Models with particle escape from our brane

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Contents

Background metric of the RSII-n model

T. Gherghetta M. Shaposhnikov 2000.

Consider $(3+n)$ - brane with n compact dimensions, embedded in a $(5+n)$ - spacetime with slice AdS_{5+n} metric:

$$ds^2 = a(z)^2(\eta_{\mu\nu}dx^\mu dx^\nu - \delta_{ij}d\theta^i d\theta^j) - dz^2, \quad (1)$$

z -is the infinite extra-dimension

θ_i - are the compact extra-dimensions $\theta_i \in [0, 2\pi R_i]$, $i = \overline{1,n}$,

n - is a number of compact extra-dimensions,

$a(z) = e^{-k|z|}$ is a warp factor from Randall-Sundrum model.

Peculiar features of the RSII-n model

- Single brane along z direction
- Orbifold geometry of the compact extra-dimensions θ_i,
  Z_2 identification: $\theta_i \to -\theta_i$
- The could be localized constant zero modes $A^{(0)} = const$ of the massless fields due to presence of the warp-factor in the overlap integral

$$\int dz\, a^n |A^{(0)}|^2 \quad - \text{is finite},$$

- Kaluza-Klein excitations of the SM particles posses a continuous mass spectrum.

$SU(2) \times U(1)$ bulk sector of the Standard Model

The action of the theory:

$$S = \int d^4x\, dz \prod_{i=1}^{n} \frac{d\theta_i}{2\pi R_i} \sqrt{g} [-\frac{1}{4} \left(F^{\alpha}_{MN}\right)^2 - \frac{1}{4} B^2_{MN} + \left(D_M \Phi\right)^{\dagger} D_M \Phi - \\ - V(\Phi^{\dagger}, \Phi) + \delta(z) \mathcal{L}_F], \tag{2}$$

F^{α}_{MN}, B_{MN}, Φ are the bulk fields.

fermions ψ are localized on the brane and depend only on four-dimensional coordinates x.

$$V(\Phi^{\dagger}, \Phi) = \frac{\lambda}{2} \left(\Phi^{\dagger} \Phi - \frac{v^2}{2} \right)^2 \tag{3}$$

$$D_M \Phi = \partial_M \Phi - i \frac{\widetilde{g}_1}{2} \hat{B}_M \Phi - i \widetilde{g}_2 \frac{\sigma_i}{2} A^{\alpha}_M \Phi \tag{4}$$

Symmetry breaking sector of the SM

$$S = \int d^4x\, dz \prod_{i=1}^{n} \frac{d\theta_i}{2\pi R_i} \sqrt{g} \Big[-\frac{1}{2} |\, W_{MN}\,|^2 + m_W^2 |\, W_M\,|^2 - \frac{1}{4} Z_{MN}^2 + \frac{1}{2} m_Z^2 Z_M^2 - \\ - \frac{1}{4} F_{MN}^2 + \frac{1}{2} (\partial_M \chi)^2 - \frac{1}{2} m_\chi^2 \chi^2 + \delta(z) \mathcal{L}_F \Big], \qquad (5)$$

where $m_W^2 = \frac{1}{4}\widetilde{g}_2^2 v^2$, $m_Z^2 = \frac{1}{4}(\widetilde{g}_2^2 + \widetilde{g}_1^2) v^2$ and $m_\chi^2 = \lambda v^2$ are the bulk masses of the gauge fields and Higgs respectively.

Redefinition of the variables

Redefinition of the gauge fields

$$Z_M = \frac{1}{\sqrt{\widetilde{g}_1^2 + \widetilde{g}_2^2}} \left(-\widetilde{g}_1 B_M + \widetilde{g}_2 A_M^3 \right) , \quad A_M = \frac{1}{\sqrt{\widetilde{g}_1^2 + \widetilde{g}_2^2}} \left(\widetilde{g}_2 B_M + \widetilde{g}_1 A_M^3 \right) ,$$

$$W_M^{\pm} = \frac{1}{\sqrt{2}} \left(A_M^1 \mp i A_M^2 \right) .$$

Redefinition of the coupling constants

$$\widetilde{e}_5 = \widetilde{g}_2 \sin\theta_W = \widetilde{g}_1 \cos\theta_W$$

At low energies, $E \ll 1/R_i$ fields are independent of θ_i,

$$\chi = \chi(x,z),\ W_M^+ = W_M^+(x,z),\ Z_M = Z_M(x,z),\ A_M = A_M(x,z).$$

Equations of motion for bulk photon

$$-p^2\widetilde{A}_5 - ip^\mu\partial_5\widetilde{A}_\mu = 0, \tag{6}$$

$$\left(-\partial_5^2 + (2+n)k\,\mathrm{sign}(z)\partial_5 - \frac{p^2}{a^2}\right)\widetilde{A}_\lambda + \frac{1}{a^2}p^\mu p_\lambda\widetilde{A}_\mu +$$
$$+ip_\lambda\left(\partial_5\widetilde{A}_5 - (2+n)k\,\mathrm{sign}(z)\widetilde{A}_5\right) = 0. \tag{7}$$

