

Extraction of saon formfactors from $K \rightarrow \mu \nu \gamma$ decay at ISTRA+ setup

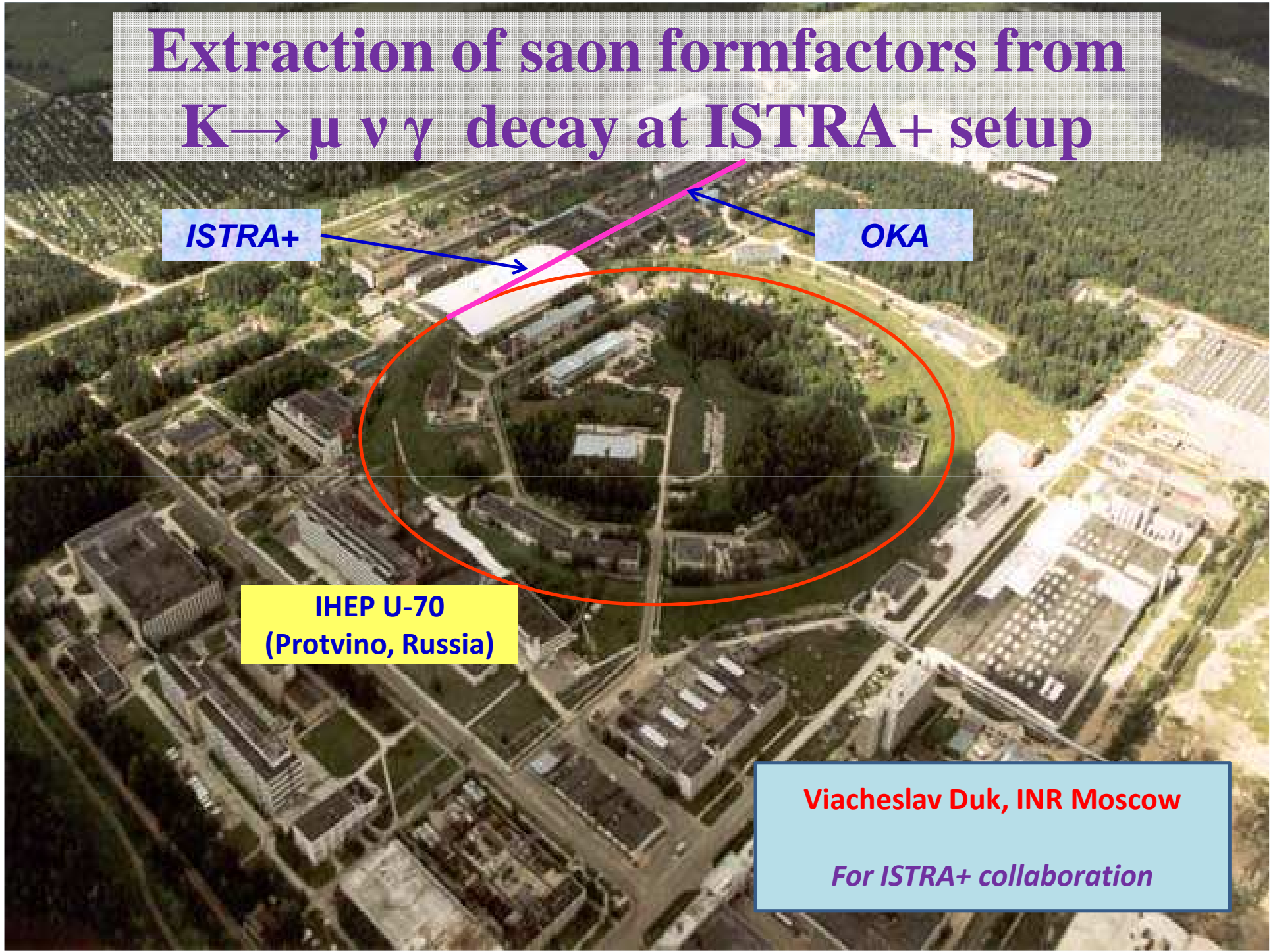
ISTRA+

OKA

IHEP U-70
(Protvino, Russia)

Viacheslav Duk, INR Moscow

For ISTRA+ collaboration



Outline

- $K \rightarrow \mu \nu \gamma$ decay
- ISTRA+ experiment
- Event selection
- Background rejection
- Signal extraction
- Spectrum fitting
- Results
- Conclusions

$K \rightarrow \mu \nu \gamma$ decay: motivation

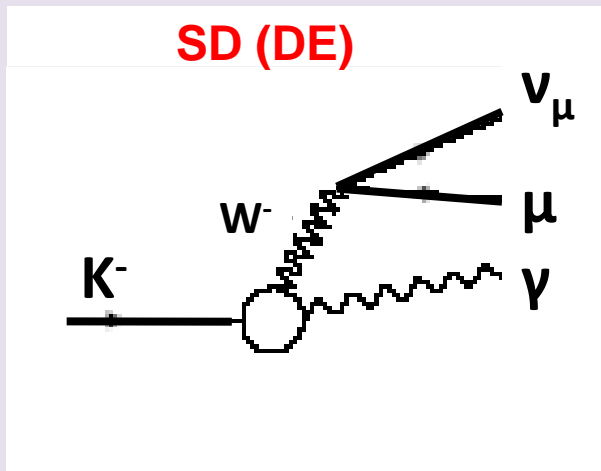
photon emitted from the vertex (SD or DE)



Probing electroweak structure of kaon



Testing predictions of ChPT, LFQM, ...



$$M = M_{IB} + M_{SD}$$

$$\Gamma = \Gamma_{IB} + \Gamma_{SD} + \Gamma_{INT}$$

$K \rightarrow \mu \nu \gamma$ theory: differential decay rate

$$\frac{d\Gamma_{K\mu\nu\gamma}}{dx dy} = A_{IB} f_{IB}(x, y) + A_{SD} [(F_V + F_A)^2 f_{SD+}(x, y) + (F_V - F_A)^2 f_{SD-}(x, y)] - A_{INT} [(F_V + F_A) f_{INT+}(x, y) + (F_V - F_A) f_{INT-}(x, y)],$$

3 main terms:
 IB – dominant
 SD \pm , INT \pm - most interesting ($\rightarrow F_V, F_A$)

Kinematical variables:

$$x = 2 * E_\nu(\text{cm}) / M_K$$

$$y = 2 * E_\mu(\text{cm}) / M_K$$

$$f_{IB}(x, y) = \left[\frac{1 - y + r}{x^2(x + y - 1 - r)} \right] \times \left[x^2 + 2(1 - x)(1 - r) - \frac{2xr(1 - r)}{x + y - 1 - r} \right],$$

$$f_{SD+} = [x + y - 1 - r][(x + y - 1)(1 - x) - r],$$

$$f_{SD-} = [1 - y + r][(1 - x)(1 - y) + r],$$

$$f_{INT+} = \left[\frac{1 - y + r}{x(x + y - 1 - r)} \right] [(1 - x)(1 - x - y) + r],$$

$$f_{INT-} = \left[\frac{1 - y + r}{x(x + y - 1 - r)} \right] [x^2 - (1 - x)(1 - x - y) - r],$$

$$r = \left[\frac{M_\mu}{M_K} \right]^2,$$

$$A_{IB} = \Gamma_{K\mu^2} \frac{\alpha}{2\pi} \frac{1}{(1 - r)^2},$$

$$A_{SD} = \Gamma_{K\mu^2} \frac{\alpha}{8\pi} \frac{1}{r(1 - r)^2} \left[\frac{M_K}{F_K} \right]^2,$$

$$A_{INT} = \Gamma_{K\mu^2} \frac{\alpha}{2\pi} \frac{1}{(1 - r)^2} \frac{M_K}{F_K}.$$

$K \rightarrow \mu \nu \gamma$ theory: differential decay rate

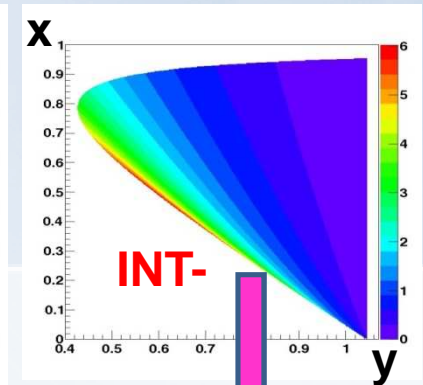
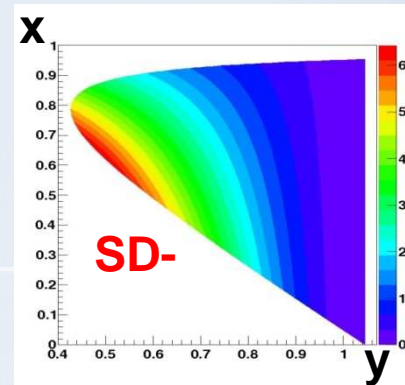
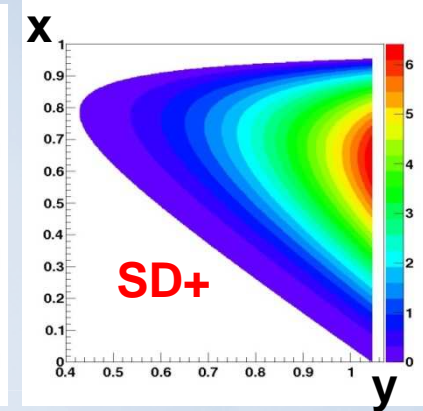
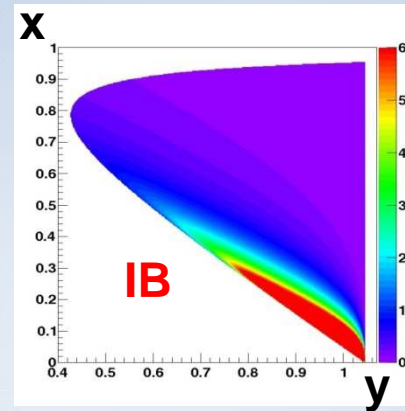
$$\frac{d\Gamma_{K\mu\nu\gamma}}{dx dy} = A_{IB} f_{IB}(x, y) + A_{SD} [(F_V + F_A)^2 f_{SD+}(x, y) + (F_V - F_A)^2 f_{SD-}(x, y)] - A_{INT} [(F_V + F_A) f_{INT+}(x, y) + (F_V - F_A) f_{INT-}(x, y)],$$

