CONDENSATION OF CHARGED BOSONS IN PLASMA PHYSICS AND COSMOLOGY

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Based on:

A.D. Dolgov, A. Lepidi, G. Piccinelli,
JCAP 0902 (2009) 027;
Phys. Rev D, 80 (2009) 125009;
arXiv: 1005.2702 [astro-ph].

Similar results but another method:

G. Gabadadze, R.A. Rosen,
Phys. Lett. B 658 (2008) 266;
JCAP 0810 (2008) 030;
JCAP 1004 (2010) 028.

Textbook formula for screening:

$$U(r) = \frac{Q}{4\pi r} \to \frac{Q\ \exp(-m_D r)}{4\pi r},$$

because the time-time component of the photon propagator acquires "mass":

$$k^2 \to k^2 + \Pi_{00}(k) = k^2 + m_D^2\,,$$

where e.g. for relativistic fermions

$$m_D^2 = e^2 \left(T^2/3 + \mu^2/\pi^2\right).$$

Strangely until recently effects on screening from condensate of a charged Bose field were not studied.
Consider electrically neutral plasma with large electric charge density of fermions compensated by charged bosons.
Bosons condense when their chemical potential reaches maximum value:

$$\mu_B = m_B \,.$$

Equilibrium distribution of condensed bosons:

$$f_B = C\delta^{(3)}(\mathrm{q}) + \frac{1}{\exp\left[(E - m_B)/T\right] \pm 1}$$

annihilates collision integral for an arbitrary constant C.

$$I_{coll} \sim \left[\Pi f_i(1 \pm f_f) - (i \leftrightarrow f)\right] d\tau \,.$$

$I_{coll} = 0$ for arbitrary T and C iff $\mu = m$. Equilibrium distributions are always determined by 2 parameters: T and μ if $\mu < m$ and by T and C if $\mu = m$.

In calculations neither imaginary time method which may be inconvenient in presence of condensate, nor Matsubara-Keldysh technique are used. We started from the quantum equations of motion, solved them up to e^2 order, and averaged the corresponding operators not only over vacuum but also over "non-empty" medium.

Operator Maxwell equations:

$$\partial_\nu F^{\mu\nu}(x) = \mathcal{J}^\mu_B(x) + \mathcal{J}^\mu_F(x)\,,$$

where bosonic current is

$$\mathcal{J}^\mu_B(x) = -i\,e[(\phi^\dagger(x)\partial^\mu\phi(x)) - (\partial^\mu\phi^\dagger(x))\phi(x)] + 2e^2 A^\mu(x)|\phi(x)|^2\,,$$

plus fermionic current:

$$\mathcal{J}^\mu_F(x) = e\bar{\psi}\gamma_\mu\psi\,.$$

Using equation of motion for quantum operator ϕ:

$$(\partial^2 + m^2)\phi(x) = \mathcal{J}_\phi(x)$$

express ϕ through A_μ:

$$\phi(x) = \phi_0(x) + \int d^4y\, G_B(x-y)\mathcal{J}_\phi(y)\,,$$

ϕ_0 is free field operator. In the lowest order in e take $\phi = \phi_0$ in $\mathcal{J}_B^\mu(x)$.

The r.h.s. of the Maxwell equations in e^2 order is linear (but non-local) in A_μ and bilinear in ϕ_0 and ψ_0.

Expand free fields as usually:

$$\phi_0(x) = \int d\tilde{q} \left[a(q) e^{-iqx} + b^\dagger(q) e^{iqx} \right] .$$

Average over medium:

$$\langle a^\dagger(\mathrm{q}) a(\mathrm{q}') \rangle = f_B(E_q) \delta^{(3)}(\mathrm{q} - \mathrm{q}'),$$
$$\langle a(\mathrm{q}) a^\dagger(\mathrm{q}') \rangle = [1 + f_B(E_p)] \delta^{(3)}(\mathrm{q} - \mathrm{q}') .$$

Solving Fourier transformed linear Maxwell equation for A_0:

$$\Pi_{00}(0,k) = \frac{e^2}{2\pi^2}\int_0^\infty \frac{dq\, q^2}{E_B}[f_B(E_B,\mu_B) + \bar{f}_B(E_B,\bar{\mu}_B)][1 + \frac{E_B^2}{kq}\ln|\frac{2q+k}{2q-k}|] \,.$$

Asymptotics of the screened potential of charged impurities is determined by the singularities of Π_{00} in complex k-plane.

Two types of singularities:

1. Poles of $[k^2 + \Pi_{00}(k)]^{-1}$. E.g. Debye pole. Necessary to check that the position of the poles are at small k, such that the infrared asymptotics of Π_{00} is valid.

2. Singularities of $\Pi_{00}(k)$, originating from the pinch of the integration contour in q-plane by poles of f and by branch points of log.

Without condensate one obtains the usual k-independent Debye screening:

$$\Pi_{00}(0, k) = m_D^2$$

originating from a pole at imaginary axis of k.

With condensate the corrections to Π_{00} at low k are infrared singular:

$$\frac{\Delta\Pi_{00}}{e^2} = \frac{m_B^2 T}{2k} + \frac{C}{(2\pi)^3 m_B}\left(1 + \frac{4m_B^2}{k^2}\right)$$

Both terms in the r.h.s. appear only if $\mu = m_B$.

Instead of exponential the screening becomes power law and oscillating, depending upon parameters, m_j:

$$\Pi_{00} = m_0^2 + m_1^3/k + m_2^4/k^2.$$

May this have something to do with confinement? Recent paper: P. Gaete, E. Spalucci, 0902.00905 – confinement in Higgs phase.

