Multiquark functions in effective models

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A number of perspective physical applications of the effective models are connected with multi-quark functions, which are the subject of present report. The basic method of calculations is a formalism of multilocal (double, triple, etc.) sources

Dahmen H.D., Jona-Lasinio G.
Variational formulation of quantum field theory.
Nuovo Cim. A52:807–838, 1967,

Rochev V.E.
Many-Particle Relativistic Equations For Fermions.
Teor. Mat. Fiz. 51:22–33, 1982 (Theor. Math. Phys. 51:330-337, 1982).

As an object of application of the method we choose Nambu - Jona-Lasinio (NJL) model

Nambu Y. and Jona-Lasinio G.
Dynamical model of elementary particles based on an analogy with superconductivity.
Phys. Rev. 122:345–358, 1961; 124:246–254, 1961.

This model is one of the most successful effective models of quantum chromodynamics for the light hadrons.The NJL model is widely used also in various aspects of the physics of light nuclei.
It is necessary to note, that this method has been successfully applied for the other field-theoretic models and can be applied also for analogous calculations in other similar effective models. The multi-quark functions arise in higher orders of the mean-field expansions (MFE) for the NJL model. To formulate the MFE we have used an iteration scheme of solution of the Schwinger-Dyson equation with the fermion bilocal source, which has been developed in works

Rochev V.E.
A nonperturbative method of calculation of Green functions.
J. Phys. A30:3671–3680, 1997;

Rochev V.E. and Saponov P.A.
The four fermion interaction in $D = 2, D = 3, D = 4$: A Nonperturbative treatment.
Int. J. Mod. Phys. A13:3649–3666, 1998;

Rochev V.E.
On nonperturbative calculations in quantum electrodynamics.
J. Phys. A33:7379–7406, 2000.

The report is organized as follows.

In Section 2 we describe the method of construction of the MFE with the fermion bilocal source for the NJL model with the $SU_V(2) \times SU_A(2)$–symmetric four-quark interaction and, for the sake of completeness, consider the well-known leading approximation results of this model. Also in this section we investigate the first-after-leading step of the iteration scheme, which gives us the equations for the leading order two-particle Green function and NLO correction to the propagator of quarks.
In Section 3 we describe the second step of the iteration scheme. As a result we obtain the equations for four-quark Green function and for the three-quark Green function. We also obtain in this step the equations for NLO two-quark function and NNLO correction to quark propagator. We discuss the structure of second step equations and obtain the solutions of four-quark and three-quark equations.

In Section 4 in base of work

Rochev V.E.
Meson contributions in the Nambu-Jona-Lasinio model.
Teor. Mat. Fiz.159:81-95,2009 (Theor. Math. Phys.159: 488-498, 2009),

a problem of calculations of corrections to mean-field approximation in NJL model is discussed, where, for solve this problem author (V.E. Rochev) used the Legendre transformation method with respect to a bilocal source. This method effectively takes into account symmetry limitations, which are originated due to chiral Ward identity. The corrections to quark propagator and two-particle quark functions are defined by proposed method.

In Section 5 we describe the third step of iteration scheme. As a result we obtain the equations for six-quark Green function and for the five-quark Green function, and, the NLO equations for four-quark and three-quark Green functions. We also obtain in this step the equations for NNLO two-particle function and NNNLO correction to quark propagator.

Jafarov R.G. and Rochev V.E.
On equations for the multi-quark bound states in Nambu-Jona-Lasinio model.
arXiv:hep-ph/0609183.

In Section 6 the modification of the MFE for the NJL model in the formalism with the multilocal diquark and triple-quark sources is briefly discussed.

Section 2.The mean-field expansion in the bilocal source formalism. Leading and first step equations

As an example of the physically interesting model we consider the chiral-symmetric NJL model. The model contains up and down quarks fields ψ, each with n_c colors. The Lagrangian of the two-flavor NJL model may be written in the well-known form

$$L = \bar{\psi} i\hat{\partial}\psi + \frac{g}{2}\left[(\bar{\psi}\psi)^2 + (\bar{\psi} i\gamma_5\tau^a\psi)^2\right]. \quad (1)$$