3 main terms:
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Kinematical variables:

$$x = 2 \cdot E_\nu(\text{cm}) / M_K$$

$$y = 2 \cdot E_\mu(\text{cm}) / M_K$$



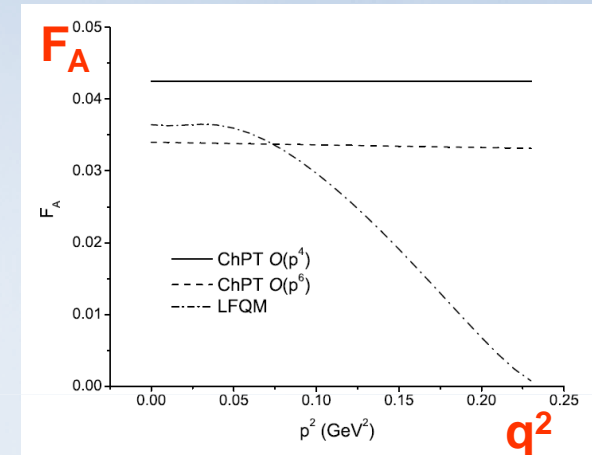
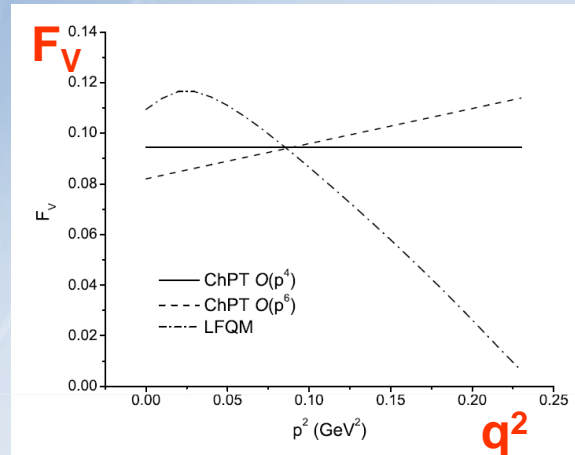
**Our strategy:
 Focusing on INT- region**

K → μνγ theory: formfactors

ChPT O(p⁴):
F_V, F_A constant

ChPT O(p⁶):
F_V, F_A linear
functions of q²

LFQM:
F_V, F_A depend on q²
in a complicated way



$$q^2 = (P_{K^-} - P_{\gamma})^2 = M_K^2 * (1-x)$$

ChPT O(p⁴)
F_V = 0.0945
F_A = 0.0425
F_V+F_A = 0.137
F_V-F_A = 0.052

ChPT O(p⁶)
F_V=F_V(0) * [1+λ*(1-x)]
F_V(0)= 0.082
λ = 0.4
F_A ≈ 0.034

$K \rightarrow \mu \nu \gamma$: main experimental results

experiment	collaboration	year	cuts	results
Barmin et al	ITEP	1988	$P_\mu < 231.5 \text{ MeV}/c$	BR(IB)
Demidov et al	ITEP	1990	$P_\mu < 231.5 \text{ MeV}/c$	BR(IB)
Akiba et al	E104 (KEK)	1985	$214.5 < P_\mu < 231.5 \text{ MeV}/c$	BR(IB)
Adler et al	E787 (BNL)	2000	$P_\mu > 218.4 \text{ MeV}/c$ $E_\gamma > 90 \text{ MeV}$	$ F_V + F_A $, BR(SD+)

Formfactors:

E787(BNL) Phys.Rev.Lett.85(2000)2256 (K $\rightarrow\mu\nu\gamma$)
 $|F_V + F_A| = 0.165 \pm 0.013$; $-0.04 < F_V - F_A < 0.24$

E865(BNL) Phys.Rev.Lett.89(2002)061803 (K $\rightarrow e\nu e^+e^-$, K $\rightarrow\mu\nu e^+e^-$)
 $F_V + F_A = 0.147 \pm 0.026$; $F_V - F_A = 0.077 \pm 0.028$

ISTRA+ collaboration



- Institute for High Energy Physics, Protvino (**IHEP**)
- Institute for Nuclear Research, Moscow (**INR**)
- Joint Institute for Nuclear Research, Dubna (**JINR**)



ISTRA+: from $\pi \rightarrow e \nu \gamma$ to $K \rightarrow \mu \nu \gamma$

1990

Study of $\pi \rightarrow e \nu \gamma$ decay: measuring $F_V, \gamma = F_A/F_V, F_T$

2003-
2007

$K \rightarrow e \nu \pi^0, K \rightarrow \mu \nu \pi^0$: high statistics, formfactor measurements.

[Phys.Lett.B589\(2004\)111](#), [Phys.Lett.B581\(2004\)31](#)

$K \rightarrow \pi^- \pi^0 \pi^0$: dalitz plot slope measurement.

[Phys.Lett.B567\(2003\)159](#)

$K \rightarrow \pi^- \pi^0 P$: search for light pseudoscalar sgoldstino.

[Phys.Lett.B602\(2004\)149](#)

$K \rightarrow e \nu \pi^0$: BR and V_{us} measurement. [arXiv:0704.2052 \[hep-ex\]](#)

$K \rightarrow e \nu \pi^0 \gamma$: T-odd correlation. [Phys.Atom.Nucl.70:734-740,2007](#)

$K \rightarrow \mu \nu \pi^0 \gamma$: first observation of the decay, T-odd correlation.

[Phys.Atom.Nucl.70:29-34,2007](#)

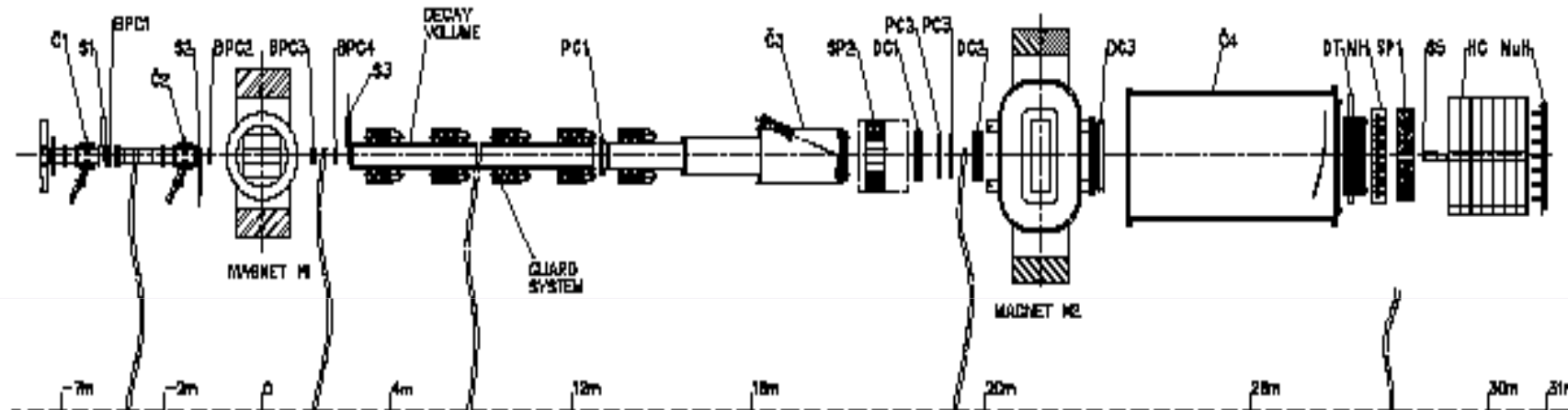
2010

Study of $K \rightarrow \mu \nu \gamma$ decay: measuring $F_V - F_A$

> 2010

ISTRA+  **OKA, see next talk by A.Khudyakov**

ISTRA+ setup



$p \sim -25 \text{ GeV}$; $\Delta p/p \sim 1.5\%$; $K^- \sim 3\%$; $I \sim 3 \cdot 10^6 / 1.9 \text{ sec}$

