Conformal Symmetry and its Breaking Effects in Gauge Theories with Fermions: New Perturbation Consequences

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Abstract

Plan of the talk:

1) Introduction - review of results of advanced QCD analytical calculations.

2) Study of the perturbative violation from the conformal symmetry of the quark-parton model - effects of non-zero RG-β-function

3) Formulation of new form of the relation between e^+e^--characteristics and DIS sum rules in Euclidean region-power series in $(\beta(a_s)/a_s)$-term.

4) Banks-Zaks relation $\beta_0(N_F) = 0$ and new constraints for Baikov, Chetyrkin, Kuhn results.

5) Conclusions.

Notations: Main quantities for e^+e^- annihilation process (Novosibirsk- Russia; Beijing- China) :

$D_A^{NS}(Q^2) = Q^2 \int_0^\infty \frac{R(s)}{(s+Q^2)^2} ds = 3 \sum_F Q_F^2 C_A^{NS}(a_s)$

$C_A^{NS}(a_s)) = 1 + \sum_n a_s^n \ \mathbf{d_n} \quad a_s = \alpha_s(\mu^2 = Q^2)/\pi$

Bjorken polarized sum rule - measured at CEBAF at intermediate and low Q^2

$\text{Bjp}(Q^2) = \int_0^1 \left[g_1^{lp}(x,Q^2) - g_1^{ln}(x,Q^2) \right] dx = \frac{1}{6} g_A C_{DIS}^{NS}(a_s)$

$C_{DIS}^{NS}(a_s) = 1 + \sum_n a_s^n \ \mathbf{k_n}$

QCD β-function- measure of the conformal symmetry breaking effects $(\beta(a_s)/a_s)$- fix renorm. of trace of energy momentum tenzor

$\beta(a_s) = \mu^2 \frac{d}{d\mu^2} a_s = -a_s^2 \left(\beta_0 + \beta_1 a_s + \beta_2 a_s^2 + O(a_s^3) \right)$

$\beta_0 = \frac{11}{12} \mathrm{C_A} - \frac{1}{3} \mathrm{T_F N_F}$

$\beta_1 = \frac{17}{24} \mathrm{C_A^2} - \frac{5}{12} \mathrm{C_A T_F N_F} - \frac{1}{4} \mathrm{C_F T_F N_F}$

$\beta_2 = \frac{2857}{3456} \mathrm{C_A^3} - \frac{1451}{1728} \mathrm{C_A^2 T_F N_F} - \frac{205}{576} \mathrm{C_A C_F T_F N_F}$
$+ \frac{79}{864} \mathrm{C_A T_F^2 N_F^2} + \frac{1}{32} \mathrm{C_F^2 T_F N_F} + \frac{11}{144} \mathrm{C_F T_F^2 N_F^2}$

$\mathbf{d_1} = C_F \frac{3}{4}$ $\mathbf{d_2} = C_F^2 \left[-\frac{3}{32}\right] + C_F C_A \left[\frac{123}{32} - \frac{11}{4}\zeta_3\right] - C_F T_F N_F \left[\frac{11}{8} - \zeta_3\right]$

Chetyrkin,Kataev,Tkachov (79)

$\mathbf{d_3} = C_F^3 \left[-\frac{69}{128}\right] + C_F^2 C_A \left[-\frac{127}{64} - \frac{143}{16}\zeta_3 + \frac{55}{4}\zeta_5\right] +$

$C_F C_A^2 \left[\frac{90445}{3456} - \frac{2737}{144}\zeta_3 - \frac{55}{24}\zeta_5\right] + C_F^2 T_F N_F \left[-\frac{29}{64} + \frac{19}{4}\zeta_3 - 5\zeta_5\right] +$

$C_F C_A T_F N_F \left[-\frac{485}{27} + \frac{112}{9}\zeta_3 + \frac{5}{6}\zeta_5\right] + C_F T_F^2 N_F^2 \left[\frac{151}{54} - \frac{19}{9}\zeta_3\right]$

