MSSM Finite-Temperature Higgs Potential and Phase Transition

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Outline

- Introduction
- **Finite-temperature corrections of squarks**
- **Thermal evolution and the critical temperature**
- Conclusions

[M. Dolgopolov, M. Dubinin, E. Rykova Threshold corrections to the MSSM finite-temperature Higgs potential. Jan 2009. 26pp. e-Print: arXiv:0901.0524v1]

Some brief T=0 history from QUARKS-2006

THDM: Fields

$$\Phi_1 = \begin{pmatrix} \phi_1^+(x) \\ \phi_1^0(x) \end{pmatrix} = \begin{pmatrix} -i\omega_1^+ \\ \frac{1}{\sqrt{2}}(v_1 + \eta_1 + i\chi_1) \end{pmatrix},$$

$$\Phi_2 = e^{i\xi} \begin{pmatrix} \phi_2^+(x) \\ \phi_2^0(x) \end{pmatrix} = e^{i\xi} \begin{pmatrix} -i\omega_2^+ \\ \frac{1}{\sqrt{2}}(v_2 e^{i\zeta} + \eta_2 + i\chi_2) \end{pmatrix}$$

$$\langle \Phi_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \qquad \langle \Phi_2 \rangle = \frac{e^{i\xi}}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 e^{i\zeta} \end{pmatrix} \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 e^{i\theta} \end{pmatrix}.$$

$$\mathrm{tg}\,\beta = \frac{v_2}{v_1}, \qquad v^2 \equiv v_1^2 + v_2^2 = (246\ \mathrm{GeV})^2.$$

[Akhmetzyanova E.N., *D M.V.*, Dubinin M.N. Higgs Bosons in the Two-Doublet Model with CP Violation Phys.Rev.D. V.71. N7. 2005. P.075008
Violation of CP invariance in the two-doublet Higgs sector of the MSSM.
E.N. Akhmetzyanova, *M.V. D* , M.N. Dubinin 2006. 58pp. Phys.Part.Nucl.37:677-734,2006.]

Effective THDM potential with explicit CP violation

General hermitian renormalized *SU(2)xU(1)* invariant potential:

$$U(\Phi_1,\Phi_2) = -\mu_1^2(\Phi_1^\dagger\Phi_1) - \mu_2^2(\Phi_2^\dagger\Phi_2) - \mu_{12}^2(\Phi_1^\dagger\Phi_2) - \mu_{12}^{*2}(\Phi_2^\dagger\Phi_1) +$$

$$+\frac{\lambda_1}{2}(\Phi_1^\dagger\Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) +$$

$$+\frac{\lambda_5}{2}(\Phi_1^\dagger\Phi_2)(\Phi_1^\dagger\Phi_2) + \frac{\lambda_5^*}{2}(\Phi_2^\dagger\Phi_1)(\Phi_2^\dagger\Phi_1) +$$

$$+\lambda_6(\Phi_1^\dagger\Phi_1)(\Phi_1^\dagger\Phi_2) + \lambda_6^*(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_1) +$$

$$+\lambda_7(\Phi_2^\dagger\Phi_2)(\Phi_1^\dagger\Phi_2) + \lambda_7^*(\Phi_2^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)$$

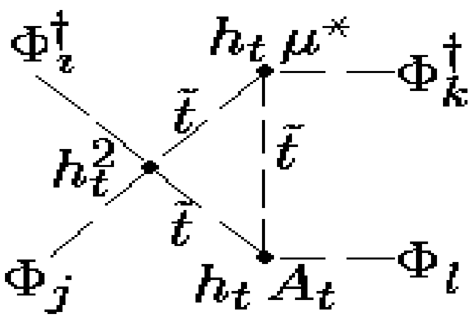
$$\Phi_1^\dagger\Phi_2 \xrightarrow{CP} \Phi_2^\dagger\Phi_1$$

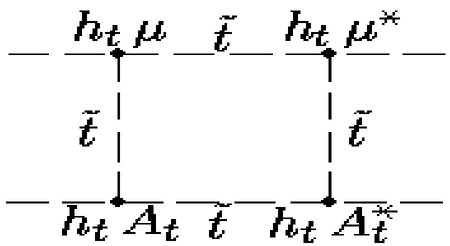
$$\lambda_{5,6,7} \xrightarrow{CP} \lambda_{5,6,7}^*$$

μ_{12}^2, λ_5, λ_6, λ_7 **complex**

$\varphi = \arg(\lambda_{6,7})$

$= \arg(\lambda_5)/2$





***U* is *CP*-invariant at the M_{SUSY} scale, because $\lambda_{5,6,7}=0$**

One-loop (t, b) contributions — ~~*CP*~~ U_{eff} — m_{top} — M_{SUSY} — *μ - mass-energy scale*

Eff. potential method or Feynman diags

Scalar sector for MSSM

The main contribution to self-couplings due to Yukawa 3rd generation couplings.

The corresponding potential with CPV sources

$$\mathcal{V}^0 = \mathcal{V}_M + \mathcal{V}_\Gamma + \mathcal{V}_\Lambda + \mathcal{V}_{\widetilde{Q}}\,,$$

$$\mathcal{V}_M = (-1)^{i+j} m_{ij}^2 \Phi_i^\dagger \Phi_j + M_{\widetilde{Q}}^2 \left(\widetilde{Q}^\dagger \widetilde{Q}\right) + M_{\widetilde{U}}^2 \widetilde{U}^* \widetilde{U} + M_{\widetilde{D}}^2 \widetilde{D}^* \widetilde{D}\,,$$