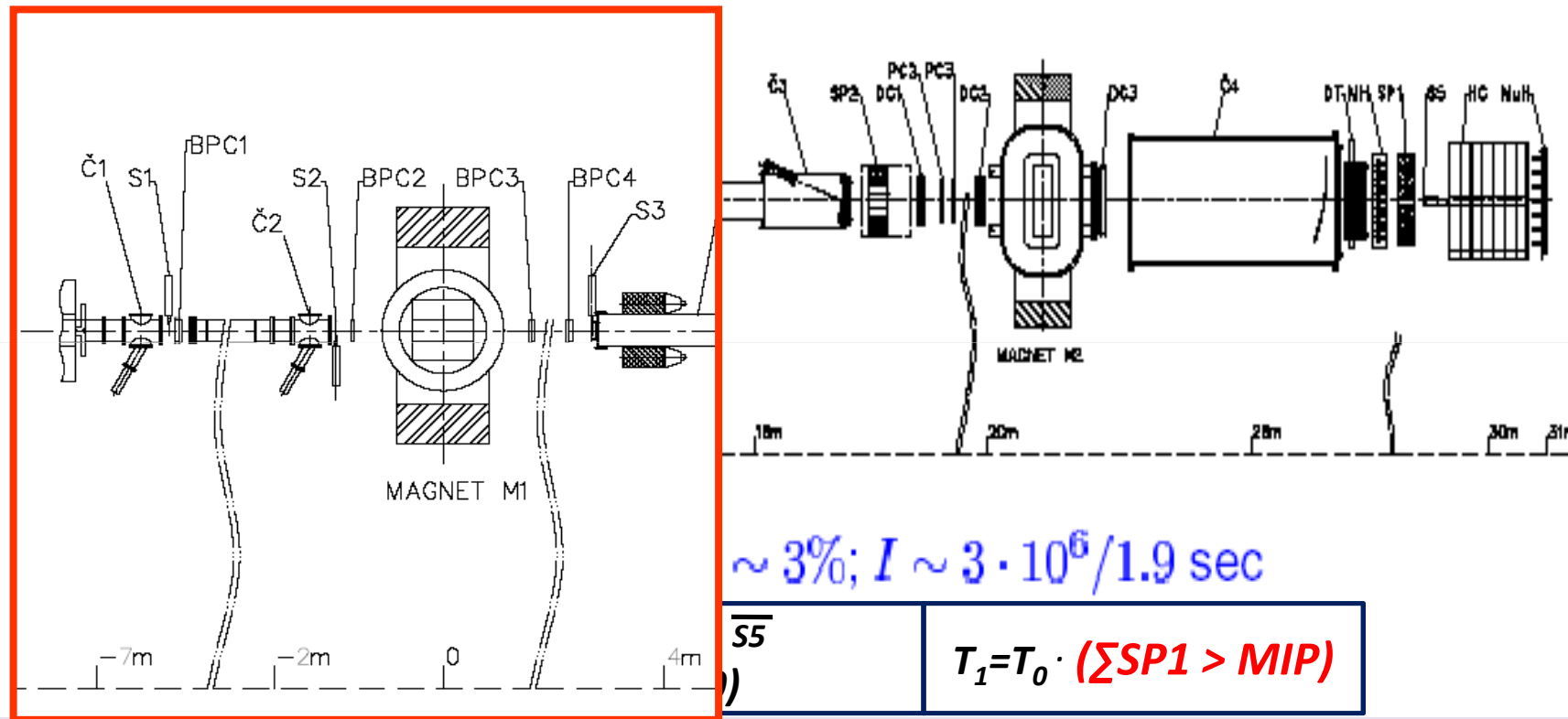
$$T_0 = S1 \cdot S2 \cdot S3 \cdot S4 \cdot C0 \cdot C1 \cdot C2 \cdot \overline{S5}$$

(prescaled by a factor of ~ 10)

$$T_1 = T_0 \cdot (\sum SP1 > MIP)$$

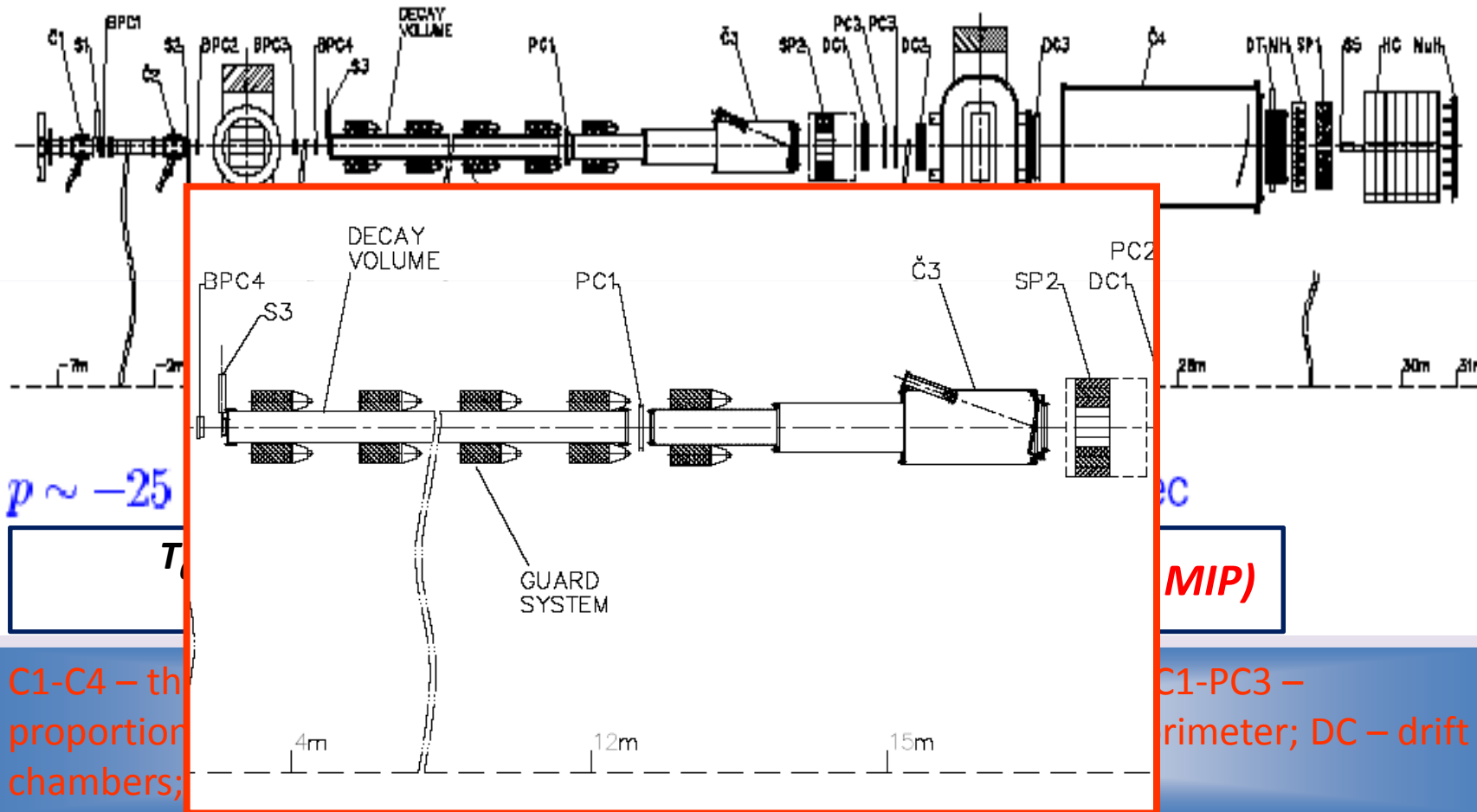
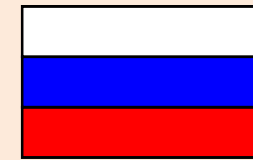
C1-C4 – thresh. cherenkov counters; S1-S5 – scintillation counters; PC1-PC3 – proportional chambers; SP2 – veto calorimeter; SP1 – lead-glass calorimeter; DC – drift chambers; DT-drift tubes; MH – matrix scintillation godoscope

ISTRA+ setup: beam part

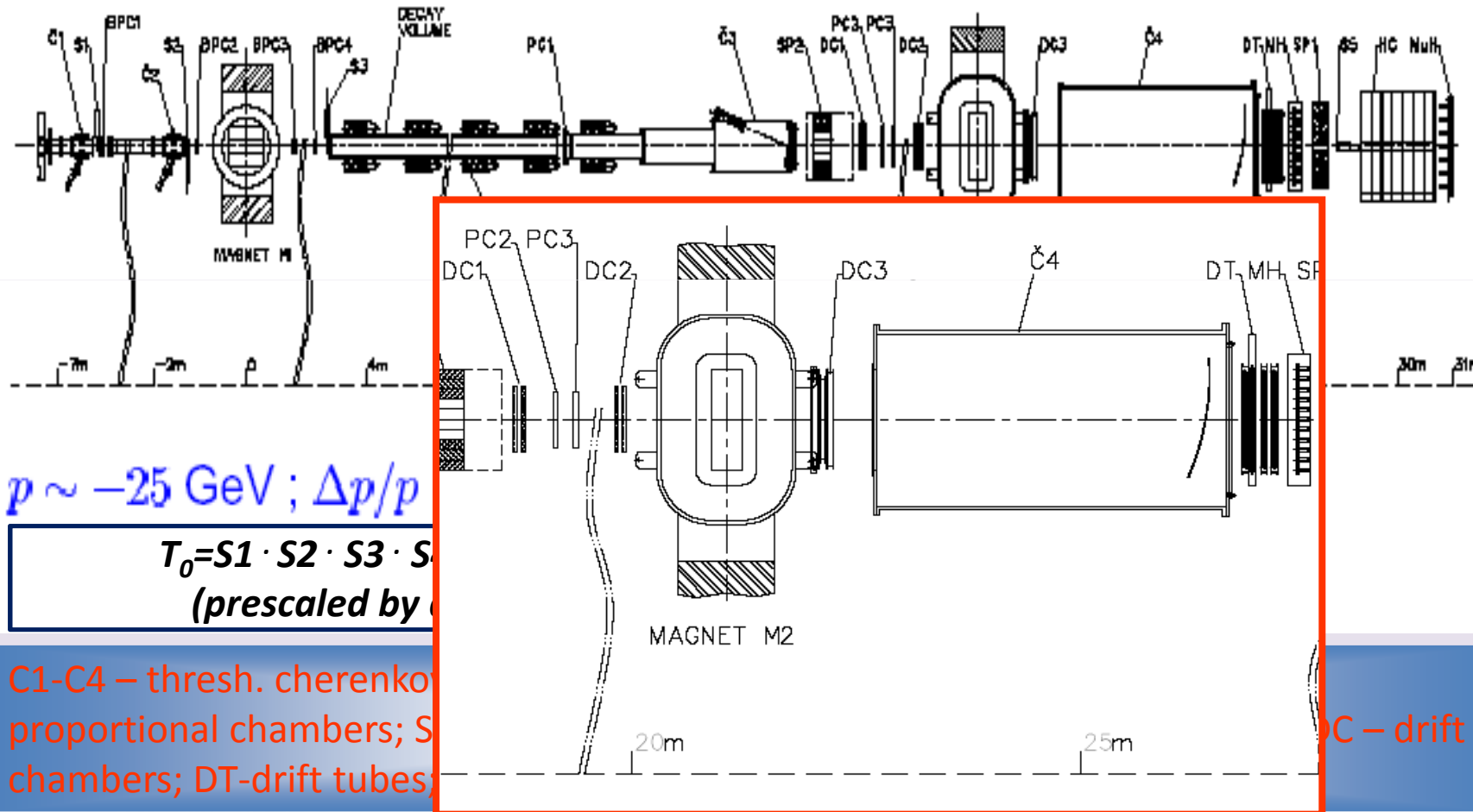
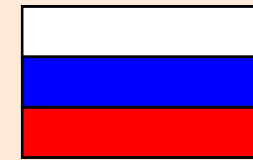