Gorishny, Kataev, Larin (88-90) $\mathbf{d_4} = \frac{d_F^{abcd} d_A^{abcd}}{d_R} \left[\frac{3}{16} - \frac{1}{4}\zeta_3 - \frac{5}{4}\zeta_5\right] +$

$N_F \frac{d_F^{abcd} d_F^{abcd}}{d_R} \left[-\frac{13}{16} - \zeta_3 + \frac{5}{2}\zeta_5\right] + C_F^4 \left[\frac{4157}{2048} + \frac{3}{8}\zeta_3\right] +$

$C_F^3 T_F N_F \left[\frac{1001}{384} + \frac{99}{32}\zeta_3 - \frac{125}{4}\zeta_5 + \frac{105}{4}\zeta_7\right] +$

$C_F^2 T_F^2 N_F^2 \left[\frac{5713}{1728} - \frac{581}{24}\zeta_3 + \frac{125}{6}\zeta_5 + 3\zeta_3^2\right] -$

$C_F T_F^3 N_F^3 \left[\frac{6131}{972} - \frac{203}{54}\zeta_3 - \frac{5}{3}\zeta_5\right]$ - Baikov,Chetyrkin,Kuhn(08-10)

$C_F^3 C_A \left[\frac{253}{32} + \frac{139}{128}\zeta_3 - \frac{2255}{32}\zeta_5 + \frac{1155}{16}\zeta_7\right] +$

$C_F^2 T_F N_F C_A \left[\frac{32357}{13824} + \frac{10661}{96}\zeta_3 - \frac{5155}{48}\zeta_5 - \frac{33}{4}\zeta_3^2 - \frac{105}{8}\zeta_7\right] +$

$C_F T_F^2 N_F^2 C_A \left[\frac{340843}{5184} - \frac{10453}{288}\zeta_3 - \frac{170}{9}\zeta_5 - \frac{1}{2}\zeta_3^2\right] +$

$C_F^2 C_A^2 \left[-\frac{592141}{18432} - \frac{43925}{384}\zeta_3 + \frac{6505}{48}\zeta_5 + \frac{1155}{32}\zeta_7\right] +$

$C_F T_F N_F C_A^2 \left[-\frac{4379861}{20736} + \frac{8609}{72}\zeta_3 + \frac{18805}{288}\zeta_5 - \frac{11}{2}\zeta_3^2 + \frac{35}{16}\zeta_7\right] +$

$C_F C_A^3 \left[\frac{52207039}{248832} - \frac{456223}{3456}\zeta_3 - \frac{77995}{1152}\zeta_5 + \frac{605}{32}\zeta_3^2 - \frac{385}{64}\zeta_7\right]$

$\mathbf{k_1} = -C_F \frac{3}{4}$ $\mathbf{k_2} = C_F^2 \left[\frac{21}{32}\right] - C_F C_A \left[\frac{23}{16}\right] + C_F T_F N_F \left[\frac{1}{2}\right]$

Gorishny, Larin (86) $\mathbf{k_3} = C_F^3 \left[-\frac{3}{128}\right] + C_F^2 C_A \left[\frac{1241}{576} - \frac{11}{12}\zeta_3\right] +$
$C_F C_A^2 \left[-\frac{5437}{864} + \frac{55}{24}\zeta_5\right] + C_F^2 T_F N_F \left[-\frac{133}{576} - \frac{5}{12}\zeta_3\right] +$
$C_F C_A T_F N_F \left[\frac{3535}{864} + \frac{3}{4}\zeta_3 - \frac{5}{6}\zeta_5\right] - C_F T_F^2 N_F^2 \left[\frac{115}{216}\right]$