We set the gauge $\widetilde{A}_5 = 0$,
the constant solution with respect to fifth z-coordinate
$\widetilde{A}^{(0)}_\mu(p,z) \equiv \widetilde{A}^{(0)} = \mathrm{const}$, is a photon localized on the brane.

$$\int_0^\infty dz\, e^{-nk|z|}\,|\widetilde{A}^{(0)}|^2 = 1 \;\Rightarrow\; \widetilde{A}^{(0)} = \sqrt{\frac{nk}{2}}. \tag{8}$$

$$\mathcal{L}_{int} = e_5\bar{\psi}\gamma_\mu\psi A^\mu(x)\widetilde{A}^{(0)} \quad \Rightarrow \quad e_5 = e_4\sqrt{\frac{2}{nk}}$$

Equations of motion for bulk Z^0 boson.

$$\widetilde{Z}_5(\widetilde{p}^2 - m_Z^2)a = -i\widetilde{p}^\mu \partial_5 \widetilde{Z}_\mu, \tag{9}$$

$$\left(\eta_{\mu\lambda} - \frac{\widetilde{p}_\mu \widetilde{p}_\lambda}{\widetilde{p}^2 - m_Z^2}\right)\left(-\partial_5^2 + (2+n)k\,\mathrm{sign}(z)\partial_5 + m_Z^2 - \widetilde{p}^2\right)\widetilde{Z}^\mu -$$

$$-2k\,\mathrm{sign}(z)\frac{\widetilde{p}_\mu \widetilde{p}_\lambda}{(\widetilde{p}^2 - m_Z^2)^2}\partial_5 \widetilde{Z}^\mu = 0, \tag{10}$$

$\widetilde{p}_\mu = \frac{p_\mu}{a}$ is a physical momentum of the particle

We split the solution of this equation by longitudial and transverse parts:

$$\widetilde{Z}_L^\mu = p^\mu Z_L(p,z), \quad \widetilde{Z}_T^\mu = \epsilon^\mu Z_T(p,z), \quad (\epsilon_\mu p^\mu) = 0.$$

Transverse component

Transverse solution $Z_T(p,z)$ obeys the equation:

$$\left(-\partial_5^2 + (2+n)k\,\mathrm{sign}(z)\partial_5 + m_Z^2 - \frac{m^2}{a^2}\right) Z_m(z) = 0, \qquad m^2 = p^2, \tag{11}$$

One easily finds:

$$Z_m(z) = \sqrt{\frac{m}{2k}}\, e^{(\frac{n}{2}+1)k|z|} \left[a_m J_\nu\left(\frac{m}{k}e^{k|z|}\right) + b_m N_\nu\left(\frac{m}{k}e^{k|z|}\right)\right],$$

normalization condition:

$$\int dz\, e^{-nk|z|} Z_m(z) Z_{m'}(z) = \delta(m-m'), \qquad a_m^2 + b_m^2 = 1,$$

boundary condition on the brane:

$$\partial_z Z_m(+0) - \partial_z Z_m(-0) = 0.$$

Classical limit

if $k \to \infty$, then $Z_m^2(0)$ tend to the delta function:

$$Z_m^2(0) = \frac{nk}{2} \cdot \frac{1}{\pi} \frac{\Gamma_Z}{2} \frac{1}{(m - M_Z)^2 + \left(\frac{\Gamma_Z}{2}\right)^2} \to \frac{nk}{2} \cdot \delta(m - M_Z). \qquad (12)$$

$$\frac{k}{m} \gg 1,$$

where

$$\Gamma_Z = \frac{2\pi}{n\Gamma^2\left(\frac{n}{2}\right)} m \left(\frac{m}{2k}\right)^n, \quad M_Z = m_Z \sqrt{\frac{n}{n+2}} \qquad (13)$$

are the invisible width decay, and the mass of Z^0 boson respectively

Z^0 boson Green function

$$G^{\mu\nu}_{Z(5)}(x-x',z,z') = \langle 0|\, T Z^{\mu}(x,z) Z^{\nu}(x',z')|0\rangle =$$

$$= \int dm\, \frac{d^4p}{(2\pi)^4} \cdot \frac{(-i)\, e^{-ip(x-x')}}{p^2-m^2+i\epsilon} Z_m(z) Z_m(z') \left(\eta^{\mu\nu} - \frac{p^{\mu}p^{\nu}}{m^2}\right).$$

Brane to brane propagator:

$$G^{\mu\nu}_{Z(5)}(x-x',0,0) = \int dm\, \frac{d^4p}{(2\pi)^4} \cdot \frac{(-i)\, e^{-ip(x-x')}}{p^2-m^2+i\epsilon} Z_m^2(0) \left(\eta^{\mu\nu} - \frac{p^{\mu}p^{\nu}}{m^2}\right).$$

Correspondence between five-dimensional and four-dimensional Green functions in a limit $k \to \infty$:

$$G^{\mu\nu}_{Z(5)}(x-x',0,0) = G^{\mu\nu}_{(4)}(x-x',0,0) \cdot \frac{nk}{2}$$

Conclusion

- We showed that the Standard Model remains a consistent in the RSII-n model with continuous mass spectrum of the particle.
- The localized zero photon mode remains a massless.
- The massive bulk mode of Z^0 boson possess a finite probability escape from our brane.