Interpreting dualities from superconformal index identities

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Superconformal index

Römelsberger; Kinney et al.

- Extension of Witten $\mathrm{tr}\,(-1)^F$ index to count BPS states in superconformal field theories
  - Pick a couple of supercharges $\{\mathcal{Q}, \mathcal{Q}^\dagger\} = \Delta > 0$
  - To improve convergence take another operator $\Xi > \Delta$ that preserves selected supercharge, $[\Xi, \mathcal{Q}] = 0$
  - It's nice to keep track of all other quantum numbers commuting with $\mathcal{Q}$ (e.g. global H and gauge G groups, R-charge)
- $\mathrm{ind}(\beta, \gamma, h) = \int_G d\mu(g)\ \mathrm{tr}\left[(-1)^F e^{-\beta\Xi}\,\gamma\, g\, h\right]$,

$$\gamma \in SU(2)_R, g \in G, h \in H$$

- Explicit expression for any symmetry group and field content
- Integration for classical groups made only recently in terms of newly discovered elliptical hypergeometric functions Spiridonov

Seiberg duality

- Different theories describe identical physics in their IR fixed points
- Generically both fixed points are non-trivial (thus not a strong/weak coupling duality)

What do we have on them in IR?

- Physical global symmetries
  Not always identical — accidental enhancement in IR is possible!
- Anomaly coefficients for global currents
  't Hooft matching conditions
- Vacuum manifold or moduli space:
  Given by holomorphic gauge invariants modulo classical relations and conditions of superpotential extremum

Indices for dual theories should coincide

Römelsberger; Dolan & Osborn

- Indices characterise symmetries and field content in deep IR
  Conjecture is very plausible but still unproven
- Checked for almost all known $\mathcal{N} = 1$ SUSY dualities
  Spiridonov, Vartanov
- New dualities are conjectured from index identities!
  (highly non-trivial relations → strong evidence)

Features

- Possible to reconstruct symmetries and field content of theory from index integral
- Various limits of indices correspond to limits for theories (decoupling of fields / gauge symmetry breaking)
- Ample transformation properties → a lot of new duals
  e.g. 72 claimed for $N_c = 2, N_f = 4$ SQCD, where only 4 were known before

$\mathcal{N} = 1$ SQCD with $N_c = 2$ colours & $N_f = 4$ flavours

- Field content:
  - $\mathcal{N} = 1$ $SU(2)_c$ SYM
  - 4 quarks Q^i with $SU(4)_L$ flavour symmetry
  - 4 antiquarks $\tilde{Q}_{\tilde{j}}$ with $SU(4)_R$ flavour symmetry
- Fundamentals and antifundamentals of $SU(2)_c$ are equivalent, possible to combine Q^i with $\tilde{Q}_{\tilde{j}}$
- Global group is $SU(8) \supset SU(4)_L \times SU(4)_R \times U(1)_B$
- Superpotential $W \equiv 0 \rightarrow$ all gauge invariants are moduli
- "Mesons" $M^i_{\tilde{j}} \equiv Q^i \cdot \tilde{Q}_{\tilde{j}}$, "baryons" $B^{ij} \equiv Q^i \cdot Q^j$ and "antibaryons" $\tilde{B}_{\tilde{i}\tilde{j}}$
- In terms of $SU(8)$ antisymmetric representation

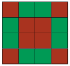
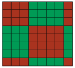
B^{ij}	$M^i_{\tilde{j}}$
$-M^j_{\tilde{i}}$	$\tilde{B}_{\tilde{i}\tilde{j}}$

