in Fe **nucleus,** $\tau(Fe) > 6.5\,10^{31}\,yr$ **(C. Berger et al. (Frejus Collab.) Phys.Lett. B240, 237 (1990)**

The general belief: the $n-\bar{n}$ transition in nuclei are strongly suppressed in comparison with the $n-\bar{n}$ transition in vacuum.

V.I.Nazaruk (1994 - 2010; JETP Lett., Phys Lett., Phys.Rev. C, Eur.Phys.J. C):
arguments based on the "true field-theoretical approach" NO suppression of the $n-\bar{n}$ transition in nuclei in comparison with vacuum.

This result has been criticized within somewhat different approaches (potential, S-matrix, diagram), and general physics arguments.
It seems to be necessary to re-analyze this problem just within the quantum field theory based approach.

The $n-\bar{n}$ transition in vacuum

The $\Delta B = 2$ interaction is

$$V = \mu_{n\bar{n}}\sigma_1/2,$$

σ_1 being the Pauli matrix. $\mu_{n\bar{n}}$ is the parameter which has the dimension of mass.

The neutron-antineutron oscillation time in vacuum is $\tau_{n\bar{n}} = 1/\delta m = 2/\mu_{n\bar{n}}$, A contact $n-\bar{n}$ coupling is assumed.

The $n-\bar{n}$ state is described by the 2-component spinor Ψ. The evolution equation is

$$i\frac{d\Psi}{dt} = (V_0 + V)\Psi \tag{1}$$

with $V_0 = m_N - i\gamma_n/2$ in the rest frame of the neutron (m_N is the nucleon mass, γ_n - the (anti)neutron normal weak interaction decay width. CP-invariance of weak interactions is assumed. Eq. (1) has solution

$$\Psi(t) = exp\left[-i\left(\mu_{n\bar{n}}t\,\sigma_1/2 + V_0t\right)\right]\Psi_0 = \left[cos\frac{\mu_{n\bar{n}}t}{2} - i\sigma_1 sin\frac{\mu_{n\bar{n}}t}{2}\right]exp(-iV_0t)\Psi_0 \tag{2}$$

Ψ_0 is the starting wave function, e.g. $\Psi_0 = (0,1)^T$. For an arbitrary time

$$\Psi(\bar{n},t) = -i\,sin\frac{\mu_{n\bar{n}}t}{2}exp(-iV_0t), \quad \Psi(n,t) = cos\frac{\mu_{n\bar{n}}t}{2}exp(-iV_0t), \tag{3}$$

which describes oscillation $n-\bar{n}$.

Since the parameter $\mu_{n\bar{n}}$ is small, the expansion of sin **and** cos **can be made in Eq.** (3) **at not too large times. In this case the average (over the time $t^{obs} \ll 1/\mu_{n\bar{n}}$) change of the probability of appearance of antineutron in vacuum is (for the sake of brevity we do not take into account the (anti)neutron natural instability which has obvious consequences)**

$$W(\bar{n}; t^{obs})/t^{obs} = |\Psi(\bar{n}, t^{obs})|^2/t^{obs} \simeq \frac{\mu_{n\bar{n}}^2 t^{obs}}{4} \tag{4}$$

In vacuum the transition $n \to \bar{n}$ is suppressed if the observation time is small, $t^{obs} \ll 1/\mu_{n\bar{n}}$. From data obtained with free neutrons from reactor