Larin, Vermaseren (91) $\mathbf{k_4} = \frac{d_F^{abcd} d_A^{abcd}}{d_R} \left[-\frac{3}{16} + \frac{1}{4}\zeta_3 + \frac{5}{4}\zeta_5\right] +$
$N_F \frac{d_F^{abcd} d_F^{abcd}}{d_R} \left[\frac{13}{16} + \zeta_3 - \frac{5}{2}\zeta_5\right] + C_F^4 \left[-\frac{4823}{2048} - \frac{3}{8}\zeta_3\right] +$
$C_F^3 T_F N_F \left[\frac{839}{2304} + \frac{451}{96}\zeta_3 - \frac{145}{24}\zeta_5\right] + C_F^2 T_F^2 N_F^2 \left[-\frac{265}{576} + \frac{29}{24}\zeta_3\right] +$
$C_F T_F^3 N_F^3 \left[\frac{605}{972}\right] + C_F^3 C_A \left[-\frac{3707}{4608} - \frac{971}{96}\zeta_3 + \frac{1045}{48}\zeta_5\right] +$
$C_F^2 C_A T_F N_F \left[-\frac{87403}{13824} - \frac{1289}{144}\zeta_3 + \frac{275}{144}\zeta_5 + \frac{35}{4}\zeta_7\right] +$
$C_F C_A T_F^2 N_F^2 \left[-\frac{165283}{20736} - \frac{43}{144}\zeta_3 + \frac{5}{12}\zeta_5 - \frac{1}{6}\zeta_3^2\right] +$
$C_F^2 C_A^2 \left[\frac{1071641}{55296} + \frac{1591}{144}\zeta_3 - \frac{1375}{144}\zeta_5 - \frac{385}{16}\zeta_7\right] +$
$C_F C_A^2 T_F N_F \left[\frac{1238827}{41472} + \frac{59}{64}\zeta_3 - \frac{1855}{288}\zeta_5 + \frac{11}{12}\zeta_3^2 - \frac{35}{16}\zeta_7\right] +$
$C_F C_A^3 \left[-\frac{8004277}{248832} + \frac{1069}{576}\zeta_3 + \frac{12545}{1152}\zeta_5 - \frac{121}{96}\zeta_3^2 + \frac{385}{64}\zeta_7\right]$ presented here
using Baikov,Cheterkin,Kuhn (2010) results.

Generalized Crewther relation in the $\overline{MS}$-scheme- explicit role of conformal symmetry breaking effects- factorization of QCD β-function in massless QCD- discovered at a_s^3-level by Broadhurst, Kataev (93). Shown by applying operator product expansion application to 3-point AVV triangle diagram in momentum p- space Gabadadze, Kataev (95) and Kataev (96)- INR-09296; Proved in coordinate x-space by Crewther (97) and D. Mueller (97)- His proof published V.Braun, G. Korchemsky, D. Mueller in the review of 2003 ! $C_A^{NS}(a_s(Q^2)) \times C_{DIS}^{NS}(a_s(Q^2)) = \mathbb{1} + \Delta_{\rm CSB}(a_s(Q^2))$ $\Delta_{\rm CSB}(a_s) = \left(\frac{\beta(a_s)}{a_s}\right) \mathcal{P}(a_s)$ where polynomial $\mathcal{P}(a_s) = \sum_{m\geq 1} K_m a_s^m$ (Note $(\beta(a_s)/a_s)$ enters into conformal anomaly- anomalous dimension of Trace of Energy Momentum Tensor!) $K_1 = K_1[1,0,0]{\rm C_F}$ $K_2 = K_2[2,0,0]{\rm C_F^2} + K_2[1,1,0]{\rm C_F C_A}$ $+ K_2[1,0,1]{\rm C_F T_F N_F}$ - **notice** $\mathbf{T_F N_F}$**-dependence (!)**

New explicit term at a_s^4-level: Baikov, Chetyrkin, Kuhn (2010)

$K_3 = K_3[3,0,0]\mathrm{C_F^3} + K_3[2,1,0]\mathrm{C_F^2 C_A} + K_3[1,2,0]\mathrm{C_F C_A^2} +$
$K_3[2,0,1]\mathrm{C_F^2 T_F N_F} + K_3[1,1,1]\mathrm{C_F C_A T_F N_F} + K_3[1,0,2]\mathrm{C_F T_F^2 N_F^2}$