Č1-Č4 – thresh. cherenkov counters; S1-S5 – scintillation counters; PC1-PC3 – proportional chambers; SP2 – veto calorimeter; SP1 – lead-glass calorimeter; DC – drift chambers; DT-drift tubes; MH – matrix scintillation godoscope

ISTRA+ setup: decay volume



ISTRA+ setup: magnetic spectrometer



$p \sim -25 \text{ GeV} ; \Delta p/p$

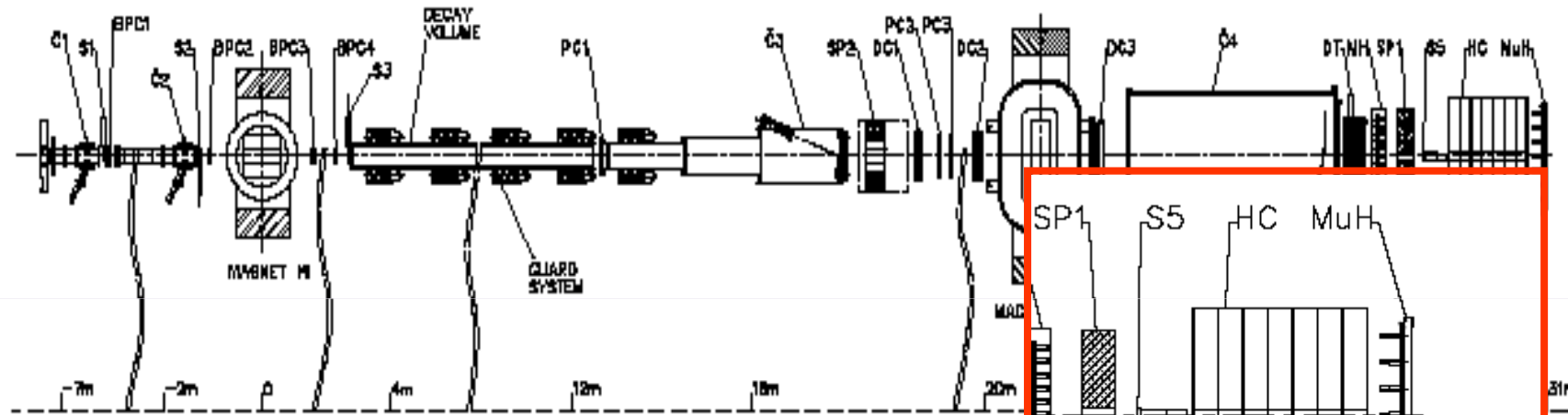
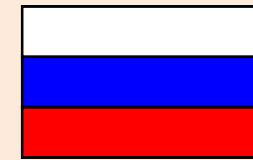
$$T_0 = S1 \cdot S2 \cdot S3 \cdot S4$$

(prescaled by)

C1-C4 – thresh. cherenko
proportional chambers; S
chambers; DT-drift tubes

C – drift

ISTRA+ setup: ECAL, HCAL



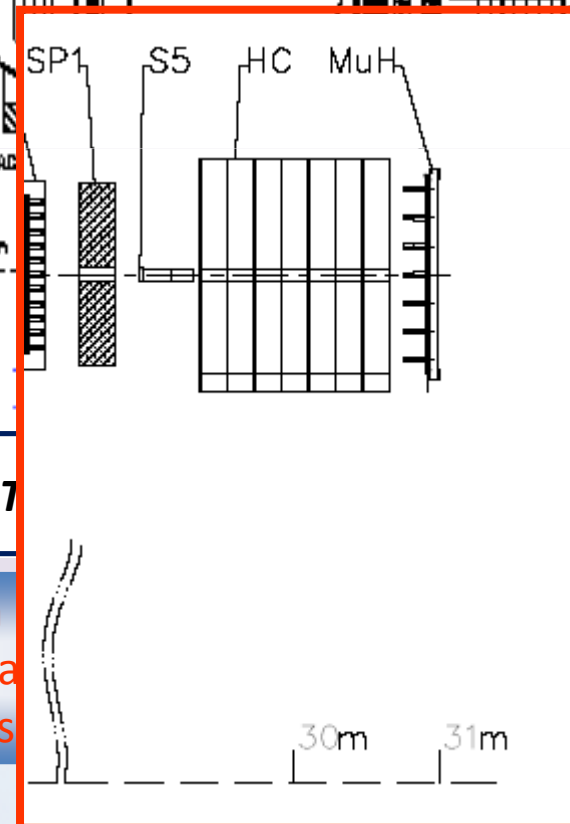
$p \sim -25 \text{ GeV}$; $\Delta p/p \sim 1.5\%$; $K^- \sim 3\%$; $I \sim 3 \cdot 10^7$

$$T_0 = S1 \cdot S2 \cdot S3 \cdot S4 \cdot C0 \cdot C1 \cdot C2 \cdot \overline{S5}$$

(prescaled by a factor of ~ 10)

$$T_1 = T_0$$

C1-C4 – thresh. cherenkov counters; S1-S5 – scintillation proportional chambers; SP2 – veto calorimeter; SP1 – lead chambers; DT-drift tubes; MH – matrix scintillation godos



Event selection

- **Track** requirements (one primary track, one secondary track, cuts on track quality)
- **Veto** requirements (no signals above threshold)
- **Vertex** requirements ($400 < z < 1600$ cm, cut on vertex fit probability)

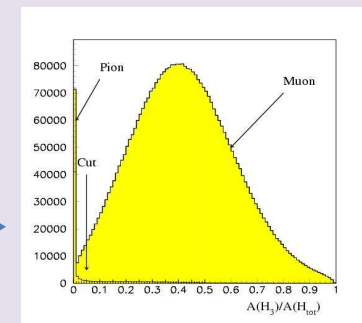
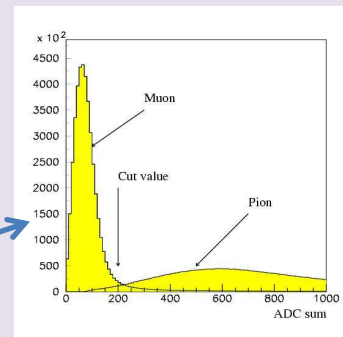
- **Particle ID :**

Photon: isolated shower in ECAL

Muon: 1) ADC sum in HCAL < 200

2) relative energy deposition in

last 3 layers of HCAL > 0.05



Trigger efficiency ε for photon

Data structure:

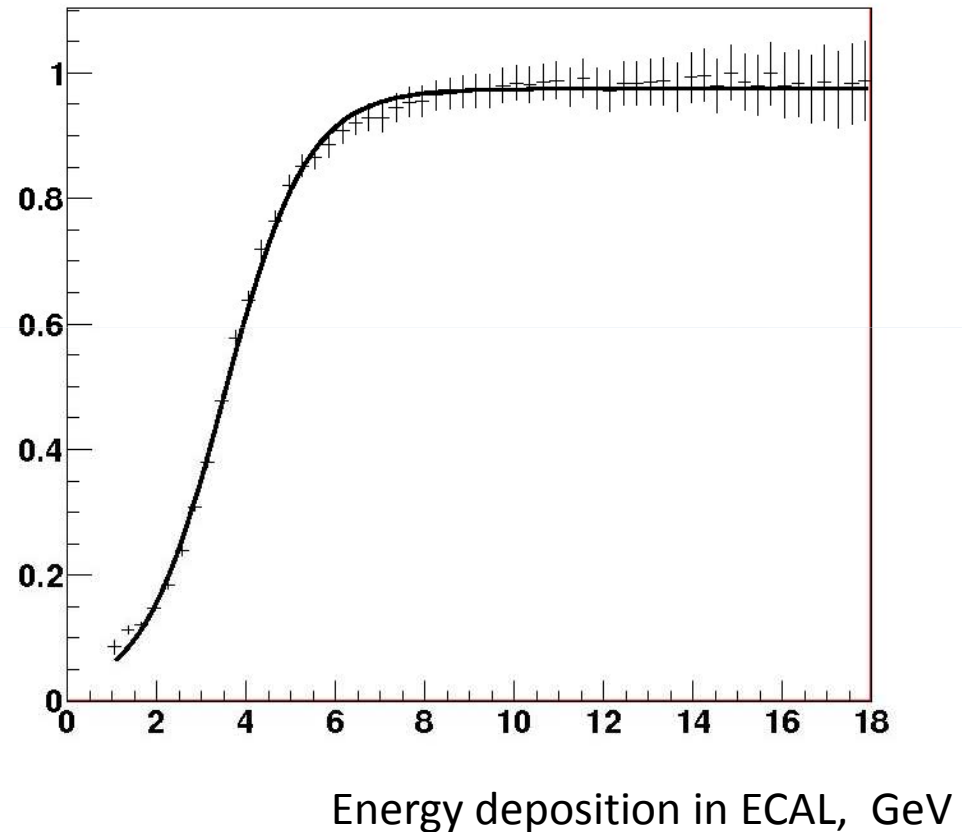
T_0 trigger: ~10%

$T_1 = T_0 \cdot (\sum ECAL > MIP) : \sim 90\%$

Trigger efficiency:

$$\varepsilon = T_0 \cdot T_1 / T_0$$

**Data with T_1 are
weighted by $1/\varepsilon$**



Background rejection and signal observation

- Main background:



with 1 gamma lost (from $\pi^0 \rightarrow \gamma\gamma$)



with 1 gamma lost (from $\pi^0 \rightarrow \gamma\gamma$) and π misidentification