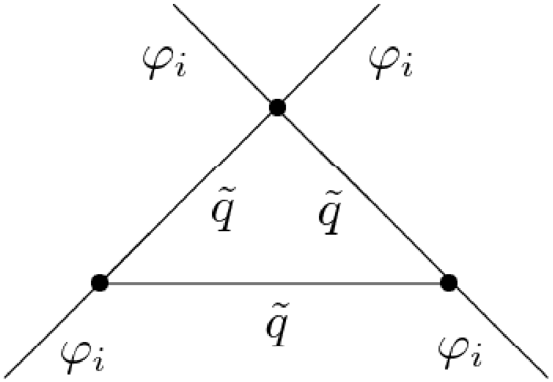
$$\mathcal{V}_\Gamma = \Gamma_i^D \left(\Phi_i^\dagger \widetilde{Q}\right) \widetilde{D} + \Gamma_i^U \left(i\Phi_i^T \sigma_2 \widetilde{Q}\right) \widetilde{U} + \overset{*}{\Gamma}{}_i^D \left(\widetilde{Q}^\dagger \Phi_i\right) \widetilde{D}^* - \overset{*}{\Gamma}{}_i^U \left(i\widetilde{Q}^\dagger \sigma_2 \Phi_i^*\right) \widetilde{U}^*$$

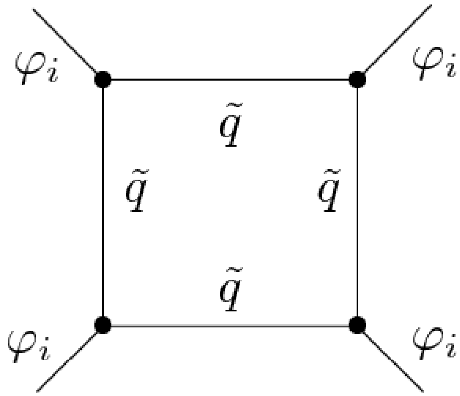
$$\mathcal{V}_\Lambda = \Lambda_{ik}^{jl} \left(\Phi_i^\dagger \Phi_j\right)\left(\Phi_k^\dagger \Phi_l\right) + \left(\Phi_i^\dagger \Phi_j\right)\left[\Lambda_{ij}^Q \left(\widetilde{Q}^\dagger \widetilde{Q}\right) + \Lambda_{ij}^U \widetilde{U}^* \widetilde{U} + \Lambda_{ij}^D \widetilde{D}^* \widetilde{D}\right] +$$

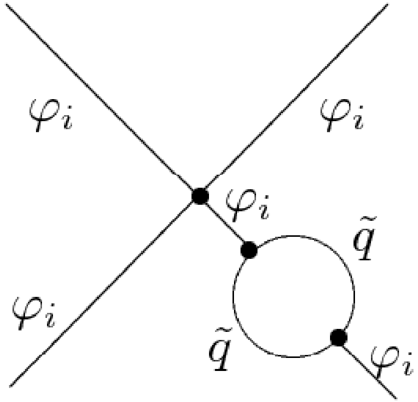
$$+\overline{\Lambda}_{ij}^Q \left(\Phi_i^\dagger \widetilde{Q}\right)\left(\widetilde{Q}^\dagger \Phi_j\right) + \frac{1}{2}\left[\Lambda \epsilon_{ij} \left(i\Phi_i^T \sigma_2 \Phi_j\right) \widetilde{D}^* \widetilde{U} + \text{э.с.}\right]\,, \quad i, j, k, l = 1, 2$$

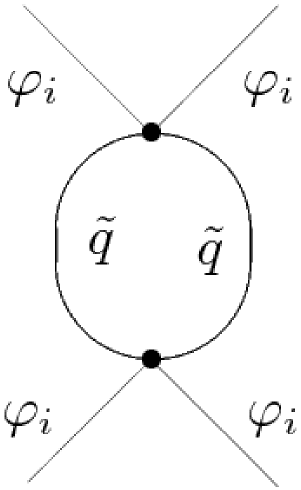
$$\Gamma_{\{1;\,2\}}^U = h_U \{-\mu^*; A_U\}, \qquad \Gamma_{\{1;\,2\}}^D = h_D \{A_D; -\mu^*\}$$

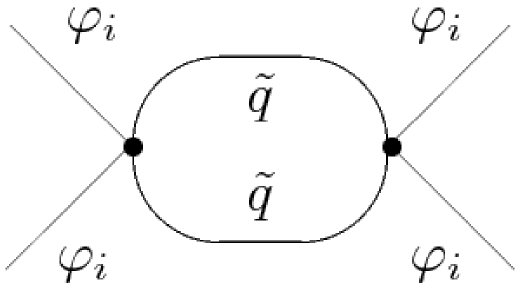
Threshold corrections (left and central diagram) and diagram contributing to the wave-function renormalization (right)