(Contain additional $\mathrm{T_F N_F}$-terms)

Validity of Generalized Crewther at a_s^3-strong check of different a_s^3 analytical calculations ! (Broadhurst,Kataev (93)) Coefficients:

$K_1[1,0,0] = -\frac{21}{8} + 3\zeta_3$ $K_2[2,0,0] = \frac{397}{96} + \frac{17}{2}\zeta_3 - 15\zeta_5$

$K_2[1,1,0] = -\frac{629}{32} + \frac{221}{12}\zeta_3$ $K_2[1,0,1] = \frac{163}{24} - \frac{19}{3}\zeta_3$

New terms Baikov,Chetyrkin, Kuhn (10):

$K_3[3,0,0] = \frac{2471}{768} + \frac{61}{8}\zeta_3 - \frac{715}{8}\zeta_5 + \frac{315}{4}\zeta_7;$

$K_3[2,1,0] = \frac{99757}{2304} + \frac{8285}{96}\zeta_3 - \frac{1555}{12}\zeta_5 - \frac{105}{8}\zeta_7$

$K_3[1,2,0] = -\frac{406043}{2304} + \frac{18007}{144}\zeta_3 + \frac{2975}{48}\zeta_5 - \frac{77}{4}\zeta_3^2$

$K_3[2,0,1] = -\frac{7729}{1152} - \frac{917}{16}\zeta_3 + \frac{125}{2}\zeta_5 + 9\zeta_3^2$

$K_3[1,1,1] = \frac{1055}{9} - \frac{2521}{36}\zeta_3 - \frac{125}{3}\zeta_5 - 2\zeta_3^2$

$K_3[1,0,2] = -\frac{307}{18} + \frac{203}{18}\zeta_3 + 5\zeta_5$ - Validity at a_s^4- strong check of advanced a_s^4 analytical calculations!

Q 1: Is it possible to unravel structure of $\Delta_{\rm CSB}(a_s)$ -term ?
Guess : Yes! Kataev,Mikhailov (09-10) CERN-PH-TH/2009-203;
PoS (RADCOR2009) 036(prior learning Baikov,Chetyrkin, Kuhn a_s^4 results) (arXiv:1001.0728)

$\Delta_{\rm CSB}(a_s) = \sum_{n\geq 1} \left(\frac{\beta(a_s)}{a_s}\right)^n \mathcal{P}_n(a_s)$, $\mathcal{P}_1(a_s) = \sum_{n\geq 1} P_1^{(n)} a_s^n$

$\mathcal{P}_2(a_s) = \sum_{n\geq 1} P_2^{(n)} a_s^n$, $\mathcal{P}_3(a_s) = \sum_{n\geq 1} P_3^{(n)} a_s^n$

$\mathcal{P}_1(a_s) = a_s \mathrm{C_F} \Big[\left(-\frac{21}{8} + \zeta_3\right)$ $(= K_1[1,0,0]$- BK-expansion coeff.) $+$

$\left[\left(-\frac{47}{48} + \zeta_3\right) \mathrm{C_A} + \left(\frac{397}{96} + \frac{17}{2}\zeta_3 - 15\zeta_5\right) \mathrm{C_F}\right] a_s \Big] + O(a_s^3)$

$\mathcal{P}_2(a_s) = a_s \mathrm{C_F} 3 \left(\frac{163}{24} - \frac{19}{3}\zeta_3\right)$ $((\dots) = K_2[1,0,1]|_{\mathrm{T_F N_F}}$ BK-expansion coeff.)$+ O(a_s^2)$, $\mathcal{P}_3(a_s) = O(a_s)$- was unfixed.

Obtained by: a) requiring $\mathrm{T_F N_F}$-independence of $\mathcal{P}_n(a_s)$ and absorption them into $\beta(a_s)$- coeff. (system of equations); b) expressions of $C_A^{NS}(a_s)$ and $C_{DIS}^{NS}(a_s)$- coeff. $\mathrm{d_n}$ and $\mathrm{k_n}$ $(1 \leq n \leq 3)$ through β_1, β_0 (S.V. Mikhailov Quarks-2004 and **JHEP** (2007)).