- Signal observation:

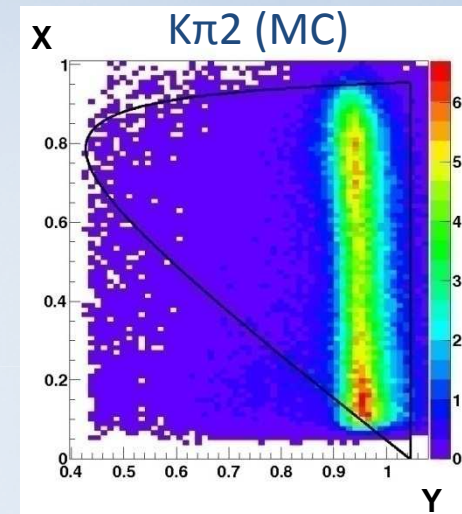
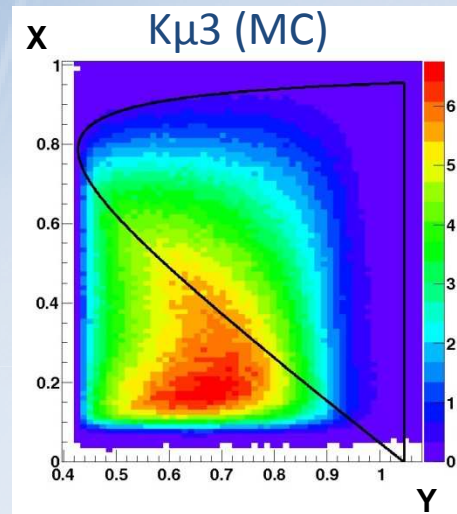
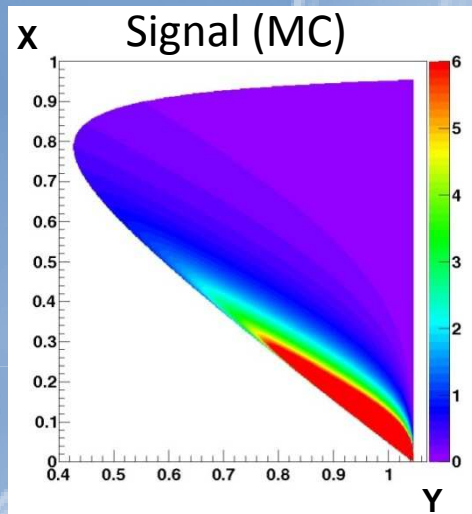
$$M(\mu \nu \gamma) = \sqrt{(P_\mu + P_\nu + P_\gamma)^2}$$
 where

$$\vec{p}_\nu = \vec{p}_K - \vec{p}_\mu - \vec{p}_\gamma ; E_\nu = |\vec{p}_\nu|$$

$M(\mu\nu\gamma)$ peaks at $M_K = 0.494$ GeV for signal

Background rejection procedure: scanning over (x,y) Dalits-plot and looking for a peak in $M(\mu\nu\gamma)$

Background rejection: (x,y) Dalits plot



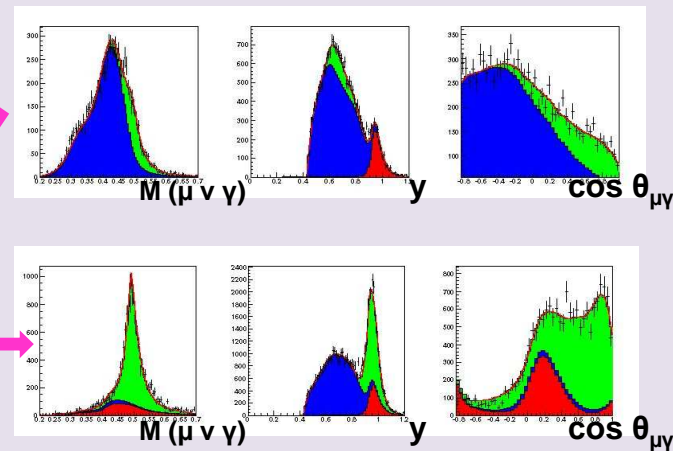
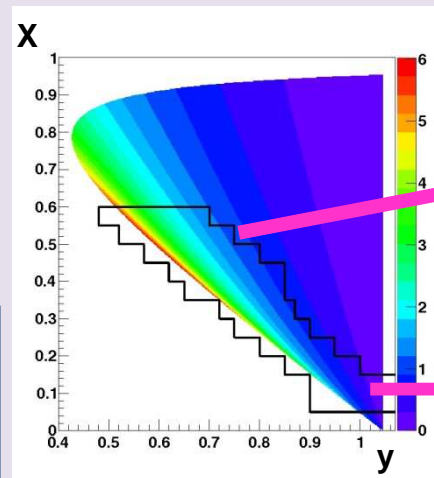
Previous experiments: looking for signal near y kinematical border

ISTRA+: looking for signal near E_ν kinematical border

Signal extraction

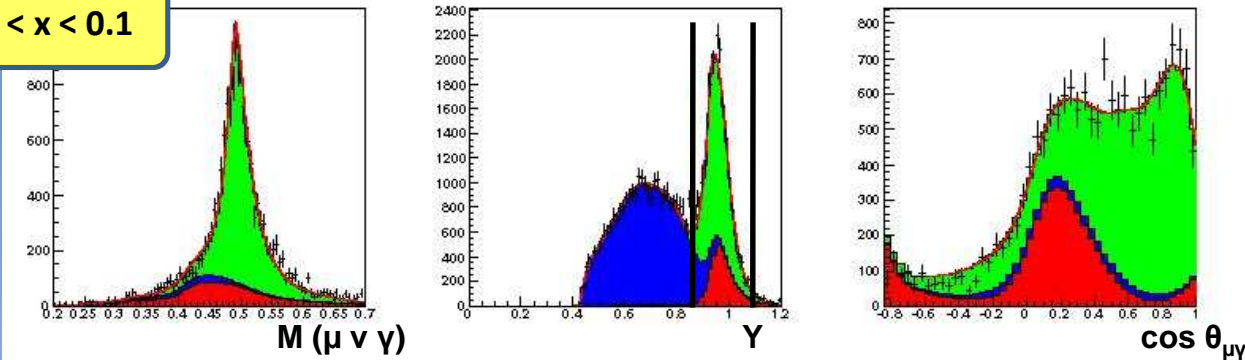
- (x,y) dalits-plot is divided into stripes with $\delta x=0.05$ width (x-stripes)
- Optimal cut on y is put in each x-stripe
- Simultaneous fit of $M(\mu \nu \gamma)$, y and $\cos \theta_{\mu\gamma}$ is done in x-stripes

**~22K events
extracted**



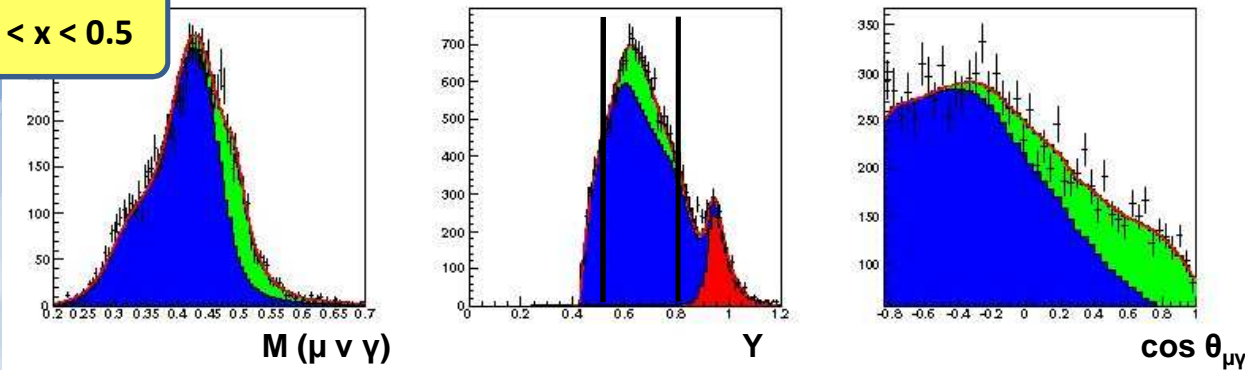
Signal extraction: simultaneous fit in x-stripes

$0.05 < x < 0.1$



fitting $M(\mu \nu \gamma)$
alone is not
sufficient
(signal and
background are
similar)