To construct the MFE we use an iteration scheme of the solution of functional-differential Schwinger-Dyson equation (SDE)

$$G + i\hat{\partial}\frac{\delta G}{\delta\eta} + ig\left\{\frac{\delta}{\delta\eta} tr\left[\frac{\delta G}{\delta\eta}\right] + i\gamma_5\tau^a\frac{\delta}{\delta\eta} tr\left[i\gamma_5\tau^a\frac{\delta G}{\delta\eta}\right]\right\} = \eta \star \frac{\delta G}{\delta\eta} \quad (2)$$

for the generating functional G of Green functions.
The generating functional G can be represented as the functional integral with bilocal fermion source η:

$$G(\eta) = \int D(\psi,\bar{\psi}) \exp i\left\{\int dxL - \int dxdy\bar{\psi}(y)\eta(y,x)\psi(x)\right\}. \quad (3)$$

We shall solve Eq. (2) employing the method which proposed in work

Jafarov R.G. and Rochev V.E.
Central Eur. J. Phys. 2:367–381, 2004
arXiv: hep-ph/0311339;

Jafarov R.G. and Rochev V.E.
arXiv:hep-ph/0406333.

The solution of the equation of leading approximation,i.e., the functional-differential SDE (2) with zero r.h.s., is the following functional $G^{(0)} = \exp\left\{\mathrm{Tr}\left(S \star \eta\right)\right\}$, where S is solution of the equation

$$1 + i\hat{\partial}S + igS \cdot tr[S(0)] = 0. \quad (4)$$

The leading approximation generates the linear iteration scheme

$$G = G^{(0)} + G^{(1)} + \cdots + G^{(n)} + \cdots ,$$

consists in the step-by-step solutions of the equations

$$G^{(n)} + i\hat{\partial}\frac{\delta G^{(n)}}{\delta\eta} + ig\left\{\frac{\delta}{\delta\eta} tr\left[\frac{\delta G^{(n)}}{\delta\eta}\right] - \gamma_5\tau^a\frac{\delta}{\delta\eta} tr\left[\gamma_5\tau^{\mathbf{a}}\frac{\delta G^{(n)}}{\delta\eta}\right]\right\} = \eta\star\frac{\delta G^{(n-1)}}{\delta\eta}. \qquad (5)$$

Functional $G^{(n)}$ is $G^{(n)} = P^{(n)}G^{(0)}$, where $P^{(n)}$ is a polynomial of $2n$ -th degree on the bilocal source η.

The unique connected Green function of the leading approximation S is the quark propagator. A solution of Eq. (4) is $S(p) = (m - \hat{p})^{-1}$, where m is the dynamical quark mass, which is a solution of the gap equation of the NJL model in the chiral limit.

The other connected Green's functions appear in the subsequent steps of the iterative scheme.

The first iteration step contains the leading-order equation for the two-particle S_2 quark function

$$S_2 = -S\cdot S + K\star S_2 \qquad (6)$$

(Here $K = ig\Big\{(S\cdot S)\star tr[S_2] - (S\gamma_5\tau^a S)\star tr[\gamma_5\tau^a S_2]\Big\}$ is the kernel of equation) and also the equation for the correction to the propagator $S^{(1)}$

$$S^{(1)} = [S^{(1)}]^0 + ig\Big\{(S \star S) \cdot tr[S^{(1)}(0)]\Big\}. \tag{7}$$

(Here $[S^{(1)}]^0 = ig\{S \star [S_2 - \gamma_5\tau^a S_2 \gamma_5 \tau^a]\}$ is inhomogeneous term of equation).

As a result of standard operation we obtain for the sigma-meson and pion amplitudes:

$$A_\sigma(p) = \frac{1}{4n_c(4m^2 - p^2)I_0(p)},$$

$$A_\pi(p) = \frac{1}{4n_c p^2 I_0(p)},$$

$I_0(p)$ – single-loop integral.

