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Chirality violating condensates in QCD and their connection with zero mode solutions of quark Dirac equations

B.L.Ioffe

Institute of Theoretical and Experimental Physics, Moscow, Russia

Light quark masses are small:

$$m_u + m_d \approx 10 \text{ MeV}, \quad m_s \approx 100 \text{ MeV}$$

The chiral symmetry is valid in perturbative QCD. In the real hadronic world the chiral symmetry is badly violated. This statement evidently follows from the existence of large proton mass.
Large value of quark condensates

$$\langle 0 \mid \bar{q}q \mid 0 \rangle \approx -(250 \text{ MeV})^3 \quad q = u, d, s$$

also indicate the violation of chiral symmetry in QCD. These two facts are deeply interconnected:

$$m_p^3 \approx -2(2\pi)^2 \langle 0 \mid \bar{q}q \mid 0 \rangle$$

In this talk I discuss the appearance of quark condensate in nonperturbative QCD. The main idea is the connection of quark condensate with zero solutions of quark Dirac equation. The QCD action (in Euclidean space-time)

$$S = \frac{1}{4}\int d^4x G^2_{\mu\nu} - \int d^4x \sum_f \left[\psi^+_f (i\gamma_\mu \nabla_\mu + im_f)\psi_f\right]$$

$$\nabla_\mu = \partial_\mu + ig\frac{\lambda^n}{2}A^n_\mu$$

The Dirac equation

$$-i\gamma_\mu \nabla_\mu \psi_n(x) = \lambda_n \psi_n(x)$$

$$\psi = \frac{1}{2}(1+\gamma_5)\psi_L + \frac{1}{2}(1-\gamma_5)\psi_R$$

$$\psi^+ = \psi^+_L \frac{1}{2}(1+\gamma_5) + \psi^+_R \frac{1}{2}(1-\gamma_5),$$

For $\lambda_n \neq 0$ the quark Lagrangian

$$L = -\int \left[\ \psi^+_L \boldsymbol{\nabla} \psi_R + \psi^+_R \boldsymbol{\nabla} \psi_L\ \right] d^4x$$

is symmetric under interchange $L \leftrightarrow R$. In case of $\lambda_0 = 0$

$$\Delta L = \int d^4x[\ \psi_L^+ + \psi_R^+\ ]\boldsymbol{\nabla}\psi_0$$

$$\Delta L = 0$$

No statement about the symmetry $L \leftrightarrow R$ can be done.
All chirality violating vacuum condensates in QCD arise from zero mode solutions of Dirac equations.
Two well known statements:

1. The Banks-Casher relation:

$$TrS(x^2) = \frac{1}{\pi}\int d\lambda\rho(\lambda)\Delta(x^2,\lambda)$$

$S(x)$ is the quark propagator. At $x^2 = 0$ $\Delta(x^2,\lambda) \to \delta(\lambda)$

$$\rho(0) = -\pi\langle 0 \mid \overline{\psi}(0)\psi(0) \mid 0\rangle \quad \text{Banks-Casher relation}$$

2. In the field of instanton:

$$\psi_{R,0}(x) = (1-\gamma_5)\psi(x) \neq 0 \quad \psi_{L,0} = 0$$

The model

$$\langle 0 \mid \overline{\psi} O_{c.v.} \psi \mid 0 \rangle \sim \psi_0^+ O_{c.v.} \psi_0$$

$$\psi_0 = \psi_0(x - x_c, \rho)$$

$$\langle 0 \mid \overline{\psi}(0) O_{c.v.} \psi(0) \mid 0 \rangle = -n \int d^4x \psi_0^+(x,\rho) O_{c.v.} \psi_0(x,\rho)$$

The unknown constant n has dimension 3. Determination of n and ρ in case of instanton

$$\psi_0(x,\rho) = \frac{1}{2}(1-\gamma_5)\frac{1}{\pi}\frac{\rho}{(x^2+\rho^2)^{3/2}}\chi_0,$$

$$\int d^4x \psi^+(x,\rho)\psi(x,\rho) = 1$$

Put $O_{c.v.} = 1$

$$n = -\langle 0 \mid \bar{q}q \mid 0 \rangle = (1.65 \pm 0.15) \times 10^{-2}\ \text{GeV}^3\ (\text{at 1 GeV})$$

$$-g\langle 0 \mid \overline{\psi}\sigma_{\mu\nu}\frac{\lambda^n}{2}G^n_{\mu\nu}\psi \mid 0 \rangle \equiv m_0^2 \langle 0 \mid \bar{q}q \mid 0 \rangle$$

$m_0^2 = 0.8\ \text{GeV}^2$.

$$G^a_{\mu\nu}(x,\rho) = \frac{4}{g}\eta_{a\mu\nu}\frac{\rho^2}{(x^2+\rho^2)^2},$$

$$\rho = \frac{1}{\sqrt{2}m_0} = 0.79\ \text{GeV}^{-1} = 0.156\text{fm}(\text{at}\ \ 1\ \text{GeV}).$$

Determination of quark condensate magnetic susceptibilities

$F_{\mu\nu}$ – a constant electromagnetic field.