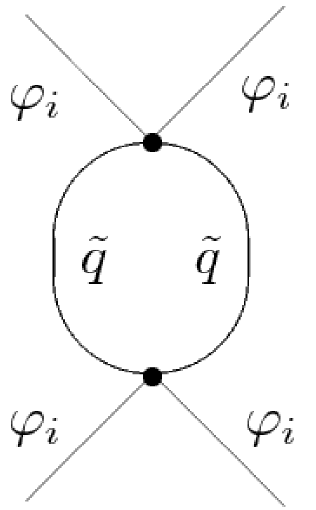










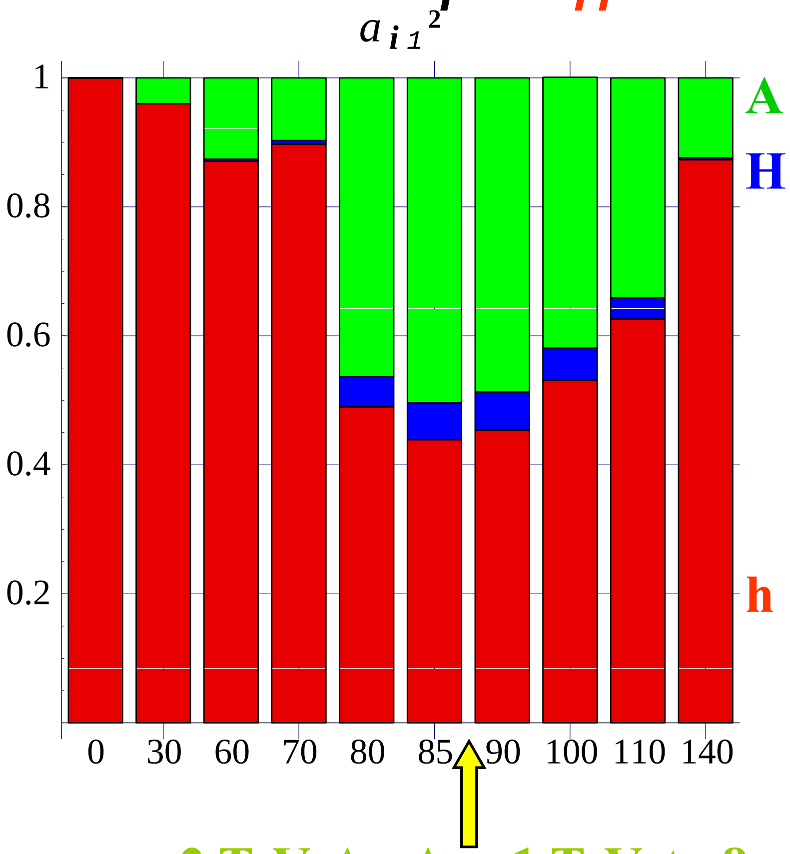


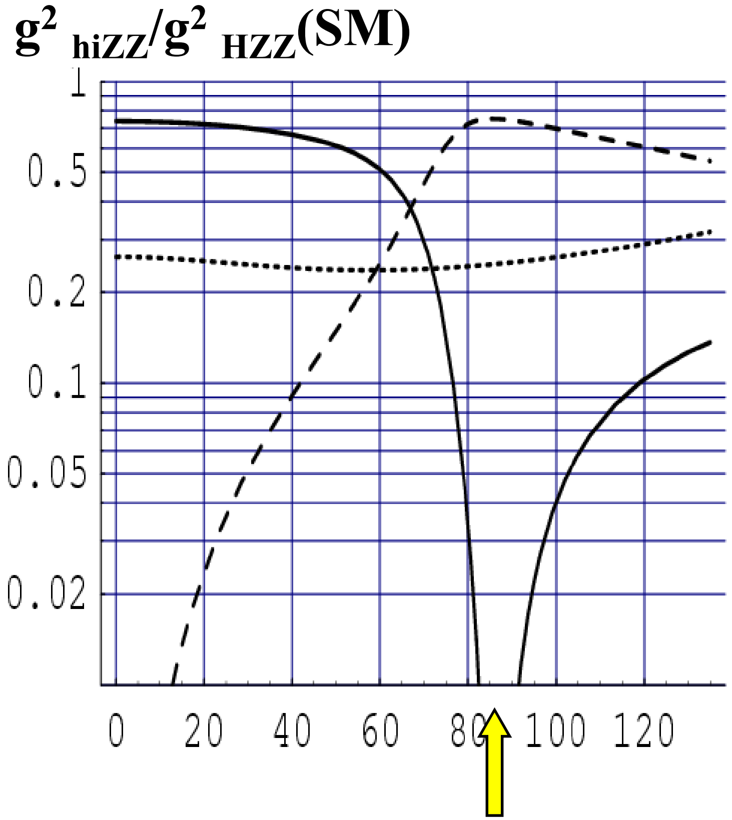
"Fish" diagrams

Matrix elements a_{i1} and coupling with Z-boson

New interesting possibility – non-observability

$$h_1 = a_{11}\, h + a_{21}\, H + a_{31}\, A$$





μ = 2 TeV, A_t=A_b= 1 TeV, tg β =5, M_{SUSY} =0.5 TeV, $m_{H^\pm}$=150 GeV

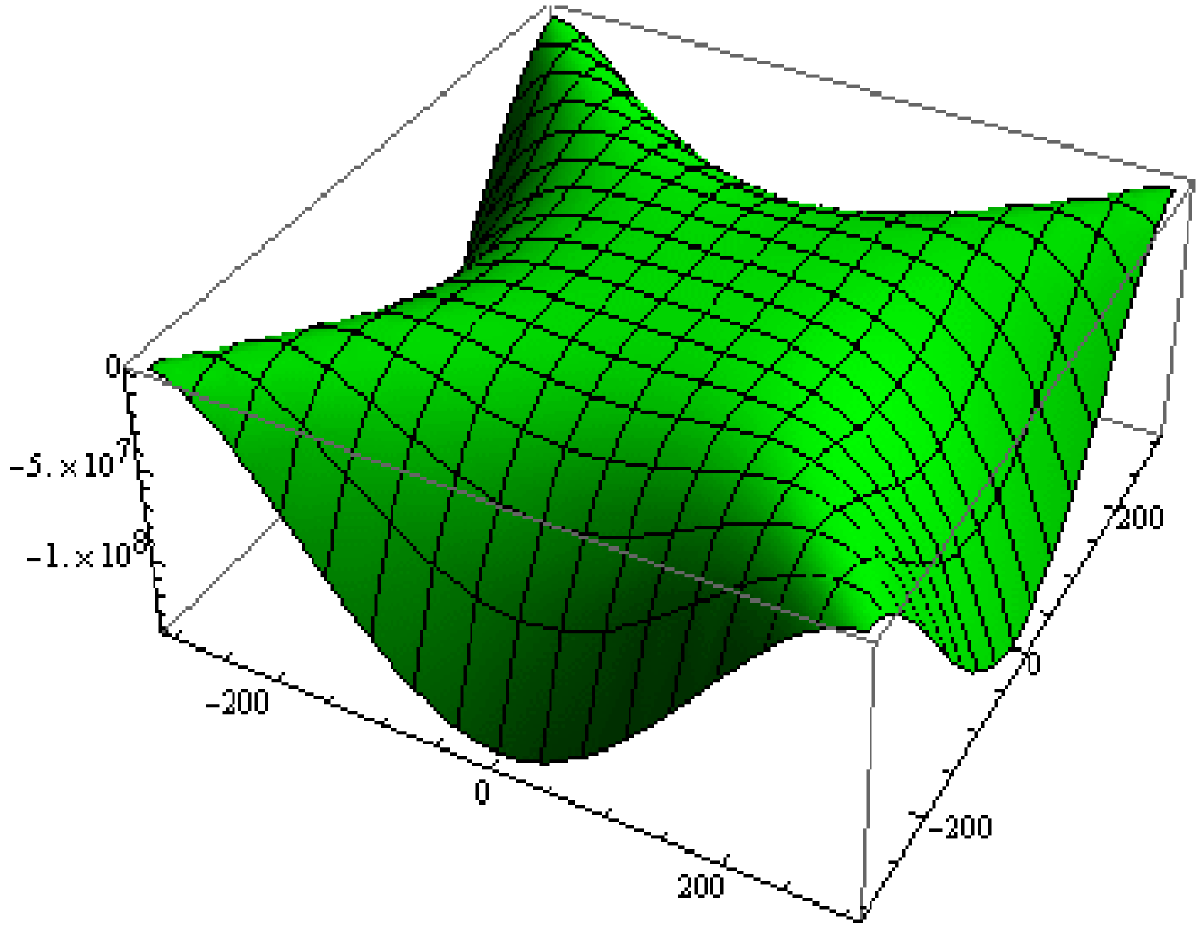
Electroweak Baryogenesis

- **Two problems in the Standard Model**
  - **First order phase transition requires m_h< 50 GeV**
  - **Need new sources of CP violation**
- **Minimal Supersymmetric Standard Model**
  - **1st order phase transition is possible if** $m_{\tilde{t}_R} < 160\text{GeV}$
  - **New CP violating phases**

Light stop window: M. Carena, M. Quiros, C.E.M. Wagner, Phys.Lett. B380 (1996) 81; M. Carena, G. Nardini, M. Quiros and C.E.M.Wagner JHEP 10 (2008) 062; M. Carena, G. Nardini, M. Quiros, C.E.M. Wagner. Nucl.Phys. B812: 243-263, 2009.

In the MSSM we calculate the 1-loop FT corrections from the squarks-Higgs bosons sector, reconstruct the effective two-Higgs-doublet potential and study possibilities of the EWPT in the full MSSM ($\mathbf{m_{H\pm}}$, **tgβ**, $\mathbf{A_{t,b}}$, **μ**, $\mathbf{m_Q}$, $\mathbf{m_U}$, $\mathbf{m_D}$) parameter space.

The *surface of minima* for zero-temperature two-doublet Higgs potential at the scale M_{SUSY}



Integration and summation method

In the finite temperature field theory Feynman diagrams with boson propagators, containing Matsubara frequencies, lead to structures of the form