Eq. (7) for NLO quark propagator $S^{(1)}$ gives us a possibility for define the meson corrections to quark mass. Introducing the NLO mass operator $\Sigma^{(1)} = S^{-1} \star S^{(1)} \star S^{-1}$, we obtain from Eq. (7):

$$\Sigma^{(1)}(x) = S(x)A_\sigma(x) + 3S(-x)A_\pi(x) + ig\delta(x)[S^{(1)}(0)]. \tag{8}$$

Section 3. Second step equations and solutions

The second step contains the equations for the four (S_4)- and three-particle S_3 functions and also the equations for the two-particle function $S_2^{(1)}$ and the second-order corrections to the quark propagator $S^{(2)}$. For these four functions we have a system of four integral equations. All these equations (and all equations of following steps of the iteration scheme) possess the structure, which is similar to the structure of Eq. (6):

$$S_n = S_n^0 + ig\Big\{(S \cdot S) \star tr[S_n] - (S\gamma_5\tau^a \cdot S) \star tr[\gamma_5\tau^a S_n]\Big\},$$

and differ from each other by the structure of inhomogeneous terms S_n^0. The inhomogeneous term in the equation for four-quark function S_4 is

$$S_4^0 = 3 \cdot \Big\{-S \cdot S \cdot S_2\Big\}, \tag{9}$$

where S_2 is defined in preceding section by Eq. (6). The inhomogeneous term in the equation for three-quark function S_3

$$S_3^0 = 2 \cdot \Big\{-S \cdot S \cdot S^{(1)}\Big\} + 2 \cdot \Big[-S \cdot S_2\Big] + ig \cdot S \star \Big\{tr[S_4] - \gamma_5\tau tr[\gamma_5\tau S_4]\Big\}.$$

Due to such structure of the system its solution should be started with the equation for four-quark function S_4, then should be solved the equation for three-quark function S_3, etc.
The equation for four-quark Green function S_4 with inhomogeneous term (9) has following simple solution

$$S_4 = 3 \cdot \left\{ S_2 \cdot S_2 \right\}.$$

This solution is disconnected, and it means the absence of physical effects due to four-particle functions in the given order of MFE. Particularly, the pion-pion scattering can not describe in the given order and will be appear only in next step of iteration scheme (third order of MFE), in principle. The solution can be obtained likewise to solving of Eq. (6) for the two-quark function. The connected part of the amputated three-quark function possesses two-meson and three-meson contributions.

Section 4.The corrections to the two-quark function and the Legendre transformation in mean-field expansion

This iterative scheme is attractive because the equations of any order have a simple analytic structure. Solving the equations of any order is actually an algebraic problem reminiscent of constructing a perturbation series.
The reverse side of this simplicity is problems arising related to including symmetry and its physical consequences. The main problem of such kind arises in calculating corrections to the two-particle function.
This Section based in work

Rochev V.E.
Teor. Mat. Fiz.159:81-95,2009 (Theor. Math. Phys.159: 488-498, 2009).

where, author use the method of the Legendre transformation with respect to a bilocal source to determine corrections the two-quark function in the NJL model.
The two-quark function of the leading approximation in the NJL model contains two single-pole terms corresponding to composite mesons. The presence of these mesons in the spectrum of the NJL model is one of the main effects of this model. The scalar meson (the sigma meson) is massive, while the pseudoscalar meson (the pion) is massless in the chiral limit according to the general NGB theorem for systems with spontaneously broken symmetry.

But in the process of calculating the corrections to the two-quark function in the framework of the above-mentioned method the terms containing second-order poles in the quark-antiquark channel appear. It is very difficult to coordinate these terms, which are artifacts of the computational scheme, with the widely accepted interpretation of results in terms of particles. Another problem arises in attempting to interpret them as the first terms of some expansions: the pion acquires mass in the chiral limit, which contradicts the NGB theorem.A way out of this dilemma is to modify the calculation method in such a manner that the pion masslessness will be preserved in the chiral limit in the theory with higher-order corrections taken into account.

The method for such a modification is to pass from the generating functional G to the generating functional of the Legendre transformation with respect to the bilocal source η.

Such a passage in terms of perturbation theory diagrams means that only those diagrams that are two-quark irreducible are taken into account. The other diagrams, just as in the usual perturbation theory, are effectively taken into account by the functional combinatorics of the Legendre transformation.