$$\mu_{n\bar{n}} < 1.5 \cdot 10^{-23}\, eV, \tag{5}$$

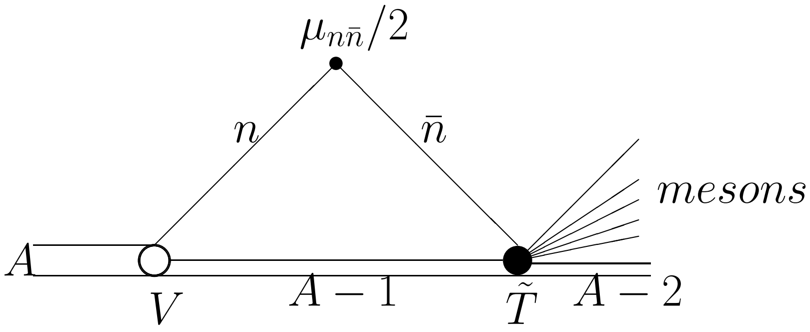
Calculation of the quantity $\mu_{n\bar{n}}$ or $\tau_{n\bar{n}}$ from existing data on nuclei stability is somewhat model dependent, and different authors obtained somewhat different results, within about 1 order of magnitude.

Analyticity based arguments for the suppression of the $n - \bar{n}$ transition in a nucleus

In the case of the nucleus decay $A \to (A-2) + mesons$, by dimension arguments

$$\Gamma(A \to (A-2) \, + \, mesons) \sim \frac{\mu_{n\bar{n}}^2}{m_0}, \tag{6}$$

where m_0 is some energy (mass) scale. For the result by VN to be correct, the mass m_0 should be very small, $m_0 \sim \mu_{n\bar{n}} \sim 10^{-23}\, eV$, but we argue that $m_0 \sim m_{hadr} \sim (10 - 100)\, MeV$ - normal hadronic or nuclear scale.



The matrix element of any Feynman diagram containing such transition

$$T(A \to (A-2) \, + \, mesons) \sim$$

$$\sim \mu_{n\bar{n}}(A-Z) \int V(A; n, (A-1)) \frac{\tilde{T}(\bar{n} + (A-1) \to (A-2) \, + \, mesons)}{(E_n - E_n^0 + i\delta)^2} dE_n \simeq$$

$$\simeq -2\pi i (A-Z) \frac{d(V\,\tilde{T})}{dE_n}(E_n = E_n^0), \tag{7}$$

according to the Cauchy theorem. E_n is the neutron (antineutron) energy - integration variable, E_n^0 is the (anti)neutron on-mass-shell energy $E_n^0 \simeq m_N + \vec{p}^2/2m_N$.

The amplitude $\tilde{T}$ and the vertex function V are of normal hadronic or nuclear scale and cannot, in principle, contain a very small factors in denominator (or very large factors, of the order of 10^{15}, in the numerator). By this reason we come to the above relation, and the resulting decay width of the nucleus is very small,

$$\Gamma(A \to (A-2) + mesons) < 10^{-30}\mu_{n\bar{n}}, \tag{8}$$

at least 30 orders of magnitude smaller than the inverse time of neutron-antineutron oscillation in vacuum $\mu_{n\bar{n}}$. We obtain

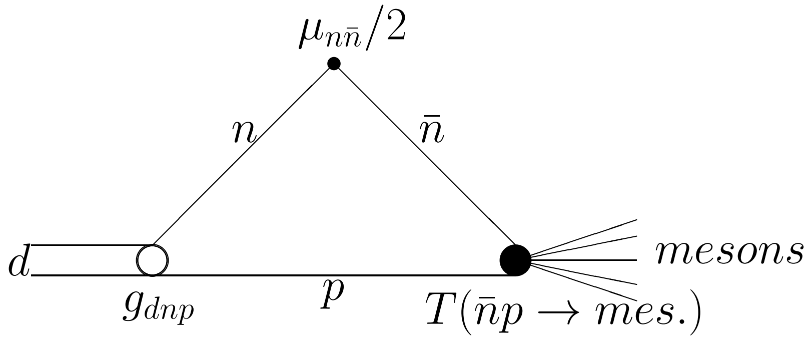
$$\mu_{n\bar{n}} \sim \sqrt{\Gamma(A \to (A-2) + mesons)m_0}, \tag{9}$$

and the restriction on $\mu_{n\bar{n}}$ from the nuclei stability data the is close to that from the vacuum experiment, somewhat smaller.