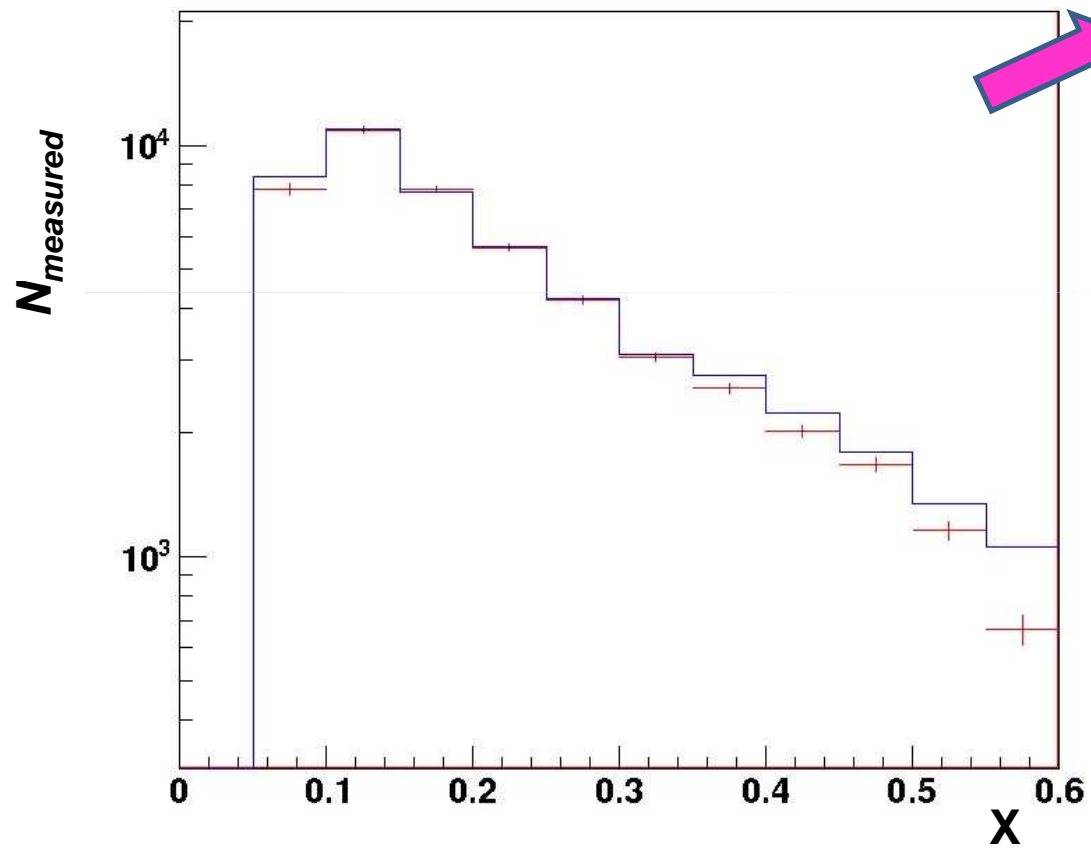
$0.45 < x < 0.5$



*Signal and
background
shapes taken
from MC*

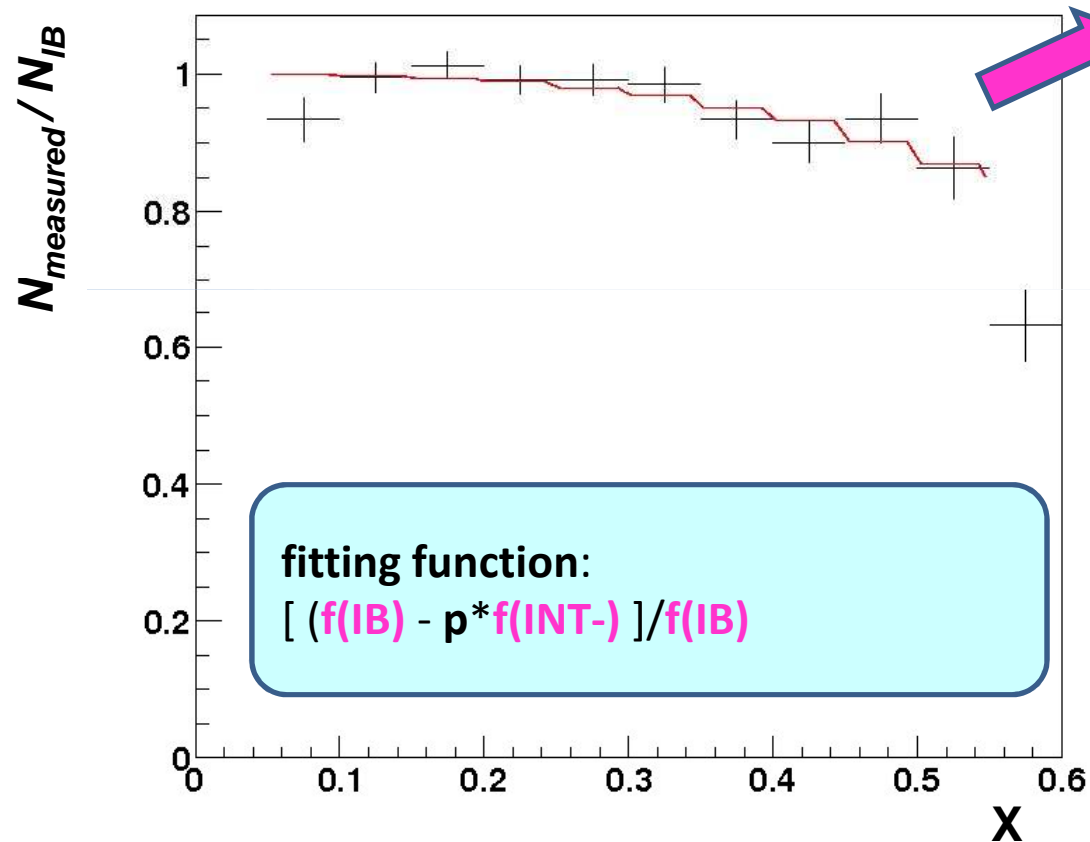
green – signal, blue – $K\mu 3$, red – $K\pi 2$

final x-spectrum



- N_{measured} taken from fits in x-stripes
- blue histogram – IB
- lack of events for large x due to INT- term
- Negative sign of INT- clearly seen

Spectrum fitting: x normalized on IB



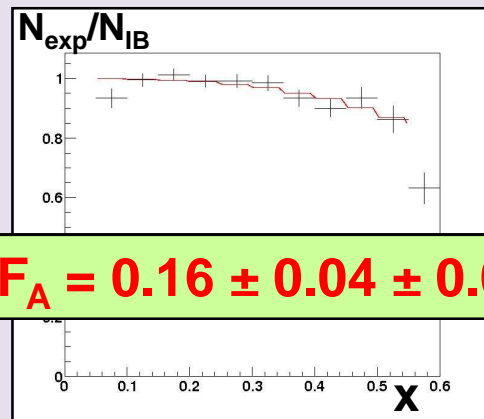
- N_{measured} taken from fits in x-strips
- N_{IB} taken from MC
- for IB only $N_{\text{measured}} / N_{\text{IB}} = 1$
- Negative sign of **INT-** clearly seen
- ~4% effect of **INT-**

Final result:

$$F_V - F_A = 0.16 \pm 0.04 \pm 0.05$$

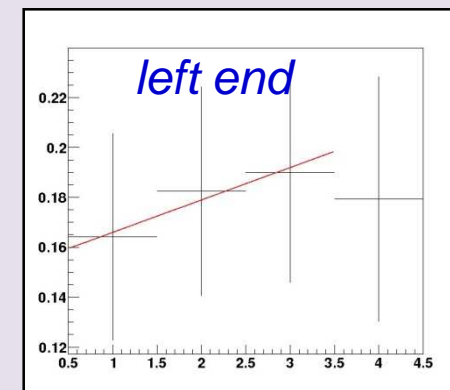
$F_V - F_A$: systematic error

Systematic error	value
cut on x	$3.1 \cdot 10^{-2}$
cut on y	-
cut on z	$2.3 \cdot 10^{-2}$
fit systematics	$3.2 \cdot 10^{-2}$
total	$5.0 \cdot 10^{-2}$