$$I[m_1, m_2, ..., m_b] = T \sum_{n=-\infty}^{\infty} \int \frac{d\mathbf{k}}{(2\pi)^3} \prod_{j=1}^{b} \frac{(-1)^b}{(\mathbf{k}^2 + \omega_n^2 + m_j^2)},$$

$$\omega_n = 2\pi n T \; (n = 0, \pm 1, \pm 2, ...),$$

T - temperature

Integration and summation method

$$I[m_1, m_2, ..., m_b] \;=\; 2T\,(2\pi T)^{3-2b}\;\frac{(-1)^b \pi^{3/2}}{(2\pi)^3}\;\frac{\Gamma(b-3/2)}{\Gamma(b)}\;S(M, b-3/2),$$

$$S(M, b-3/2) \;=\; \int\{\mathrm{d}x\}\sum_{n=1}^{\infty}\frac{1}{(n^2+M^2)^{b-3/2}}, \qquad M^2 \equiv \left(\frac{m}{2\pi T}\right)^2.$$

Parameters of the effective potential
(forms of contribitions)

$$\Delta\lambda_1 = 3h_t^4|\mu|^4 I_2[m_Q,m_t] + 3h_b^4|A|^4 I_2[m_Q,m_b] +$$
$$+h_t^2|\mu|^2(\frac{g_1^2-3g_2^2}{2}I_1[m_Q,m_t] + 2g_1^2 I_1[m_t,m_Q]) +$$
$$+h_b^2|A|^2(\frac{12h_b^2-g_1^2-3g_2^2}{2}I_1[m_Q,m_b] + (6h_b^2-g_1^2)I_1[m_b,m_Q])$$

$$\Delta\lambda_2 = 3h_t^4|A|^4 I_2[m_Q,m_t] + 3h_b^4|\mu|^4 I_2[m_Q,m_b] +$$
$$+h_b^2|\mu|^2(\frac{g_1^2+3g_2^2}{2}I_1[m_Q,m_b] + g_1^2 I_1[m_b,m_Q]) +$$
$$+h_t^2|A|^2(\frac{12h_t^2+g_1^2-3g_2^2}{2}I_1[m_Q,m_t] + (6h_t^2-2g_1^2)I_1[m_t,m_Q])$$

$$\Delta\lambda_3 + \Delta\lambda_4 = 6h_t^4|\mu|^2|A|^2 I_2[m_Q,m_t] + 6h_b^4|\mu|^2|A|^2 I_2[m_Q,m_b] +$$
$$+h_t^2((|\mu|^2\frac{12h_t^2+g_1^2-3g_2^2}{4} - |A|^2\frac{g_1^2-3g_2^2}{4})I_1[m_Q,m_t] +$$
$$+(|A|^2 g_1^2 - |\mu|^2(g_1^2-3h_t^2))I_1[m_t,m_Q]) +$$
$$+h_b^2((|\mu|^2\frac{-12h_t^2+g_1^2+3g_2^2}{4} - |A|^2\frac{g_1^2+3g_2^2}{4})I_1[m_Q,m_b] +$$
$$+\frac{1}{2}(|A|^2 g_1^2 - |\mu|^2(g_1^2-6h_b^2))I_1[m_b,m_Q])$$