"Classical" duals for $SU(2)_c$ $N_f = 4$ SQCD

Field content	Quark gauge invariants	Superpotential	Moduli
$Q^i \quad (\mathbf{4},\mathbf{1})^{+1}$ $\tilde{Q}_{\tilde{j}} \quad (\mathbf{1},\mathbf{4})^{-1}$	$B^{ij} \equiv Q^i \cdot Q^j \quad (\mathbf{6},\mathbf{1})^{+2}$ $\tilde{B}_{\tilde{i}\tilde{j}} \equiv \tilde{Q}_{\tilde{i}} \cdot \tilde{Q}_{\tilde{j}} \quad (\mathbf{1},\mathbf{6})^{-2}$ $M^i_{\tilde{j}} \equiv Q^i \cdot \tilde{Q}_{\tilde{j}} \quad (\mathbf{4},\mathbf{4})^0$		$\begin{pmatrix} B^{ij} & M^i_{\tilde{j}} \\ -M^j_{\tilde{i}} & \tilde{B}_{\tilde{i}\tilde{j}} \end{pmatrix}$
$q^i \quad (\mathbf{4},\mathbf{1})^{-1}$ $\tilde{q}_{\tilde{j}} \quad (\mathbf{1},\mathbf{4})^{+1}$ $B^{ij} \quad (\mathbf{6},\mathbf{1})^{+2}$ $\tilde{B}_{\tilde{i}\tilde{j}} \quad (\mathbf{1},\mathbf{6})^{-2}$	$C^{ij} \equiv q^i \cdot q^j \quad (\mathbf{6},\mathbf{1})^{-2}$ $\tilde{C}_{\tilde{i}\tilde{j}} \equiv \tilde{q}_{\tilde{i}} \cdot \tilde{q}_{\tilde{j}} \quad (\mathbf{1},\mathbf{6})^{+2}$ $N^i_{\tilde{j}} \equiv q^i \cdot \tilde{q}_{\tilde{j}} \quad (\mathbf{4},\mathbf{4})^0$	$\frac{1}{4\mu}\epsilon_{ijkl} B^{ij}\, q^k \cdot q^l +$ $\frac{1}{4\mu}\epsilon^{\tilde{i}\tilde{j}\tilde{k}\tilde{l}} \tilde{B}_{\tilde{i}\tilde{j}}\, \tilde{q}_{\tilde{k}} \cdot \tilde{q}_{\tilde{l}}$	$\begin{pmatrix} B^{ij} & N^i_{\tilde{j}} \\ -N^j_{\tilde{i}} & \tilde{B}_{\tilde{i}\tilde{j}} \end{pmatrix}$
$q_i \quad (\bar{\mathbf{4}},\mathbf{1})^{+1}$ $\tilde{q}^{\tilde{j}} \quad (\mathbf{1},\bar{\mathbf{4}})^{-1}$ $M^i_{\tilde{j}} \quad (\mathbf{4},\mathbf{4})^0$	$C_{ij} \equiv q_i \cdot q_j \quad (\bar{\mathbf{6}},\mathbf{1})^{+2}$ $\tilde{C}^{\tilde{i}\tilde{j}} \equiv \tilde{q}^{\tilde{i}} \cdot \tilde{q}^{\tilde{j}} \quad (\mathbf{1},\bar{\mathbf{6}})^{-2}$ $N_i^{\tilde{j}} \equiv q_i \cdot \tilde{q}^{\tilde{j}} \quad (\bar{\mathbf{4}},\bar{\mathbf{4}})^0$	$\frac{1}{\mu} M^i_{\tilde{j}}\, q_i \cdot \tilde{q}^{\tilde{j}}$	$\begin{pmatrix} \epsilon^{ijkl} C_{kl} & M^i_{\tilde{j}} \\ -M^j_{\tilde{i}} & \epsilon_{\tilde{i}\tilde{j}\tilde{k}\tilde{l}} \tilde{C}^{\tilde{k}\tilde{l}} \end{pmatrix}$
$q_i \quad (\bar{\mathbf{4}},\mathbf{1})^{-1}$ $\tilde{q}^{\tilde{j}} \quad (\mathbf{1},\bar{\mathbf{4}})^{+1}$ $B^{ij} \quad (\mathbf{6},\mathbf{1})^{+2}$ $\tilde{B}_{\tilde{i}\tilde{j}} \quad (\mathbf{1},\mathbf{6})^{-2}$ $M^i_{\tilde{j}} \quad (\mathbf{4},\mathbf{4})^0$	$C_{ij} \equiv q_i \cdot q_j \quad (\bar{\mathbf{6}},\mathbf{1})^{-2}$ $\tilde{C}^{\tilde{i}\tilde{j}} \equiv \tilde{q}^{\tilde{i}} \cdot \tilde{q}^{\tilde{j}} \quad (\mathbf{1},\bar{\mathbf{6}})^{+2}$ $N_i^{\tilde{j}} \equiv q_i \cdot \tilde{q}^{\tilde{j}} \quad (\bar{\mathbf{4}},\bar{\mathbf{4}})^0$	$\frac{1}{\mu} M^i_{\tilde{j}}\, q_i \cdot \tilde{q}^{\tilde{j}} +$ $\frac{1}{2\mu} B^{ij}\, q_i \cdot q_j +$ $\frac{1}{2\mu} \tilde{B}_{\tilde{i}\tilde{j}}\, \tilde{q}^{\tilde{i}} \cdot \tilde{q}^{\tilde{j}}$	$\begin{pmatrix} B^{ij} & M^i_{\tilde{j}} \\ -M^j_{\tilde{i}} & \tilde{B}_{\tilde{i}\tilde{j}} \end{pmatrix}$

$SU(4)_L \times SU(4)_R$ structure is inherent in these dualities!?

Novel duals from index identities

- Discriminate "Seiberg-type" and "Csáki-type" magnetic theories (called so because expressions for indices similar to those of corresponding "classical" duals)
- Seem to break $SU(4)^2$ pattern to smaller subgroups e.g. $SU(2)^4 \times U(1)^3$  or $(SU(3)^{\times} U(1))^2 \times U(1)$ 
- Patterns given by overlap of two independent splittings of 8 quarks in two groups of 4
  - Composite vs Elementary moduli
  - Quarks vs Antiquarks ("baryon" charge)
- Latter one is artificial and could be rearranged by field redefinitions
- All new dualities per se are equivalent to the "classical" ones!

Physical meaning of new duals

- Are there any physical interpretation of index multiplicity?
- On index side comes from discrimination of quark flavours
- On physical side one can couple flavour currents to external field
- Duals will differ by currents coupled to the same external field
- e.g. "baryon" current $B = \text{diag}(1,1,1,1,-1,-1,-1,-1)$ breaks $SU(8)$ to $SU(4) \times U(1) \times SU(4)$ in electric theory
- $B' = \text{diag}(-1,1,1,1,1,-1,-1,-1)$, $B'' = \text{diag}(-1,-1,1,1,1,1,-1,-1)$ equivalent to B in electric theory but lead to different patterns of symmetry breaking in magnetic duals (precisely those from previous slide)!

Other findings and puzzles

- Full symmetry group of index indeed correspond to the set of dual theories
  - One may break $SU(8)$ down to $U(1)^7$ introducing asymmetrical enough currents
  - Assigning 7 charges to 28 mesons/baryons could be quite a lot of options
  - Only 35! Charges of magnetic elementary quarks define everything else
  - Charges of magnetic and electric quarks related by $W(E_7)$ transformations, found to be the full symmetry group of index for these theories
- Various limits of index correspond to limits in dual theories e.g. large masses or vevs for quarks/elementary hadrons
  - Integration out of quarks — flow towards $N_f = 3$
  - Gauge group breaking
- Superconformal index — powerful tool for non-perturbative exploration of SUSY gauge theories