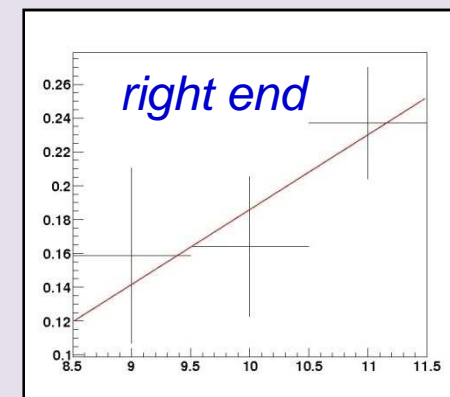


$$F_V - F_A = 0.16 \pm 0.04 \pm 0.05$$

x-systematics

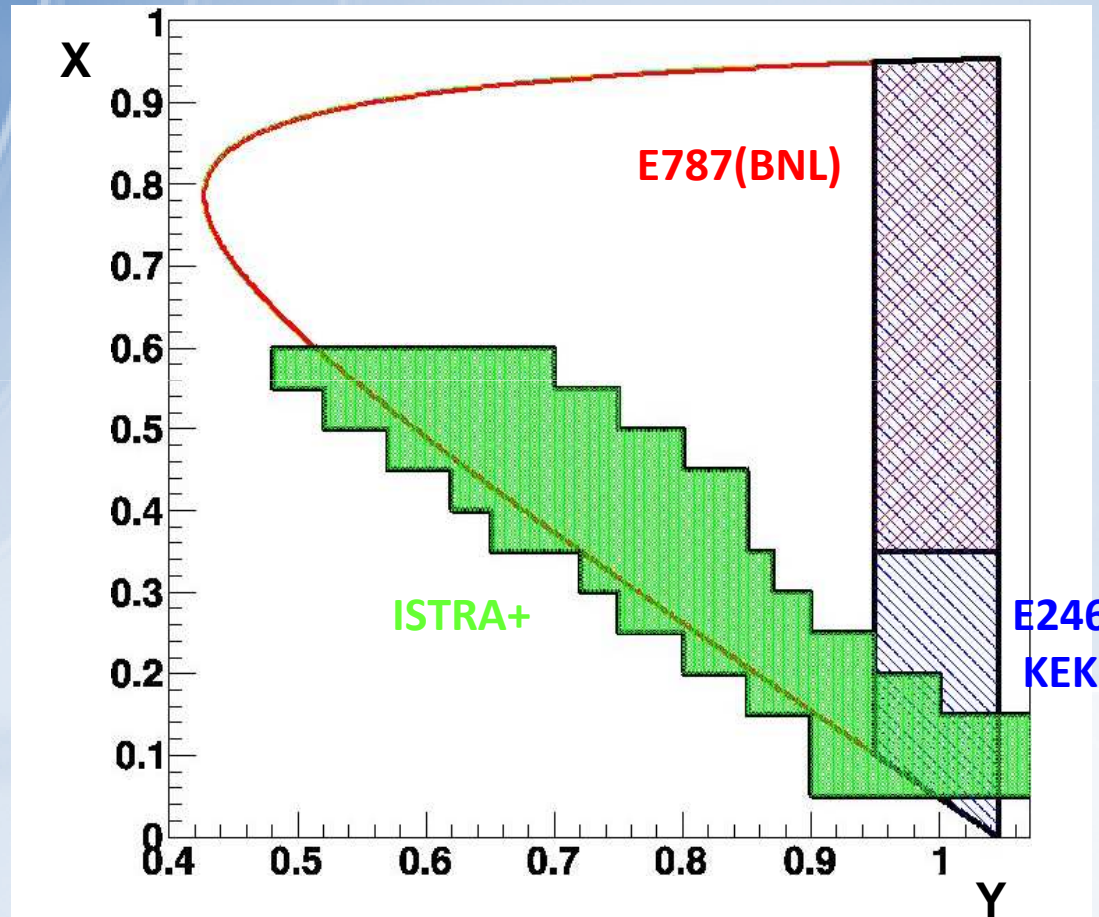


x-stripe (cut)



x-stripe (cut)

ISTRA+ kinematical region: complementary with previous experiments



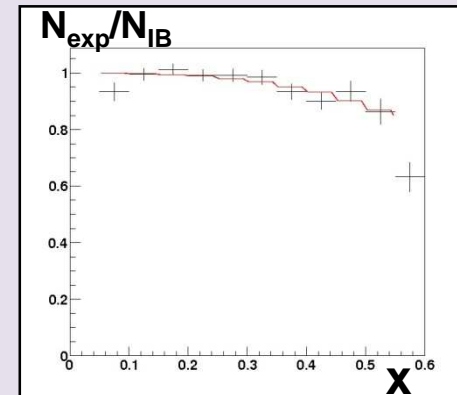
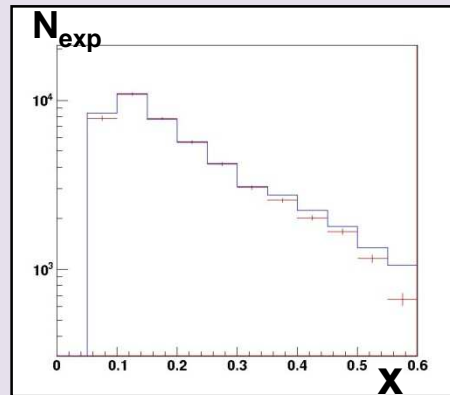
$F_V - F_A$: comparison with ChPT $O(p^4)$

$$F_V - F_A = 0.16 \pm 0.04(\text{stat}) \pm 0.05(\text{syst})$$

$$\chi^2/\text{ndf} = 7.7 / 8$$

ChPT $O(p^4)$

$$F_V - F_A = 0.052$$



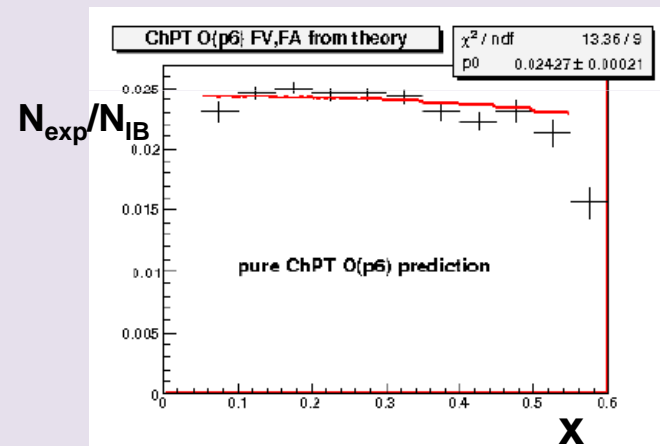
1.7 σ above ChPT $O(p^4)$

$F_V - F_A$: comparison with ChPT $O(p^6)$

F_V, F_A taken from theory

pure ChPT
 $O(p^6)$

$$\chi^2/\text{ndf} = 13.4 / 9$$



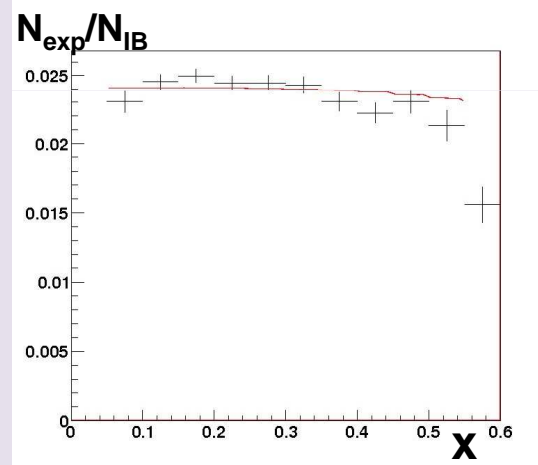
1σ from $\chi^2=1$

$F_V - F_A$: comparison with LFQM

F_V, F_A taken from theory

pure LFQM

$$\chi^2/ndf = 15.8 / 9$$



1.6 σ from $\chi^2=1$

$F_V - F_A$: comparison with ChPT $O(p^6)$

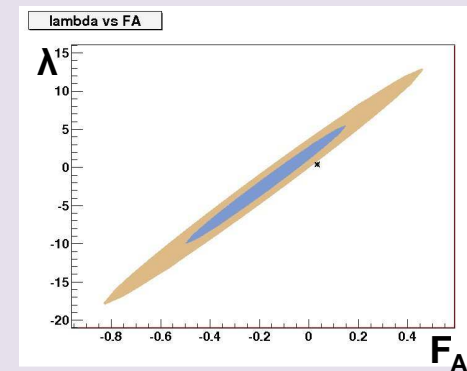
ChPT $O(p^6)$

$$F_V = F_V(0) * [1 + \lambda(1-x)]$$
$$F_V(0) = 0.082$$
$$F_A \approx 0.034$$
$$\lambda = 0.4$$

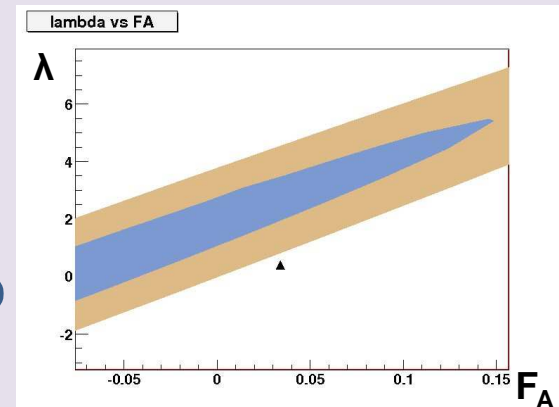
ISTRA+

$$F_V(0) \text{ from theory}$$
$$F_A = -0.17 \pm 0.32$$
$$\lambda = -2.1 \pm 7.7$$

F_A and λ : strong correlation



$$\chi^2/ndf = 7.6 / 8$$



ISTRA+ result: complementary with previous experiments

experiment	collaboration	year	cuts	results
Barmin et al	Itep	1988	$P_\mu < 231.5 \text{ MeV}/c$	BR(IB)
Demidov et al	Itep	1990	$P_\mu < 231.5 \text{ MeV}/c$	BR(IB)
Akiba et al	E104 (KEK)	1985	$214.5 < P_\mu < 231.5 \text{ MeV}/c$	BR(IB)
Adler et al	E787 (BNL)	2000	$P_\mu > 218.4 \text{ MeV}/c$ $E_\gamma > 90 \text{ MeV}$	$ F_V + F_A $
Akimenko et al	ISTRA+	2010	$E_\gamma < 136 \text{ MeV}$ ($x < 0.55$)	$F_V - F_A$

E787: 2800 events
ISTRA+: 22000 events

***~8 times larger
statistics***

conclusion

- The $K \rightarrow \mu \nu \gamma$ decay is observed at ISTRA+ setup in a new kinematical region
- The **event number** observed is **22K** (the largest statistics in the world)
- First measurement of INT- term gives **$F_V - F_A = 0.16 \pm 0.04(\text{stat}) \pm 0.05(\text{syst})$**
- The sign of INT- is negative