Generating functional of the Legendre transformation depending on the functional variable S is:

$$W[S] = Z[\eta[S]] + iTr[S \star \eta[S]],$$

$S(\eta) = i\frac{\delta Z[\eta]}{\delta \eta}, \quad \frac{1}{i}\frac{\delta W}{\delta S} = \eta.$
In view of these relations, the SDE for the generating functional of the Legendre transformation becomes

$$\frac{1}{i}\frac{\delta W}{\delta S} = S^{-1} + i\hat{\partial} + ig\{tr[S] + i\gamma_5\tau^a tr[i\gamma_5\tau^a S]\} + ig\left\{S_2 \star S^{-1} + i\gamma_5\tau^a S_2 \star i\gamma_5\tau^a S^{-1}\right\},$$

where $S_2 = i\frac{\delta^2 Z}{\delta\eta\delta\eta} = \frac{\delta S}{\delta \eta}$ is two-quark function.
The two-quark function, as a functional of S, is defined by the relation

$$S_2 \star \frac{\delta \eta}{\delta S} = S_2 \star \frac{1}{i}\frac{\delta^2 W}{\delta S \delta S} = 1. \quad (10)$$

This equations in essentially the equation for two-particle function (Bethe-Salpeter equation(BSE)) in the formalism of the Legendre transformation with respect to the bilocal source η.

The kernel K of the BSE is the connected part of the second derivative of the generating functional of the Legendre transformation and is defined by the relation $\frac{\delta\eta}{\delta S} = -S^{-1} \cdot S^{-1} + K$. Two-quark amplitude is

$$F_2^c = -K + S \star F_2^c \star S \star K. \quad (11)$$

In terms of the generating functional of the Legendre transformation, chiral Ward identity has the form

$$\tau^a \gamma_5 i\hat{\partial} = tr\left[\frac{\delta\eta}{\delta S} \star S\gamma_5\tau^a + \tau^a\gamma_5 S \star \frac{\delta\eta}{\delta S}\right] \quad (12)$$

Easily determine that chiral Ward identity (12) holds in the leading approximation, and this guarantees the existence of a massless pion in the given approximation. With the switched-off source, the solution of the equation for two-quark amplitude has the form of solution for leading order two-quark equation

$$F_0^c = \{1 \cdot 1 A_\sigma + i\gamma_5\tau^a \cdot i\gamma_5\tau^a A_\pi\},$$

In the leading order, there are no differences in the descriptions of physical effects between the Legendre transformation formalism and the mean-field expansion considered in Section 2.

The equation for the generating functional of the Legendre transformation with the corrections included has the form

$$\eta = S^{-1} + i\widehat{\partial} + ig\{tr[S] + i\gamma_5\tau^a tr[i\gamma_5\tau^a S] + \frac{1}{i}\frac{\delta W_1}{\delta S},$$

where

$$\frac{1}{i}\frac{\delta W_1}{\delta S} = S \star F_0^c. \tag{13}$$

In Eq. (13) F_0^c is the functional of S, whose derivatives can be calculated using relation (10). The derivative of Eq. (13) with respect to the functional variable S yields the correction to the kernel of the BSE for the two-particle function:

$$K_1 = F_0^c + S \star \frac{\delta F_0^c}{\delta S} \tag{14}$$

Differentiating relation (11) leads to the equation for the three-particle function $\delta F_0^c/\delta S$

$$\frac{\delta F_0^c}{\delta S} =$$

$$= -\left\{A_\sigma \cdot F_0^c \star S + A_\sigma \cdot S \star F_0^c + (i\gamma_5\tau^a)\left[A_\pi \cdot F_0^c \star S(i\gamma_5\tau^a) + A_\pi \cdot (i\gamma_5\tau^a)S \star F_0^c\right]\right\} \tag{15}$$

Formula (15) together with (14) yields the correction to the kernel of the BSE for the two-particle function.

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As can be seen, this correction consists of the one-meson (the first term in (14))and two-meson contributions of three-particle function (15).
The main problem in calculating the corrections to the two-particle amplitude is the requirement that these corrections correspond to the NGB theorem. The chiral Ward identity holds in the determinations of these corrections by the Legendre transformation method, which ensures the validity of the NGB theorem in this approach.
Verifying the chiral Ward identity (12) for the kernel of the BSE including corrections (14) and ((15) is a less trivial procedure than similarly verifying the leading approximation, nevertheless the result is positive for this case too. This result shows that the approximation under consideration is physically acceptable, i.e. it is a symmetry-preserving approximation for calculating the corrections to the two-particle function.