More general structure : $\Delta_{\rm CSB} = \sum_{n\geq 1}\left(\frac{\beta(a_s)}{a_s}\right)^n \mathcal{P}_n(a_s)$ with

$\mathcal{P}_n(a_s) = \sum_{r\geq 1} P_n^{(r)}[\mathrm{k},\mathrm{m}] C_F^{\mathrm{k}} C_A^{\mathrm{m}}\ a_s^r$ where $r = \mathrm{k}+\mathrm{m}$

After learning Baikov, Chetyrkin, Kuhn results (09-10) the guess was confirmed at a_s^4-level. We get additional 3 contributions:

$$\mathcal{P}_1^{(3)}(a_s) = \Bigg[\mathrm{C_F^3}\left(\frac{2471}{768} + \frac{61}{8}\zeta_3 - \frac{715}{8}\zeta_5 + \frac{315}{4}\zeta_7\right)$$

$$+ \mathrm{C_F^2 C_A}\left(\frac{16649}{1536} - \frac{11183}{192}\zeta_3 + \frac{1015}{24}\zeta_5 - \frac{105}{8}\zeta_7 + \frac{99}{4}\zeta_3^2\right)$$

$$+ \mathrm{C_F C_A^2}\left(\frac{2107}{192} + \frac{2503}{72}\zeta_3 - \frac{355}{18}\zeta_5 - 33\zeta_3^2\right)\Bigg] a_s^3$$

$$\mathcal{P}_2^{(2)}(a_s) = \Bigg[\mathrm{C_F^2}\left(-\frac{13597}{384} - \frac{2523}{16}\zeta_3 + \frac{375}{2}\zeta_5 + 27\zeta_3^2\right) +$$

$$\mathrm{C_F C_A}\left(\frac{1433}{32} - \frac{1}{4}\zeta_3 - \frac{170}{4}\zeta_5 - 6\zeta_3^2\right)\Bigg] a_s^2$$

$$\mathcal{P}_3^{(1)}(a_s) = 9\mathrm{C_F}\left(-\frac{307}{18} + \frac{203}{18}\zeta_3 + 5\zeta_5\right) a_s \ \Big(\ \dots\ \Big) = K_3[1,0,2]|_{\mathrm{T_F^2 N_F^2}}$$

BChK (09-10) term. Agrees with coeff. of BK(93) $\mathrm{N_F}$-expansion.

$$\sum_{n<10} S_n x^n = \left[-\frac{21}{2}+12\zeta_3\right]x+\left[\frac{326}{3}-\frac{304}{3}\zeta_3\right]x^2+\left[-\frac{9824}{9}+\frac{6496}{9}\zeta_3+320\zeta_5\right]x^3+\left[\frac{2760448}{243}-\frac{1268480}{243}\zeta_3-\frac{48640}{9}\zeta_5\right]x^4+\left[-\frac{280736320}{2187}+\frac{89300480}{2187}\zeta_3+\frac{5196800}{81}\zeta_5+17920\zeta_7\right]x^5+\left[\frac{10320047360}{6561}-\frac{2327111680}{6561}\zeta_3-\frac{507392000}{729}\zeta_5-\frac{1361920}{3}\zeta_7\right]x^6+\left[-\frac{3723517199360}{177147}+\frac{611395563520}{177147}\zeta_3+\frac{50008268800}{6561}\zeta_5+\frac{203714560}{27}\zeta_7+\frac{48742400}{27}\zeta_9\right]x^7+\left[\frac{485484017500160}{1594323}-\frac{59933178265600}{1594323}\zeta_3-\frac{5212730163200}{59049}\zeta_5-\frac{79559065600}{729}\zeta_7-\frac{14817689600}{243}\zeta_9\right]x^8+\left[-\frac{7616109282344960}{1594323}+\frac{726735764193280}{1594323}\zeta_3+\frac{195646580326400}{177147}\zeta_5+\frac{1120185221120}{729}\zeta_7+\frac{316630630400}{243}\zeta_9+\frac{7821721600}{27}\zeta_{11}\right]x^9$$

Large $\mathrm{N_F}$-dependence of polynom, multiplied by $\beta(a_s)/a)s$ factor - BK (93); Here $x = \mathrm{T_F N_F} a_s/4$.