According to VN the probability of the nucleus decay is proportional to $W(t^{obs}) \sim \mu_{n\bar{n}}^2 \left(t^{obs}\right)^2$ (the process proceeds similar to the vacuum case), where t^{obs} is the large observation time, of the order of ~ 1 year or greater. By this reason the extracted value of $\mu_{n\bar{n}}$ is much smaller, by about 15 orders of magnitude. Technical reason for this strange result is the wrong interpretation of the second order pole structure of any amplitude containing the $n - \bar{n}$ transition. Instead of using the well developed Feynman diagram technique, the author tries to construct the space-time picture of the process by analogy with the vacuum case, which is misleading.

The case of the deuteron

In this case there is no final state containing antineutron, by the charge and energy conservation. Therefore, if the $n-\bar{n}$ **transition took place within the deuteron, the final state could be only some amount of mesons. The**



amplitude of the process is equal to

$$T(d \to mesons) = i g_{dnp} m_N \mu_{n\bar{n}} \int \frac{T(\bar{n}p \to mesons)}{(p^2 - m_N^2)[(d-p)^2 - m_N^2]^2} \frac{d^4p}{(2\pi)^4}. \tag{10}$$

The constant g_{dnp} **is normalized by the condition**

$$\frac{g_{dnp}^2}{16\pi} = \frac{\kappa}{m_N} = \sqrt{\frac{\epsilon_d}{m_N}} \simeq 0.049, \tag{11}$$

$\kappa = \sqrt{m_N \epsilon_d}$, $\epsilon_d \simeq 2.22\, MeV$ **being the binding energy of the deuteron.**

The integration over internal 4-momentum d^4p **in** (11) **can be made easily taking into account the nearest singularities in the energy** $p_0 = E$, **in the nonrelativistic approximation for nucleons. The integral over** d^3p **converges at small** $p \sim \kappa$ **which corresponds to large distances,** $r \sim 1/\kappa$.

By this reason the annihilation amplitude can be taken out of the integration in some average point, and we obtain the approximate equality

$$T(d \to mesons) = g_{dnp} m_N \mu_{n\bar{n}} I_{dNN} T(\bar{n}p \to mesons) \qquad (10a)$$

with

$$I_{dNN} = \frac{i}{(2\pi)^4} \int \frac{d^4p}{(p^2 - m_N^2)[(d-p)^2 - m_N^2]^2} \simeq$$

$$\simeq \frac{i}{(2\pi)^4 (2m)^3} \int \frac{d^4p}{[p_0 - m_N - \vec{p}^2/(2m_N) + i\delta]\,[m_d - m_N - p_0 - \vec{p}^2/(2m_N) - }$$

$$= \int \frac{d^3p}{(2\pi)^3 8 m_N [\kappa^2 + \vec{p}^2]^2} = \frac{1}{64\pi m_N \kappa}, \qquad (12)$$

The integral I_{dNN} enters also the deuteron charge form-factor at zero momentum transfer. The decay width (probability) is

$$\Gamma(d \to mesons) \simeq \mu_{n\bar{n}}^2 g_{dnp}^2 I_{dNN}^2 m_N \int |T(\bar{n}p \to mesons)|^2 d\Phi(mesons), \qquad (13)$$

$\Phi(mesons)$ **is the final states phase space. Final result for the width of the deuteron decay into mesons is**

$$\Gamma_{d \to mesons} \simeq \frac{\mu_{n\bar{n}}^2}{16\pi\kappa} m_N^2 \left[v_0 \sigma^{ann}(\bar{n}p)\right]_{v_0 \to 0} \simeq \frac{\mu_{n\bar{n}}^2}{8\pi\kappa} m_N \left[p_{c.m.} \sigma_{\bar{n}p}^{ann}\right]_{p_{c.m.} \to 0}, \qquad (14)$$

where $p_{c.m.}$ is the (anti)nucleon momentum in the center of mass system. This result is very close to that obtained by L.Kondratyuk.