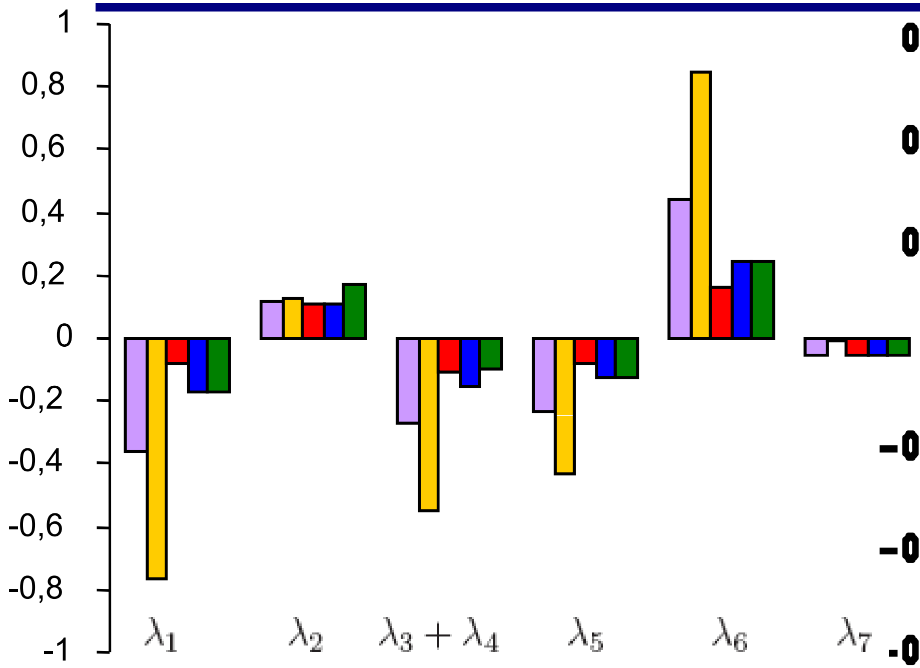
Parameters of the effective potential
(forms of contribitions)

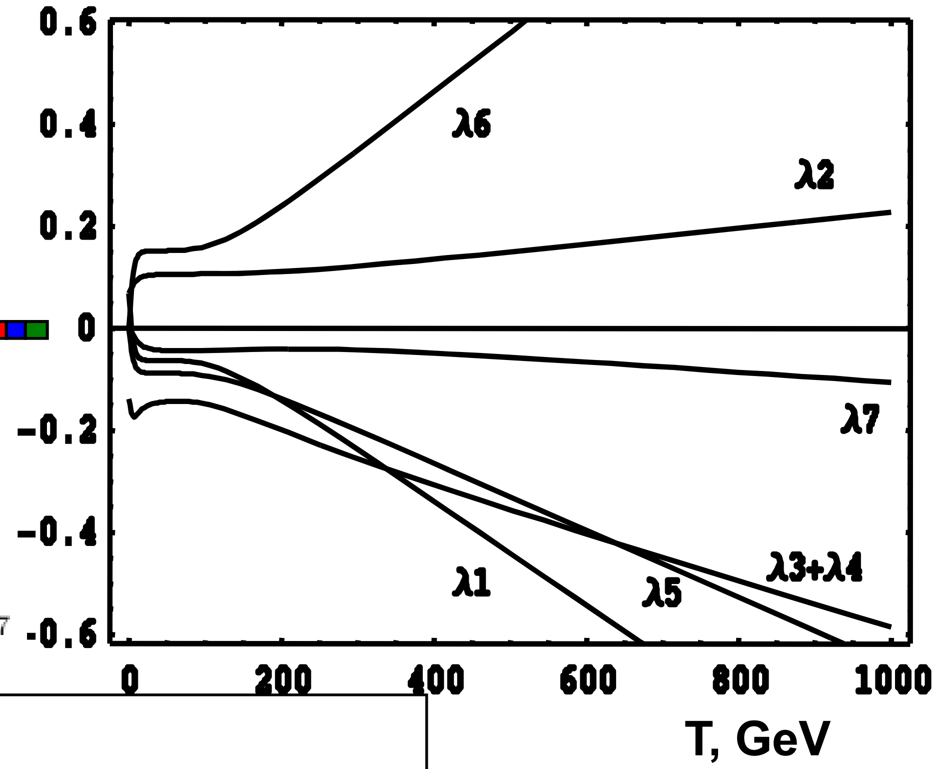
$$\Delta\lambda_5 = 3h_t^4\mu^2 A^2 I_2[m_Q, m_t] + 3h_b^4\mu^2 A^2 I_2[m_Q, m_b]$$

$$\Delta\lambda_6 = -3h_t^4\mu A|\mu|^2 I_2[m_Q, m_t] - 3h_b^4\mu A|A|^2 I_2[m_Q, m_b] +$$
$$+h_t^2\mu A(\frac{g_1^2 - 3g_2^2}{4} I_1[m_Q, m_t] - g_1^2 I_1[m_t, m_Q]) +$$
$$+h_b^2\mu A(\frac{-12h_b^2 + g_1^2 + 3g_2^2}{4} I_1[m_Q, m_b] - \frac{6h_b^2 - g_1^2}{2} I_1[m_b, m_Q])$$

$$\Delta\lambda_7 = -3h_t^4\mu A|A|^2 I_2[m_Q, m_t] - 3h_b^4\mu A|\mu|^2 I_2[m_Q, m_b] +$$
$$+h_b^2\mu A(-\frac{g_1^2 + 3g_2^2}{4} I_1[m_Q, m_b] - \frac{g_1^2}{2} I_1[m_b, m_Q]) +$$
$$+h_t^2\mu A(\frac{12h_t^2 + g_1^2 - 3g_2^2}{4} I_1[m_Q, m_t] - (3h_t^2 - g_1^2) I_1[m_t, m_Q])$$