This gives us first coefficients of the expansion:

$$\boldsymbol{\Delta}_{\mathrm{CSB}} = \sum_{\mathbf{n}\geq\mathbf{1}} \left(\frac{\beta(\mathbf{a_s})}{\mathbf{a_s}}\right)^{\mathbf{n}} \mathbf{P_n^{(1)} C_F a_s} \text{ , where } \mathbf{P_n^{(1)}} = \frac{\mathbf{S_n}}{\mathbf{4^n}}\mathbf{3^{(n-1)}}.$$

Important Question:
Is it possible to apply this NEW EXPRESSION for the conformal symmetry breaking term in practical QCD applications ?
Answer: give NEW constraints between 5-loop results of advanced complicated computer calculations by Baikov, Chetyrkin, Kuhn (08-10)
Consider Mikhailov (07) representations for the $C_A^{NS}(a_s)$ coefficients $\mathrm{d}_2 = \beta_0\, d_2[1] + d_2[0]$
$\mathrm{d}_3 = \beta_0^2\, d_3[2,0] + \beta_1\, d_3[0,1] + \beta_0\, d_3[1,0] + d_3[0,0]$ $\quad \mathrm{d}_4 = \beta_0^3\, d_4[3] + \beta_1\, \beta_0\, d_4[1,1] + \beta_2\, d_4[0,0,1] + \beta_0^2\, d_4[2] + \beta_1\, d_4[0,1] + \beta_0\, d_4[1] + d_4[0]$
and the similar representations for the $C_{DIS}^{NS}(a_s)$ coefficients $\mathrm{k_n}$. At the order a_s^3 it is possible to define all terms (see Mikhalov (07), Kataev-Mikhailov (10- this work)) From the original Crewther relation $C_A^{NS}(a_s) \times C_{DIS}^{NS}|_{ci} = 1\!\!1$ we get constraint:
$k_4[0] + d_4[0] = 2d_1 d_3[0] - 3d_1^2 d_2[0] + (d_2[0])^2 + d_1^4$

Application: New cross-checks for Baikov, Chetyrkin, Kuhn results from Banks-Zaks Anzats:

$\beta_0(\mathrm{T_F N_F}) = 0$ gives relation $\mathrm{T_F N_F} = \frac{11}{4}\mathrm{C_A}$

Our new representation for Δ_{CSB}, which is **POLYNOMIAL** in $(\beta(a_s)/a_s)$, leads to the identity:

$$\mathbf{d_4} + \mathbf{k_4}|_{BZ} = \mathbf{k_4}[\mathbf{0}] + \mathbf{d_4}[\mathbf{0}](\mathbf{CI})$$

$$-\beta_1(BZ)\left[d_4[0,1] + k_4[0,1]\right] - \beta_2(BZ)\left[d_4[0,0,1] + k_4[0,0,1]\right]$$

where $\beta_1(BZ) = -\frac{1}{16}\left[7\mathrm{C_A^2} + 11\mathrm{C_F C_A}\right]$

$$\beta_2(BZ) = -\mathrm{C_A^3}\frac{1127}{1536} - \mathrm{C_F C_A^2}\frac{77}{192} + \frac{11}{128}\mathrm{C_F^2 C_A}$$

Thus $\mathbf{k_4}[\mathbf{0}] + \mathbf{d_4}[\mathbf{0}](\mathbf{CI}) = 2d_1 d_3[0] - 3d_1^2 d_2[0] + (d_2[0])^2 + d_1^4$

$$= -\frac{333}{1024}\mathrm{C_F^4} + \mathrm{C_F^2 C_A^2}\left[\frac{525}{512} - \frac{81}{16}\zeta_3\right] + \mathrm{C_F^3 C_A}\left[\frac{99}{64}\right]$$