As we have showed above the equation for the four-quark function S_4 has a simple exact solution which is the product of first-order two-quark functions (see Eq. (12)). As it seen from this solution, the pion-pion scattering in NJL model is suppressed, i.e. in the second order of MFE this scattering is absent. This process arises in the third order of our iterative scheme, i.e. in NLO four-quark function $S_4^{(1)}$.
The third-step generating functional is

$$G^{(3)}[\eta] = \left\{ \frac{1}{6!} \mathrm{Tr}\left(S_6 * \eta^6\right) + \frac{1}{5!} \mathrm{Tr}\left(S_5 * \eta^5\right) + \frac{1}{4!} \mathrm{Tr}\left(S_4^{(1)} * \eta^4\right) + \right.$$

$$\left. \frac{1}{3!} \mathrm{Tr}\left(S_3^{(1)} * \eta^3\right) + \frac{1}{2} \mathrm{Tr}\left(S_2^{(2)} * \eta^2\right) + \mathrm{Tr}\left(S^{(3)} * \eta\right) \right\} G^{(0)}.$$

After standard operations we obtain the equations for six-quark function S_6 and for five-quark function S_5. Inhomogeneous terms are following:

$$S_6^0 = 5 \cdot \left\{ -S \cdot S \cdot S_4 \right\} \quad (16)$$

and

$$S_5^0 = 4 \cdot \left\{ -S \cdot S \cdot S_3 \cdot \right\} + 4 \cdot \left[-S \cdot S_4 \right] + ig \left\{ \left[S \star S_6 \right] - \left[S\gamma_5\tau^a \star S_6\gamma_5\tau^a \right] \right\}, \quad (17)$$

accordingly. The equations for six-quark function and for the five-quark function with inhomogeneous term (16) and (17) in our iteration scheme are new.
The third step of iterative scheme gives us the equation for NLO four-quark function ($S_4^{(1)}$). As we note above the structure of this equation have are the form (11) with following inhomogeneous term

$$(S_4^{(1)})^0 = 3 \cdot \left\{ -S \cdot S \cdot S_2^{(1)} \right\} + 3 \cdot \left[-S \cdot S_3 \right] + ig \left\{ \left[S \star S_5 \right] - \left[S\gamma_5\tau^a \star S_5\gamma_5\tau^a \right] \right\}. \quad (18)$$

The equation for NLO four-quark function $S_4^{(1)}$ gives us possibility to describe the pion-pion scattering in quark fields context. The inhomogeneous term (18) of equations for four-quark NLO function $S_4^{(1)}$ contains five-quark function S_5, three-quark function S_3 and NLO two-quark function $S_2^{(1)}$. The inhomogeneous term (17) for five-quark equation include the six-quark function S_6, four-quark function S_4 and three-quark function S_3.

Before the investigation of NLO four-quark function $S_4^{(1)}$ it is necessary to find the solution of equation for six-quark function S_6, because the inhomogeneous part (17) includes function S_6. Also it is necessary to find a solution of equation for NLO two-quark function $S_2^{(1)}$ (see Section 4).
The solution of six-quark equation is the sum of products of two-quark functions S_2 and four-quark functions S_4:

$$S_6 = 5 \cdot \left\{ S_2 \cdot S_4 \right\}$$

In this step we obtain also the equations for NLO three-quark function $S_3^{(1)}$, NNLO two-quark function $S_2^{(2)}$ and the equation for NNNLO correction to the quark propagator $S^{(3)}$, which matter the forms (16), at $n = 3$, $n = 2$, $n = 1$, accordingly.

Section 6. The generalization of the method for other types of multi-quark sources

In this last Section we consider the generalization of MFE of Section 2, which includes other types of multi-quark sources except of bilocal source η. Such generalization can be useful for the description of baryons in the framework of MFE. Firstly, consider the formalism with diquark sources. For this purpose, we add two diquark-source terms ξ and $\bar{\xi}$ in the exponent of Eq. (3) for generating functional G:

$$G(\eta,\xi,\bar{\xi}) = \int D(\psi,\bar{\psi}) \exp i\Big\{ \int dxL - \int dxdy\bar{\psi}(y)\eta(y,x)\psi(x) +$$

$$+ \int dx_1 dx_2 \bar{\psi}(x_1)\bar{\psi}(x_2)\xi(x_1,x_2) + \int dx_1 dx_2 \bar{\xi}(x_1,x_2)\psi(x_1)\psi(x_2)\Big\}.$$