Temperature-dependent parameters with various quantum corrections in CPX-like scenario
A_t=A_b=1000 GeV, μ=2000 GeV

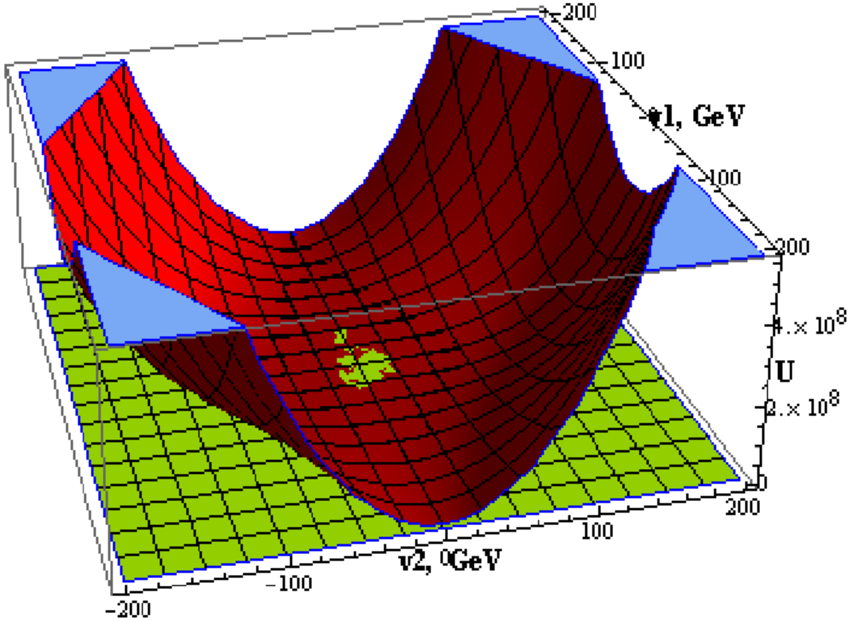


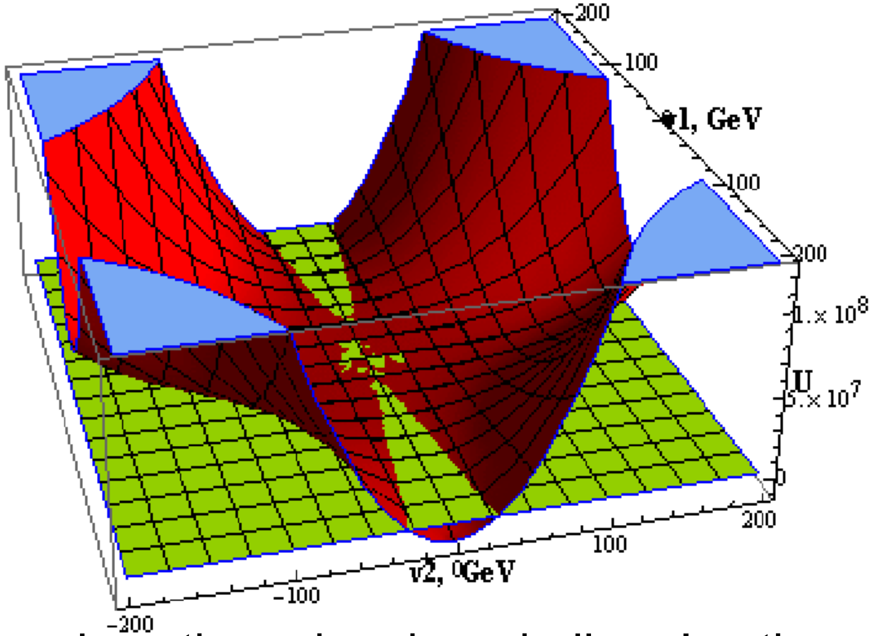


- Msusy=500GeV, T=0.
- Msusy=500GeV, T=200GeV.
- mQ=500GeV, mU=800 GeV, mD=200GeV,T=0.
- mQ=500GeV, mU=800 GeV, mD=200GeV,T=200GeV.
- mQ=500GeV, mU=800 GeV, mD=200GeV,T=200GeV, Log

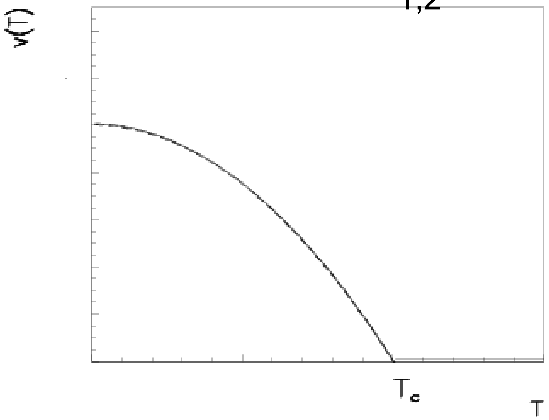
CPX: M.Carena, J.Ellis, A.Pilaftsis, C.Wagner, PL B495 (2000) 155