Contribution from poles in the limit of large $m_2 r$ but when power law terms are subdominant:

$$U(r)_{pole} = \frac{Q}{4\pi r} \exp\left(-\sqrt{e/2}\, m_2 r\right) \times \cos\left(\sqrt{e/2}\, m_2 r\right).$$

Oscillating screening is known for degenerate fermions, Friedel oscillations. Observed in experiment.

Comment.

Friedel oscillations are usually considered at $T = 0$. In this case the integral over q is in finite interval and the singularity in k appears when log branch point coinsides with the upper limit of the integration.

$T = 0$ limit can be obtained in the "pinch" method by summing all the singularities.

Contribution from the integral along imaginary axis, which is nonzero because Π_{00} contains an odd in k term. If $m_2 \neq 0$, the dominant term is

$$U(r) = -\frac{12 Q m_1^3}{\pi^2 e^2 r^6 m_2^8}.$$

If $T \neq 0$, $\mu = m_B$, but the condensate is not yet formed, the asymptotic decrease of the potential becomes:

$$U(r) = -\frac{Q}{\pi^2 e^2 r^4 m_1^3} = -\frac{2Q}{\pi^2 e^2 r^4 m_B^2 T}.$$

Contribution from logarithmic cuts (analogous to Friedel oscillations for fermions).

If the first "pinch" (between the poles of $f(q)$ and logarithmic branch point) dominates:

$$U_1(r) = -\frac{32\pi Q}{e^2 m_B r^2} \frac{e^{-z}}{\ln^2(2\sqrt{2}z)} \sin z \,,$$

where $z = 2r\sqrt{2\pi T m_B}$.
NB: $U_1(r)$ is inversely proportional to e^2 and formally vanishes at $T \to 0$, but remains finite if $\sqrt{T m_B} r \neq 0$.

All pinches are comparable:

$$U(r) \approx -\frac{3Q}{2e^2T^2m_B^3r^6\ln^3(\sqrt{8m_BTr})}.$$

$U \sim T^{-2}$ **valid if** $r \ll 1/\sqrt{16\pi T m_B}$, **i.e. if** $T = 0.1$**K and** $m_B = 1GeV$ **the distance should be bounded from above as** $r \ll 3 \cdot 10^{-8}$ **cm.**

Condensation of vector bosons.

$W^{\pm}$ would condense in the early universe if lepton asymmetry was sufficiently high. Plasma neutrality was maintained by quarks and leptons.

Depending on the sign of the pairwise spin-spin coupling W's would condense either in $S = 0$ (scalar) state or in $S = 2$ (ferromagnetic) state.

Magnetic spin-spin interaction through one photon exchange (similar to Breit equation):

$$U_{em}^{spin}(r) = \frac{e^2\rho^2}{4\pi m_W^2}\left[\frac{(S_1 \cdot S_2)}{r^3} - 3\,\frac{(S_1 \cdot r)(S_2 \cdot r)}{r^5} - \frac{8\pi}{3}(S_1 \cdot S_2)\delta^{(3)}(r)\right].$$

Here ρ is the ratio of magnetic moment of W to the standard one.

For S-wave the energy is shifted by the last term only.

Local quartic self-coupling of W:

$$U_{4W}^{(spin)} = \frac{e^2}{8m_W^2 \sin^2\theta_W}(S_1 S_2)\delta^{(3)}(r).$$

The net result $U_{em}+U_{4W}$ is negative, so $S=2$ state is energetically favorable and spontaneous magnetization in the early universe is possible.

Suppression due to screening.

The ij component of W propagator probably remains massless: $\Pi_{ij} \sim 1/q^2$. In Abelian QED it is true in perturbation theory, while in non-Abelian theories the screening may occur in higher orders of perturbation theory due to infrared singularities. The screening would diminish the long-range ferromagnetic spin-spin coupling while the local W^4 coupling is not screened.

If the propagator is modified, and the wave function of W-bosons is constant in space, the spin-spin energy shift is:

$$\delta E \sim \int \frac{d^3 q \delta(q)}{(2\pi)^3} \frac{q^2 (S_1 S_2) - (\mathrm{q}\mathrm{S}_1)(\mathrm{q}\mathrm{S}_2)}{q^2 + \Pi_{ss}(\mathrm{q})}$$

$\delta E = 0$, **if** $\Pi_{ss} \neq 0$ **at** $q = 0$.

However, the integration over space should be done with an upper limit, l, equal to the average distance between the W bosons so instead of $\delta^{(3)}(q)$, we obtain:

$$\int_0^l d^3 r e^{iqr} = \frac{4\pi}{q^3}\left[\sin\left(ql\right) - ql\cos\left(ql\right)\right].$$

and the energy shift is non-zero:

$$\delta E = -\frac{(S_1 S_2) e^2}{l^3 m_W^2} F(l)\,,$$

$$F(l) = \int_0^\infty \frac{dx\,\left[x\ \sin x + l^2 \Pi_{ss} \cos x\right]}{x^2 + l^2 \Pi_{ss}(x/l)}\,.$$

If $l^2\Pi_{ss}$ is nonnegligible the e.m. part of the spin-spin interaction would be suppressed and the ferromagnet turns into an antiferromagnet. This might happen at T above the EW phase transition when the Higgs condensate is destroyed and $m_{W,Z}$ appear as a result of temperature and density corrections and are relatively small.

The quantitative statement depends upon the (unknown) modification of the space-space part of the photon propagator in presence of the Bose condensate of charged W.

Problem of large scale magnetic fields: $B \sim \mu G$ **at several kpc. In the intergalactic space the fields are probably 2-3 orders of magnitude weaker, but still non-vanishing.**

Dynamo operates only in galaxies.

Maybe ferromagnetism of W **might create seeds for large scale magnetic fields.**