Quark condensate magnetic susceptibility of dimension 3:

$$\langle 0 \mid \bar{q}\sigma_{\mu\nu}q \mid 0 \rangle_F = e_q \langle 0 \mid \bar{q}q \mid 0 \rangle \chi F_{\mu\nu}$$

Let us put

$$\psi(x,\rho) = \psi_0(x,\rho) + \psi_1(x,\rho), \quad \psi_1(x,\rho) \sim F_{\mu\nu}$$

$$\psi_1(x,\rho) = \frac{1}{16} e_q \eta_{a\mu\nu} \sigma_a F_{\mu\nu} x^2 \left(1 + \frac{1}{2}\frac{x^2}{\rho^2}\right) \psi_0(x,\rho),$$

$$\psi^+ \sigma_{\mu\nu} \psi = -\frac{1}{2} e_q F_{\mu\nu} \psi_0^+ x^2 \left(1 + \frac{1}{2}\frac{x^2}{\rho^2}\right) \psi_0,$$

$$\langle 0 \mid \overline{\psi} \sigma_{\mu\nu} \psi \mid 0 \rangle_F = e_q F_{\mu\nu} n \frac{1}{\pi^2} \int d^4x x^2 \left(1 + \frac{1}{2}\frac{x^2}{\rho^2}\right) \frac{\rho^2}{(x^2+\rho^2)^3},$$

$$\chi = -\rho^2 \int\limits_0^{R^2} dr^2 r^4 \left(1 + \frac{1}{2}\frac{r^2}{\rho^2}\right) \frac{1}{(r^2+\rho^2)^3}$$

The cut-off is determined by the assumption, that the volume occupied by one zero mode in 3-dimensional space $V_3 \sim 1/n$. The result

$$\chi = -3.52 \text{ GeV}^{-2}$$

The QCD sum rules value:

$$\chi = -(3.0 \pm 10\%) \text{ GeV}^{-2}$$ Ball, Braun, Kivel 2003

Rohrwild, 2007

Quark condensate magnetic susceptibilities of dimension 5:

$$g\langle 0 \mid \bar{q}\frac{1}{2}\lambda^n G^n_{\mu\nu}\bar{q} \mid 0\rangle_F = e_q \kappa F_{\mu\nu}\langle 0 \mid \bar{q}q \mid 0\rangle,$$

$$-ig\varepsilon_{\mu\nu\rho\tau}\langle 0 \mid \bar{q}\gamma_5\frac{1}{2}\lambda^n G^n_{\rho\tau} q \mid 0\rangle_F = e_q \xi F_{\mu\nu}\langle 0 \mid \bar{q}q \mid 0\rangle$$

Calculation of κ:

In the field of instanton the total spin-colour isospin J is equal to zero

$$\boldsymbol{\sigma\tau}\chi_0 = -3\chi_0, \quad \sigma^a\tau^b\chi_0 = -\delta^{ab}\chi_0$$

The result is

$$\kappa = -\int_0^z u^2 du \frac{1}{(u+1)^4}\left(1+\frac{1}{2}u\right) =$$

$$-\frac{1}{2}\left[\ln(z+1) - \frac{13}{6} + \frac{1}{z+1} + \frac{1}{2}\frac{1}{(z+1)^2} - \frac{1}{3}\frac{1}{(z+1)^3}\right],$$

$$\kappa = -0.26 \qquad \xi = 2\kappa = -0.52$$

QCD sum rules values:

$$\kappa = -0.34 \pm 0.1 \qquad \xi = -0.74 \pm 0.2$$ Kogan, Wyler, 1992

Turn now to the temperature dependence of quark condensate, considered as an order parameter. It is expected that quark condensate vanishes at high temperatures and the chiral symmetry is restored. Two possibilities are discussed: the second order phase transition and the crossover. In our approach the normalization condition takes place at any temperature. So, the temperature dependence of quark condensate reduces to

$$\alpha(T) \equiv \langle 0 \mid \bar{q}q \mid \rangle_T = n(T)$$

One may expect that $n(T)$ vanishes only in the case, when the quantum number, curried by instanton is vanishing. This quantum number is the topological charge, which is temperature independent. So, in our approach $n(T)$ never vanishes and the phase transition is of the type of crossover. The order parameter temperature dependence of the second order phase transition near the critical point T_c is smeared by fluctuations. (The same statement refers, surely, to the crossover.) The fluctuation are determined by long wave oscillations of the fields. In QCD they are given by small frequency gluonic field $G^n_{\mu\nu}$, which is a constant. We have:

$$\frac{\Delta\alpha(T_c)}{\alpha(T_c)} \sim \langle 0 \mid G^2_{\mu\nu} \mid 0 \rangle T_c^2 \rho^2,$$

where $\Delta\alpha(T_c)$ is of the order of magnitude of the variation of $\alpha(T)$ near the crossover critical point. It is evident, that the left hand side should vanish, if T_c would be zero. I guess, that the factor, corresponding to this circumstance is T_c^2. The factor ρ^2 is added for dimensional reasons.

All the discussion of the $\alpha(T)$ temperature dependence is based on implicit assumption, that this object has a physical meaning at finite T. It is unclear, however, how it can be defined. I did not succeed to formulate the gedanken-experiments, in which this object could be measured at finite T. Therefore, may be, such an object has no physical sence at $T \neq 0$.

Summary

It is demonstrated, that chirality violating condensates in massless QCD arise from zero mode solutions of Dirac equations in arbitrary gluon fields. Basing of this idea, the model is suggested, which allows one to calculate quark condensate magnetic susceptibilities in the external constant electromagnetic field. The temperature dependence of the quark condensate is discussed. It is shown that the phase transition, corresponding to the T-dependence of the quark condensate $\alpha(T)$ as an order parameter, is of the type of crossover. The variation of the quark condensate $\alpha(T)$ in the crossover domain is proportional to gluon condensate. The question is put in, if $\alpha(T)$ has any physical meaning at finite T.