Next: $d_4[0,1] + k_4[0,1] = P_1^{(2)} + \left(k_3[0,1] - d_3[0,1]\right)d_1$

$$= \left[\mathrm{C_F C_A}\left(-\frac{47}{48} + \zeta_3\right) - \mathrm{C_F^2}\left(\frac{323}{96} + \frac{61}{4}\zeta_3 - 15\zeta_5\right)\right]$$

$$d_4[0,0,1] + k_4[0,0,1] = +P_1^{(1)} = \mathrm{C_F}\left(-\frac{21}{8} + 3\zeta_3\right)$$

$-\beta_1(BZ)\,[d_4[0,1] + k_4[0,1]] = \mathrm{C_F C_A^3}\left[-\frac{329}{768} + \frac{7}{16}\zeta_3\right]$ (no ζ_5- but they exist in d_4 and k_4 !)

$+\mathrm{C_F^2 C_A^2}\left[-\frac{3295}{1536} + \frac{471}{64}\zeta_3 - \frac{105}{16}\zeta_5\right] + \mathrm{C_F^3 C_A}\left[-\frac{3553}{1536} + \frac{671}{64}\zeta_3 - \frac{165}{16}\zeta_5\right]$

$-\beta_2(BZ)\,[d_4[0,0,1] + k_4[0,0,1]] =$

$\mathrm{C_F C_A^3}\left[-\frac{7889}{4069} + \frac{1127}{512}\zeta_3\right] + \mathrm{C_F^2 C_A^2}\left[-\frac{539}{512} + \frac{77}{64}\zeta_3\right] + \mathrm{C_F^3 C_A}\left[\frac{231}{1024} - \frac{33}{128}\zeta_3\right]$

Finally:

$\mathrm{d_4}(BZ) + \mathrm{k_4}(BZ) = -\frac{333}{1024}\mathrm{C_F^4} + \mathrm{C_A C_F^3}\left(-\frac{1661}{3072} + \frac{1309}{128}\zeta_3 - \frac{165}{16}\zeta_5\right) + \mathrm{C_A^2 C_F^2}\left(-\frac{3337}{1536} + \frac{7}{2}\zeta_3 - \frac{105}{16}\zeta_5\right) + \mathrm{C_A^3 C_F}\left(-\frac{28931}{12288} + \frac{1351}{512}\zeta_3\right)$

This expression, corrected and finally checked at **28.06. 2010** after the original presentation of the talk at **11.06. 2010 is satisfied** for the Baikov-Chetyrkin-Kuhn 5-loop symbolic analytical results.

Conclusions: 1) We present new QFT (QCD and QED) expression for the conformal symmetry breaking term in the relation between e^+e^- annihilation and DIS sum rules - addition to Crewther unity: $\Delta_{\rm CSB} = \sum_{n\geq 1}\left(\frac{\beta(a_s)}{a_s}\right)^n \sum_{r\geq 1} P_n^{(r)}[{\rm k},{\rm m}]C_F^{\rm k}C_A^{\rm m}\ a_s^r$ where $r = {\rm k}+{\rm m}$ and fix there coefficients at a_s^4- level and in the large $\rm N_F$ expansion up to a_s^9.

2) Applications: new constraints between ${\rm d}_4 + {\rm k}_4$ BChK new results. for $\beta_0 = 0$ - Banks-Zaks limit.

3) Odd ζ-function studies- confirm that in BZ approximation ζ_7 and ζ_3^2 disappear- proportional to β_0. ζ_{2n+1}-studies - possible link to SUSY-oriented theoretical problems ?

4) In QED diagrammatic representations of new generalisations is straightforward.

5) QCD applications- form-factors ? Summations of power series with expansion parameter being RG β-function ? **Analytical "experiments" detect detailed structures of QFT models**