The annihilation cross section of the antineutron with velocity v_0 on the proton at rest equals

$$\sigma(\bar{n}p \to mesons) = \frac{1}{4m_N^2 v_0} \int |T(\bar{n}p \to mesons)|^2 d\Phi(mesons). \tag{15}$$

According to PDG at small v_0, roughly, $\left[v_0 \sigma_{\bar{n}p}^{ann}\right]_{v_0 \to 0} \simeq (50 - 55) mb \simeq (130 - 140)\, GeV^{-2}$**.**

We obtain $\mu_{n\bar{n}} \leq 2.5\, 10^{-24} eV$, or $\tau_{n\bar{n}} > 5{\cdot}10^8\, sec$ if we take optimistically the same restriction for the deuteron stability as it was obtained for the Fe nucleus, $\tau_d \simeq \tau_{Fe} > 6.5{\cdot}10^{31} yr$. This result is valid up to numerical factor of the order ~ 1, since we did not consider explicitly the spin dependence of the annihilation cross section and the spin structure of the incident nucleus. .

Suppression factor in comparison with the case of a free neutron is

$$\mu_{n\bar{n}}/\kappa \sim 10^{-31}$$

disappears when the binding energy becomes zero.

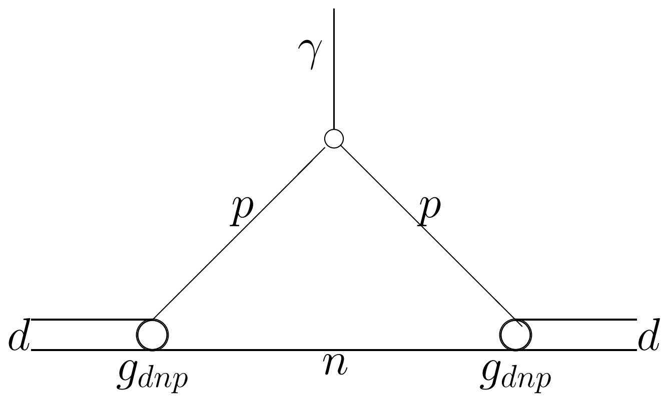
The deuteron charge formfactor

In the zero range approximation it can be written as

$$F_d(q) = \frac{i(2mg_{dnp})^2}{(2\pi)^4} \int \frac{d^4p}{(p^2 - m_N^2)[(d-p)^2 - m_N^2][(d-p+q)^2 - m_N^2]}. \tag{16}$$

Behind the zero range approximation g_{dnp} should be considered as a function of the relative $n-p$ momentum, not as a constant. For $q=0$ second order pole appears:

$$F_d(0) = (2m_N g_{dnp})^2 I_{dNN}. \tag{17}$$



In the nonrelativistic approximation,

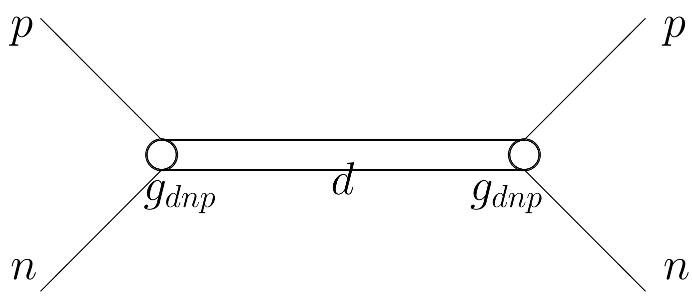
$$I_{dNN} \sim \int \frac{dE}{(E-a+i\delta)(E-b-i\delta)^2} = \frac{-2\pi i}{(a-b)^2}, \tag{18}$$

$a = m_N + \vec{p}^2/2m_N$. $b = m_d - m_N - \vec{p}^2/2m_N$, $a - b = \epsilon_d + \vec{p}^2/m_N$, **and can be calculated using the lower contour which includes the po[e at $E = a - i\delta$, or the upper contour, including the second order pole at $E = b + i\delta$.**

