Possible similarities in structures of analytical multiloop effects in pqQED and in $N = 4$ SYM

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β function in quenched QED (pqQED) and $\zeta(3)$

4-Loop result of **Gorishny–Kataev–Larin (1988–90, reported at Quarks'90)** and 5-loop result of **Baikov–Chetyrkin–Kuhn (2008)**:

$$\beta^{[1]}_{\text{QED}}(A) = \frac{4}{3}A + 4A^2 - 2A^3 - 46A^4 + \left[\frac{4157}{6} + 128\zeta(3)\right]A^5 + O(A^6) = \frac{4}{3}A \times C^{\text{ns}}_{D}(A)\,.$$

The QED perturbation-theory expansion parameter is normalized as $A = \alpha/(4\pi)$ with α being the renormalized QED coupling constant.

β function in pqQED and $\zeta(3)$

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Is there any other RG function for a gauge-invariant operator, which contain $\zeta(3)$-function in high order corrections?

PT series for mass AD in pqQED and $\zeta(3)$

It differs from AD of $\overline{\Psi}\Psi$ by overall sign only.
Consider massless conformal-invariant limit of the QED series for the AD $\gamma_{\overline{\Psi}\Psi}(A) = -\gamma_m(A)$.
It can be obtained from the 4-loop QCD calculations of mass AD $\gamma_m(\alpha_s)$, done by **Larin–van Ritbergen–Vermaseren (1997)** and **Chetyrkin (1997)** independently.
More convenient to use the results of **Larin et al.**, since they have explicit dependence on Casimir operators C_F and C_A, normalization factor T_F and the number of quarks flavours N_F.

PT series for mass AD in pqQED and $\zeta(3)$

The choice $C_F = 1$,$C_A = 0$, $T_F = 1$ and $N_F = 0$ corresponds to the case of the conformal-invariant pqQED limit.

The pqQED expression for the AD of the gauge-invariant operator $\overline{\Psi}\Psi$ has the following form

$$\gamma^{pqQED}_{\overline{\Psi}\Psi}(A) = -3A - \frac{3}{2}A^2 - \frac{129}{2}A^3 + \left(\frac{1261}{8} + 336\,\zeta(3)\right)A^4 + O(A^5)$$

pqQED series for NS CFs and $\zeta(3)$

From 5-loop pqQED β-function calculations it follows

$$C_D^{\text{ns}}(A) = 1 + 3A - \frac{3}{2}A^2 - \frac{69}{2}A^3 + \left[\frac{4157}{8} + 96\zeta(3)\right]A^4 + O(A^5)\,.$$

In the conformal-invariant limit Crewther relation holds:

$$C_D^{\text{ns}}(A) \times C_{\text{Bjp}}^{\text{ns}}(A) = 1$$

pqQED series for NS CFs and $\zeta(3)$

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Thus, as reported by **Kataev at Quarks'08**

$$C_{\rm Bjp}^{\rm ns}(A) = 1 - 3A + \frac{21}{2}A^2 - \frac{3}{2}A^3 - \left[\frac{4823}{8} + 96\zeta(3)\right]A^4 + O(A^5)\,.$$

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$$-\left[\frac{4823}{8} + 96\zeta(3)\right]A^4 + O(A^5)\,.$$

It was confirmed by **Baikov–Chetyrkin–Kuhn** at **Quarks'10** by diagrammatic calculations.

pqQED series, Crewther relation and Ellis–Jaffe SR

Ellis–Jaffe SR in QCD:

$$EJp(Q^2) = \int_0^1 g_1^{lp}(x, Q^2)dx$$

$$= C^{\rm ns}_{\rm Bjp}(A_s(Q^2))\left[\frac{1}{12}a_3 + \frac{1}{36}a_8\right] + C^{\rm s}_{\rm EJp}(Q^2)\frac{1}{9}\Delta\Sigma(Q^2)$$

where $a_3 = \Delta u - \Delta d$, $a_8 = \Delta u + \Delta d - 2\Delta s$, Δu, Δd and Δs are the polarized distributions and $\Delta\Sigma$ depends from the scheme choice. In the $\overline{MS}$-scheme it is defined as $\Delta\Sigma = \Delta u + \Delta d + \Delta s$, while in the Adler–Bardeen scheme it contains the additional additive contribution from polarized gluon distribution ΔG.

pqQED series, Crewther relation and Ellis–Jaffe SR

In the pqQED limit the singlet CF has the following form:

$$C^{\mathsf{s}}_{\mathsf{EJp}} = \overline{C}^{\mathsf{s}}_{\mathsf{EJp}}(A)/Z^{\mathsf{s}}_{5}(A)\,,$$

where

$$\overline{C}^{\mathsf{s}}_{\mathsf{EJp}}(A) = 1 - 7A + \frac{89}{2}A^2 - \left[\frac{1397}{6} - 96\zeta(3)\right]A^3 + O(A^4)$$

obtained in **Larin–van Ritbergen–Vermaseren (1997)**, and Z^{s}_{5} finite renormalization constant, containing $\zeta(3)$ as well:

$$Z^{\mathsf{s}}_{5}(A) = 1 - 4A + 22A^2 + \left[-\frac{370}{3} + 96\zeta_3\right]A^3 + O(A^4)$$

extracted from **Larin–Vermaseren (1991)**.