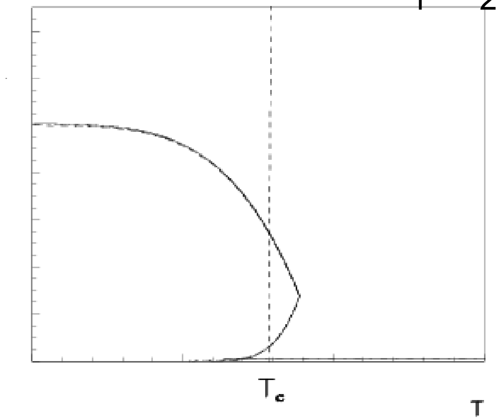
The *surfaces of minima* for effective potential U(v1,v2) at the critical temperature T=120 GeV, $\lambda_6 = \lambda_7 = 0$



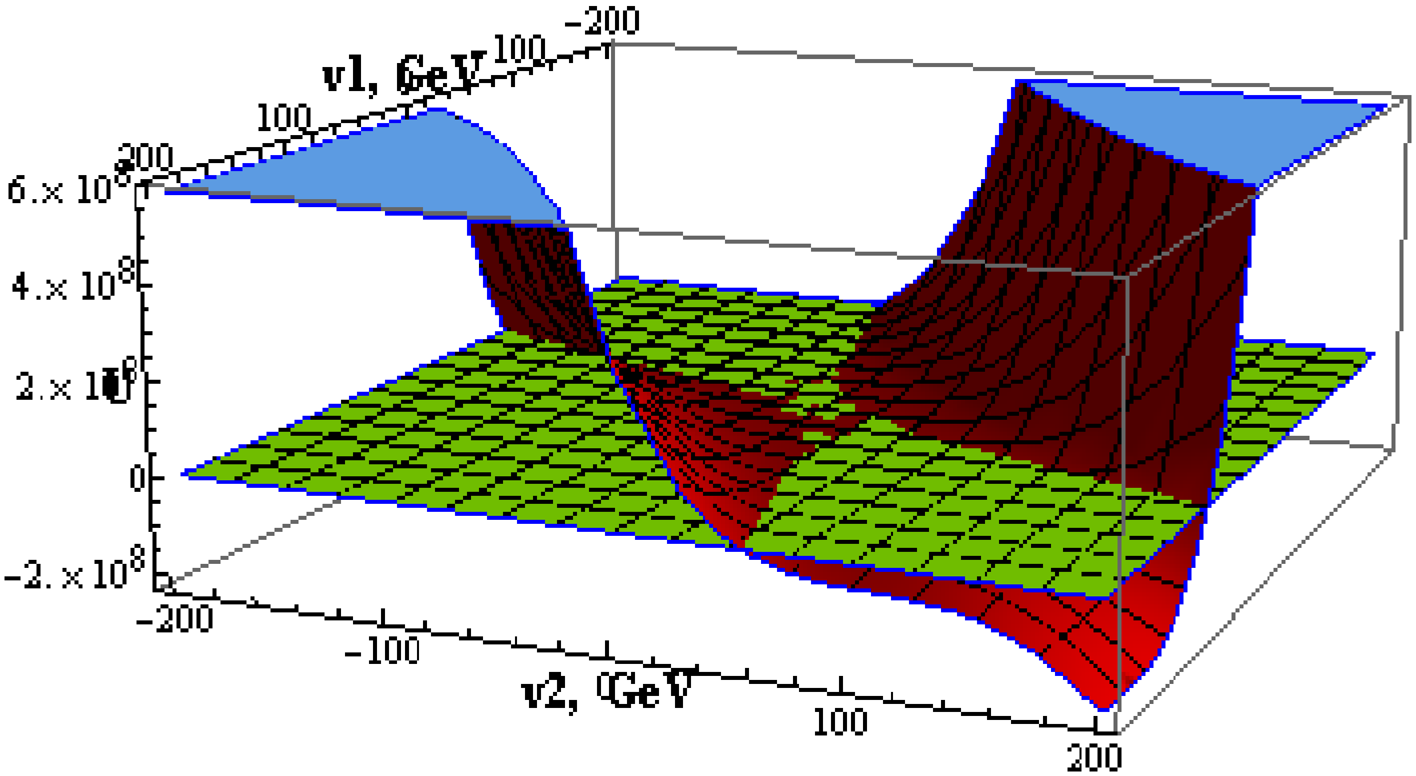


Phase transitions of the first order can occur along the going down hollow. In other directions minima at nonzero $v_{1,2}$ will be above the minimum at $v_1=v_2=0$.





The *surfaces of minima* for effective potential U(v1,v2) at the critical temperature T=120 GeV and nonzero λ_6, λ_7



At nonzero λ_6, λ_7 there are directions always along which the first order phase transition exists.

Effective potential at finite temperature

$$v_1(T) = v(T)\cos\bar{\beta}(T), \quad v_2(T) = v(T)\sin\bar{\beta}(T)$$

Mass term

$$U_{mass}(v,\bar{\beta}) = -\frac{v^2}{2}(\mu_1^2\cos^2\bar{\beta} + \mu_2^2\sin^2\bar{\beta}) - \frac{v^2}{2}\mu_{12}^2\sin 2\bar{\beta}$$