After this we obtain

$$F_d(q=0) = \frac{g_{dnp}^2 m_N}{16\pi^3} \int \frac{d^3p}{(\kappa^2 + \vec{p}^2)^2} = \frac{g_{dnp}^2 m_N}{16\pi\kappa}. \tag{19}$$

Since $F_d(0) = 1$, this leads to the normalization condition $g_{dnp}^2/(16\pi) = \sqrt{\epsilon_d/m_N}$. (Fainberg and Fradkin, 1955; Chew and Low, 1959). It can be obtained from the consideration of the pole contribution to the two-particle scattering amplitude, the np-scattering in our case (Landau, 1961).



The contribution of the pole diagram to the relativistic invariant scattering amplitude due to the presence of the bound state (the deuteron in our case) equals

$$T_{np\to np}^{pole} = \frac{(2m_N g_{dnp})^2}{s - m_d^2}, \tag{20}$$

where the Mandelstam variable $s = (p_n + p_p)^2$. At the threshold, $s = 4m_N^2$, we have

$$T_{np\to np}^{pole}(s = 4m_N^2) = \frac{m_N^2 g_{dnp}^2}{\kappa^2}, \tag{21}$$

since at the threshold $s - m_d^2 = 4m_N\epsilon_d = 4\kappa^2$ and we assume for simplicity that both the proton and neutron masses are equal to m_N.

The known quantum-mechanical expression for the scattering amplitude in the zero range approximation

$$f(k) = \frac{1}{\kappa + ik}, \tag{22}$$

k **being the value of the nucleon 3-momentum in the center of mass frame.**

Since $T(s) = 8\pi\sqrt{s}f(k)$**, at the threshold (**$k = 0$**)**

$$\frac{m_N^2 g_{dnp}^2}{\kappa^2} = \frac{16\pi m_N}{\kappa},$$

which is the former relation.

Parity violating $np \to d\gamma$ - capture amplitude

The field-theoretical description of nuclear reactions and processes is potentially useful: the effects can be studied which is not possible, in principle, to study in other way, e.g. relativistic corrections to different observables.

Analytical properties of contributing amplitudes should be considered properly.

In the case of the parity violating amplitude of the $np \to d\gamma$ - reaction relativistic contributions change the nonrelativistic weak interaction isospin selection rules for the parity violating observables: photon circular polarization (neutrons unpolarized) and photon asymmetry in the capture of polarized neutrons (VK, 1982).

In this case it was necessary to take into account contributions of all singularities (poles) of the amplitude in the complex energy plane of the virtual nucleon, not only contributions of the nearest poles in the energy variable, as it is made usually in the nonrelativistic calculations. Besides, and it is the spesifics of the processes with photon emission, the contact terms should be reconstructed to ensure the gauge invariance of the whole amplitude of the photon radiation .

The nonrelativistic diagram technique developed up to that time turned out to be misleading for this case. The cancellation between contributions of different poles takes place. As a result, the relativistic contributions to the observables turned out to be not greater than nonrelativistic values, in spite of the change of the isospin selection rules.

Conclusions

1. The 2-d order pole structure of an amplitude of $n-\bar{n}$ transition in nuclei does NOT lead to any dramatic consequences (similar to the nucleus charge formfactor).

2. Restriction on the parameter $\mu_{n=barn}$ from nuclear data are few times stronger than from reactor experiment.

3. VN tries to reconstruct the space-time picture of the process, but the correspondence of this picture to the well justified amplitude, as it appears from the Feynman diagrams, is questionable. The infrared divergence discussed in is an artefact of this inadequate space-time picture of the whole process of $n-\bar{n}$ transition in nucleus.

4. Application of the quantum field theory methods to nuclear processes demands a special care.

Examples: the Landau-Pomeranchuk effect (hadron formation length)
the parity violating $np \to d\gamma$ - amplitude, etc.