With these sources SDE (2) is modified as follows:

$$G + i\hat{\partial}\frac{\delta G}{\delta\eta} + ig\Big\{\frac{\delta}{\delta\eta} tr\left[\frac{\delta G}{\delta\eta}\right] - \gamma_5\tau^a \frac{\delta}{\delta\eta} tr\left[\gamma_5\tau^a \frac{\delta G}{\delta\eta}\right]\Big\} =$$

$$= \eta \star \frac{\delta G}{\delta\eta} + 2\cdot\frac{\delta G}{\delta\xi} \star \xi. \tag{19}$$

We have, apart from SDE (19), the additional SDE, which generates by new sources:

$$i\hat{\partial}\frac{\delta G}{\delta\bar{\xi}} + ig\left\{\frac{\delta}{\delta\bar{\xi}} tr\left[\frac{\delta G}{\delta\eta}\right] - \gamma_5\tau^a\frac{\delta}{\delta\bar{\xi}} tr\left[\gamma_5\tau^a\frac{\delta G}{\delta\eta}\right]\right\} =$$

$$= \eta\star\frac{\delta G}{\delta\bar{\xi}} - 2\cdot\bar{\xi}\star\frac{\delta G}{\delta\eta}. \tag{20}$$

It should be noted, that the presence of the new diquark source leads to the connection condition for derivatives of generating functional:

$$\frac{\delta^2 G}{\delta\bar{\xi}(x_2, x_1)\delta\eta(y, x)} = -\frac{\delta^2 G}{\delta\bar{\xi}(x_1, x)\delta\eta(y, x_2)}. \tag{21}$$

Due to this connection condition SDE (20) can be rewritten in the alternative forms. These alternative forms, being fully equivalent from the point of view of an exact solution of SDE's, can lead to different approximations in the MFE. The choice of the suitable forms for the construction of MFE in the case should be made with an assistance of corresponding physical reasons.
In the very similar manner one can introduce three-quark, or baryon sources. These sources can be used for the direct description of nucleons and other baryons omitting the intermediate diquark modelling.

The generating functional with anti-commutative three-quark sources ζ and $\bar{\zeta}$ is

$$G(\eta,\zeta,\bar{\zeta}) = \int D(\psi,\bar{\psi}) \exp i\Big\{ \int dxL - \int dxdy\bar{\psi}(y)\eta(y,x)\psi(x) +$$

$$+ \int dx_1 dx_2 dx_3 \bar{\psi}(x_1)\bar{\psi}(x_2)\bar{\psi}(x_3)\zeta(x_1,x_2,x_3) + \int dx_1 dx_2 dx_3 \bar{\zeta}(x_1,x_2,x_3)\psi(x_1)\psi(x_2)\psi(x_3)\Big\}.$$

SDE (2) with three-quark sources is modified as follows:

$$G + i\hat{\partial}\frac{\delta G}{\delta \eta} + ig\Big\{\frac{\delta}{\delta\eta} tr\Big[\frac{\delta G}{\delta\eta}\Big] - \gamma_5\tau^a\frac{\delta}{\delta\eta} tr\Big[\gamma_5\tau^a\frac{\delta G}{\delta\eta}\Big]\Big\} = \eta\star\frac{\delta G}{\delta\eta} - 3\cdot\frac{\delta G}{\delta\xi}\star\zeta. \qquad (22)$$

As above, apart from SDE (22), the additional SDE exists, which generates by the three-quark sources:

$$i\hat{\partial}\frac{\delta G}{\delta \bar{\zeta}} + ig\Big\{\frac{\delta}{\delta\bar{\zeta}} tr\Big[\frac{\delta G}{\delta\eta}\Big] - \gamma_5\tau^a\frac{\delta}{\delta\bar{\zeta}} tr\Big[\gamma_5\tau^a\frac{\delta G}{\delta\eta}\Big]\Big\} = \eta\star\frac{\delta G}{\delta\bar{\zeta}} + 3i\cdot\frac{\delta^2 G}{\delta\eta\delta\eta}\bar{\zeta}. \qquad (23)$$

The connection condition for the derivatives of the generating functional, which is very similar to the condition (21), also exists in the three-quark-source formalism, and also leads to alternative forms of SDE (23).
The method of the construction of MFE for these system of equations is similar to that of Section 2. An analysis of this construction is the object of future investigations.

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