pqQED series, Crewther relation and Ellis–Jaffe SR

$$C^{\rm s}_{\rm EJp}(A) = \overline{C}^{\rm s}_{\rm EJp}(A)/Z^{\rm s}_5(A)\,,$$

$$\overline{C}^{\rm s}_{\rm EJp}(A) = 1 - 7A + \frac{89}{2}A^2 - \left[\frac{1397}{6} - 96\zeta(3)\right]A^3 + O(A^4)$$

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Thus $\zeta(3)$ in $C^{\rm s}_{\rm EJp}(A)$ cancels:

$$C^{\rm s}_{\rm EJp}(A) = 1 - 3A + \frac{21}{2}A^2 - \frac{3}{2}A^3 + O(A^4)\,.$$

It **coincides** with the similar expression for the pqQED series of $C^{\rm ns}_{\rm Bjp}(A) = 1 - 3A + \frac{21}{2}A^2 - \frac{3}{2}A^3$.

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Question: Why $C^{s}_{EJp}(A) = C^{ns}_{Bjp}(A)$? (in all orders of PT)

pqQED series, Crewther relation and Ellis–Jaffe SR

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It **coincides** with the similar expression for the pqQED series of $C^{\rm ns}_{\rm Bjp}(A) = 1 - 3A + \frac{21}{2}A^2 - \frac{3}{2}A^3$.

Question: Why $C^{\rm s}_{\rm EJp}(A) = C^{\rm ns}_{\rm Bjp}(A)$? (in all orders of PT)

Answer: Crewther relations in the singlet + nonsinglet channels

$$C^{\rm s}_{\rm EJp}(A) \times C^{\rm s}_{D}(A)|_{\rm conf\text{-}sym} = 1$$

derived in **Kataev (1996)**,

$$C^{\rm ns}_{\rm Bjp}(A) \times C^{\rm ns}_{D}(A)|_{\rm conf\text{-}sym} = 1\,.$$

pqQED series, Crewther relation and Ellis–Jaffe SR

Therefore at the 5-loop level

$$C^{\mathrm{s}}_{\mathrm{EJp}}(A) = 1 - 3A + \frac{21}{2}A^2 - \frac{3}{2}A^3$$

$$-\left[\frac{4823}{8} + 96\zeta(3)\right]A^4 + O(A^5)$$

and contains $\zeta(3)$.

AD of Konishi operator N=4 SYM

Konishi operator is defined as

$$O_{\mathrm{K}} = \mathrm{tr}\, \overline{\Phi}_i \Phi^i \,,$$

where Φ^i is the complex adjoint scalar field.
Direct quantum field theory perturbative calculation **Velizhanin (2008)** gave the following result

$$\gamma_{\mathrm{K}}(\lambda) = 12\lambda - 48\lambda^2 + 336\lambda^3$$

$$-\lambda^4 \Big[2496 - 576\zeta(3) + 1440\zeta(5) \Big] + O(\lambda^5) \,,$$

where $\lambda = g^2 N_c/(4\pi)^2$ and N_c is the "number of colours" of $SU(N_c)$ gauge group.
Interesting feature of $N = 4$ SYM theory is that the property of **AdS/CFT** correspondence links $N = 4$ SYM with the theory of superstrings in **AdS**$_5 \times S^5$.

AD of Konishi operator N=4 SYM

$\gamma_{\mathrm{K}}(\boldsymbol{\lambda})$ can be also calculated using Bethe Ansatz quantization, from one side, and superstring theory methods, from the other side. Both give:

$$\gamma_{\mathrm{K}} = \gamma_{\mathrm{asymp}}(\boldsymbol{\lambda}) + \gamma_{\mathrm{L\ddot{u}sher}}(\boldsymbol{\lambda})\,,$$

where **Kotikov, Lipatov, Rej, Staudacher and Velizhanin (2007)**:

$$\gamma_{\mathrm{asymp}} = \mathbf{12\lambda - 48\lambda^2 + 336\lambda^3 - \Big[2820 + 288\zeta(3)\Big]\lambda^4}$$

Bajnok & Janik (2009), Fiamberti, Santambrogio, Sieg and Zanon (2008):

$$\gamma_{\mathrm{L\ddot{u}sher}}(\boldsymbol{\lambda}) = \mathbf{\Big[324 + 864\zeta(3) - 1440\zeta(5)\Big]\lambda^4}\,.$$

1st Conclusion

Question:

What is the origin of $\zeta(5)$ absence in pqQED?

At present I do not know whether it is possible to calculate **Lüscher-type** corrections in pqQED.

If **Yes** — **Lüscher-type** corrections in pqQED may cancel to zero.

2nd Conclusion

The analytical form of **5-loop** pqQED singlet contribution to **Ellis–Jaffe SR** is obtained using the **Crewther relation**.