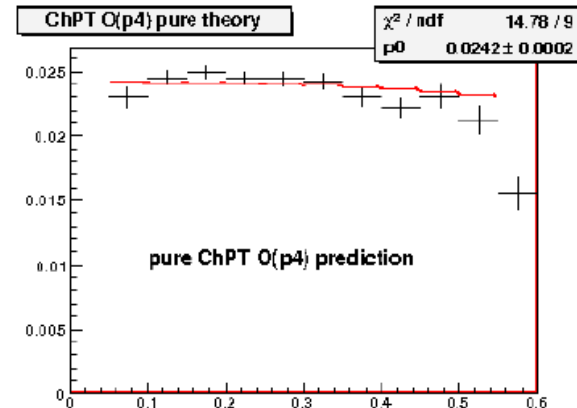
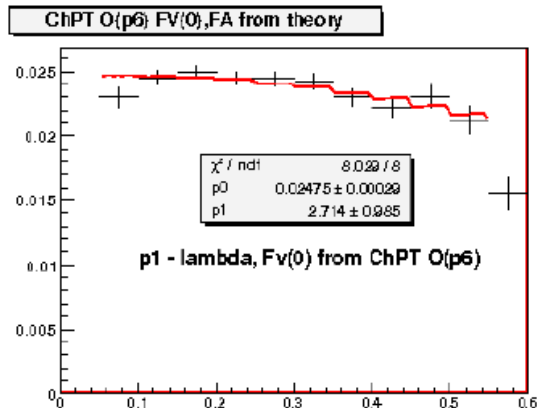
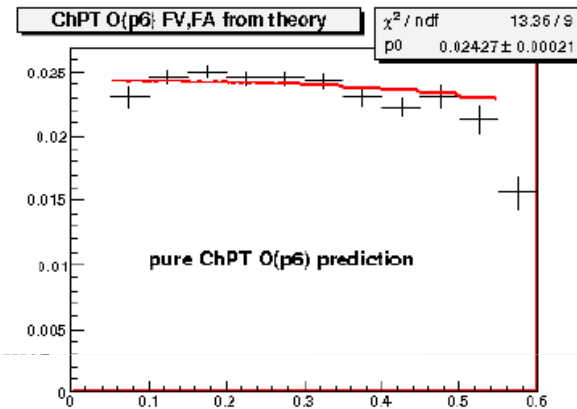
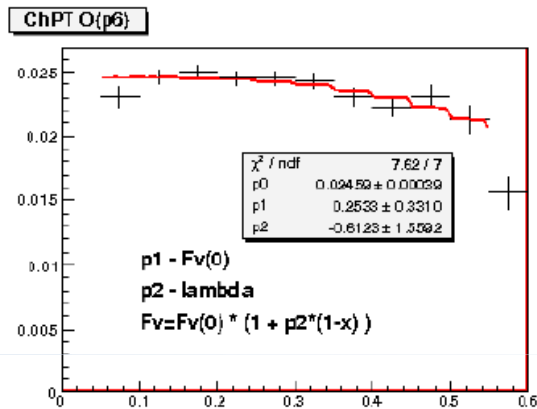
THANK YOU!!!



06-12.06.2010

V.Duk, Quarks-2010

ChPT $O(p^6)$: additional fits



Decay amplitude : general formulae

$$M = M_{IB} + M_{SD},$$

$$M_{IB} = ie \frac{G_F}{\sqrt{2}} \sin\theta_c F_K m_e \epsilon_\alpha^* K^\alpha,$$

$$M_{SD} = -ie \frac{G_F}{\sqrt{2}} \sin\theta_c \epsilon_\mu^* L_\nu H^{\mu\nu},$$

$$\langle 0 | \bar{s} \gamma^\mu \gamma_5 u | K^+(p_K) \rangle = -i F_K p_K^\mu,$$

$$\langle \gamma(q) | \bar{u} \gamma^\mu \gamma_5 s | K(p_K) \rangle = e \frac{F_A}{m_K} [(p \cdot q) \epsilon^{*\mu} - (\epsilon^* \cdot p) q^\mu],$$

$$\langle \gamma(q) | \bar{u} \gamma^\mu s | K(p_K) \rangle = ie \frac{F_V}{m_K} \epsilon^{\mu\alpha\beta\nu} \epsilon_{*\alpha} q_\beta p_\nu,$$

$$K^\alpha = \bar{u}(p_\nu) (1 + \gamma_5) \left(\frac{p_K^\alpha}{p_K \cdot q} - \frac{2p_e^\alpha + \not{q} \gamma^\alpha}{2p_e \cdot q} \right) v(p_e),$$

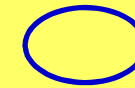
$$L_\nu = \bar{u}(p_\nu) \gamma_\nu (1 - \gamma_5) v(p_e),$$

$$H^{\mu\nu} = \frac{F_A}{m_K} (-g^{\mu\nu} p_K \cdot q + p_K^\mu q^\nu) + i \frac{F_V}{m_K} \epsilon^{\mu\nu\alpha\beta} q_\alpha p_{K\beta},$$

formfactors in ChPT $O(p^6)$: general formulae

$$\begin{aligned}
 \underline{F_V(p^2)} &= \frac{m_K}{4\sqrt{2}\pi^2 F_K} \left\{ 1 - \frac{256}{3}\pi^2 m_K^2 C_7^r + 256\pi^2(m_K^2 - m_\pi^2)C_{11}^r + \frac{64}{3}\pi^2 p^2 C_{22}^r \right. \\
 &\quad - \frac{1}{16\pi^2(\sqrt{2}F_K)^2} \left[\frac{3}{2}m_\eta^2 \ln\left(\frac{m_\eta^2}{\mu^2}\right) + \frac{7}{2}m_\pi^2 \ln\left(\frac{m_\pi^2}{\mu^2}\right) + 3m_K^2 \ln\left(\frac{m_K^2}{\mu^2}\right) \right. \\
 &\quad - 2 \int [xm_\pi^2 + (1-x)m_K^2 - x(1-x)p^2] \ln\left(\frac{xm_\pi^2 + (1-x)m_K^2 - x(1-x)p^2}{\mu^2}\right) dx \\
 &\quad - 2 \int [xm_\eta^2 + (1-x)m_K^2 - x(1-x)p^2] \ln\left(\frac{xm_\eta^2 + (1-x)m_K^2 - x(1-x)p^2}{\mu^2}\right) dx \\
 &\quad \left. \left. - 4 \int m_\pi^2 \ln\left(\frac{m_\pi^2}{\mu^2}\right) dx \right] \right\}, \quad (9)
 \end{aligned}$$

$$\begin{aligned}
 \underline{F_A(p^2)} &= \frac{4\sqrt{2}m_K}{F_K} (L_9^r + L_{10}^r) + \frac{m_K}{6F_K^3(2\pi)^8} [142.65(m_K^2 - p^2) - 198.3] \\
 &\quad - \frac{m_K}{4\sqrt{2}F_K^3\pi^2} \left\{ (4L_3^r + 7L_9^r + 7L_{10}^r)m_\pi^2 \ln\left(\frac{m_\pi^2}{m_\rho^2}\right) + 3(L_9^r + L_{10}^r)m_\eta^2 \ln\left(\frac{m_\eta^2}{m_\rho^2}\right) \right. \\
 &\quad \left. + 2(8L_1^r - 4L_2^r + 4L_3^r + 7L_9^r + 7L_{10}^r)m_K^2 \ln\left(\frac{m_K^2}{m_\rho^2}\right) \right\} \\
 &\quad - \frac{4\sqrt{2}m_K}{3F_K^3} \left\{ 2m_\pi^2(18y_{18}^r - 2y_{81}^r - 6y_{82}^r + 2y_{83}^r + 3y_{84}^r - y_{85}^r + 6y_{103}^r) \right. \\
 &\quad + 2m_K^2(18y_{17}^r + 36y_{18}^r - 4y_{81}^r - 12y_{82}^r + 4y_{83}^r + 6y_{84}^r + 4y_{85}^r - 3y_{100}^r \\
 &\quad \left. + 6y_{102}^r + 12y_{103}^r - 6y_{104}^r + 3y_{109}^r) + \frac{3}{2}(m_K^2 - p^2)(2y_{100}^r - 4y_{109}^r + y_{110}^r) \right\}, \quad (10)
 \end{aligned}$$



ChPT $O(p^4)$ value

One-loop diagrams in ChPT $O(p^6)$

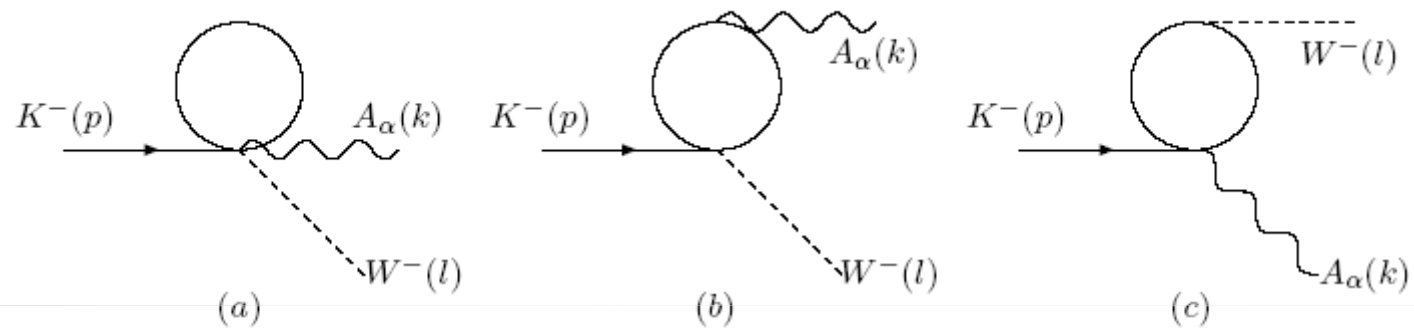


Figure 1: One-loop diagrams that contribute to F_V in $P_{\ell 2\gamma}$