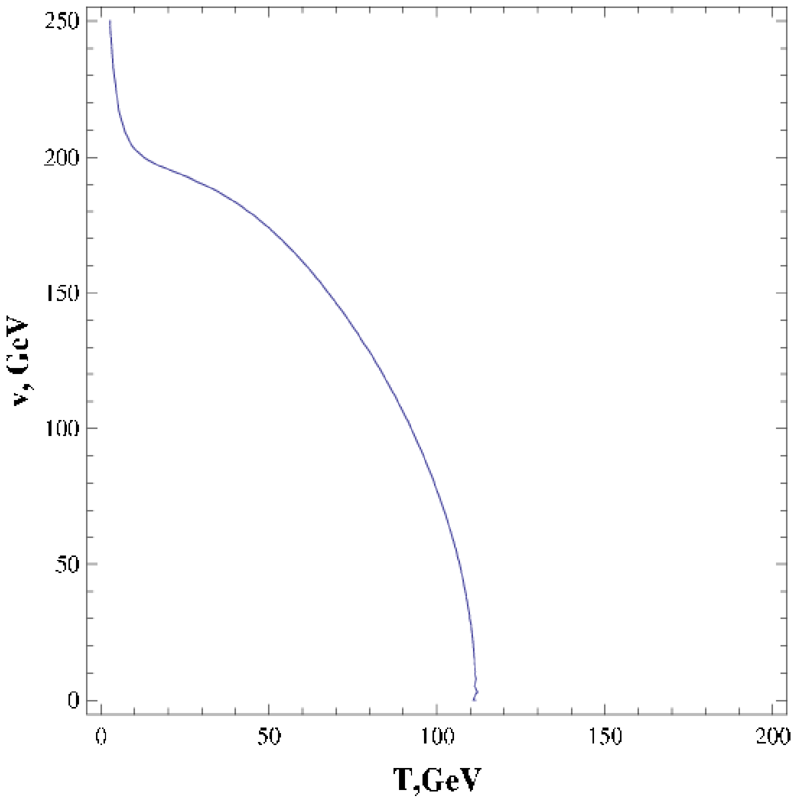
Critical temperature determination

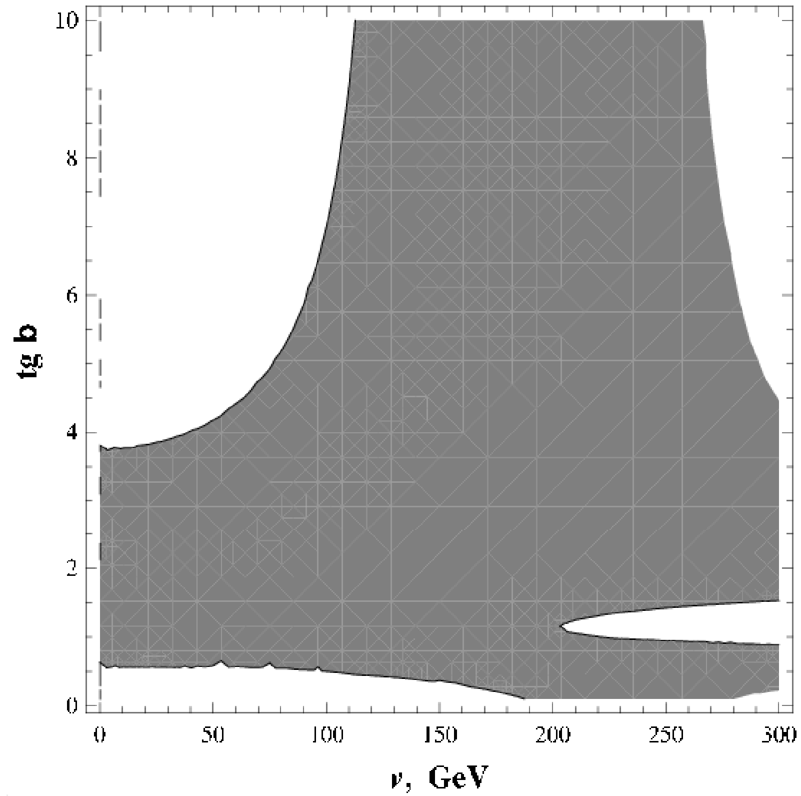
$$\partial U_{mass}/\partial v = 0 \qquad 1/v\ \partial U_{mass}/\partial\bar{\beta} = 0$$

$$\mathtt{tg}2\bar{\beta} = \frac{2\mu_{12}^2}{\mu_1^2 - \mu_2^2}, \qquad (\mu_1^2\mu_2^2 - \mu_{12}^4)[(\mu_1^2 - \mu_2^2)^2 + 4\mu_{12}^4] = 0$$

$$\mu_1^2\mu_2^2 = \mu_{12}^4$$

Evolution of the critical parameters





Effective potential U(v1,v2) at the critical temperature and nonzero λ_6 λ_7

$$\mathrm{tg}2\bar{\beta} = \mathrm{tg}2\beta \frac{1}{\left(\frac{v^2}{2m_A^2} - \alpha_1\right)} \frac{1}{\frac{2\lambda_1\cos^2\beta - 2\lambda_2\sin^2\beta}{\cos 2\beta} - \lambda_{345} + \frac{2m_A^2}{v^2} + \alpha_2}$$

$$\alpha_1 = \frac{\lambda_5}{2} + \frac{1}{4}(\lambda_6 \mathrm{ctg}\beta + \lambda_7 \mathrm{tg}\beta),$$

$$\alpha_2 = \lambda_6(\mathrm{tg}2\beta - \mathrm{ctg}\beta) - \lambda_7(\mathrm{tg}\beta + \mathrm{tg}2\beta).$$

⇓

$$-\frac{m_A^2}{v^2}(2\lambda_5 + \lambda_6\,\mathrm{ctg}\beta + \lambda_7\,\mathrm{tg}\beta) +$$

$$+\frac{v^2}{m_A^2}\left[\frac{2\lambda_1 - 2\lambda_2\mathrm{tg}^2\beta + \lambda_6(3\mathrm{tg}\beta - \mathrm{ctg}\beta) + \lambda_7(\mathrm{tg}^3\beta - 3\mathrm{tg}\beta)}{1 - \mathrm{tg}^2\beta} - \lambda_{345}\right] = 0.$$

⇓

$$\lambda_1(2\lambda_2 - \lambda_{345})^2 + \lambda_2(2\lambda_1 - \lambda_{345})^2 + \lambda_{345}(2\lambda_1 - \lambda_{345})(2\lambda_2 - \lambda_{345}) = 0$$

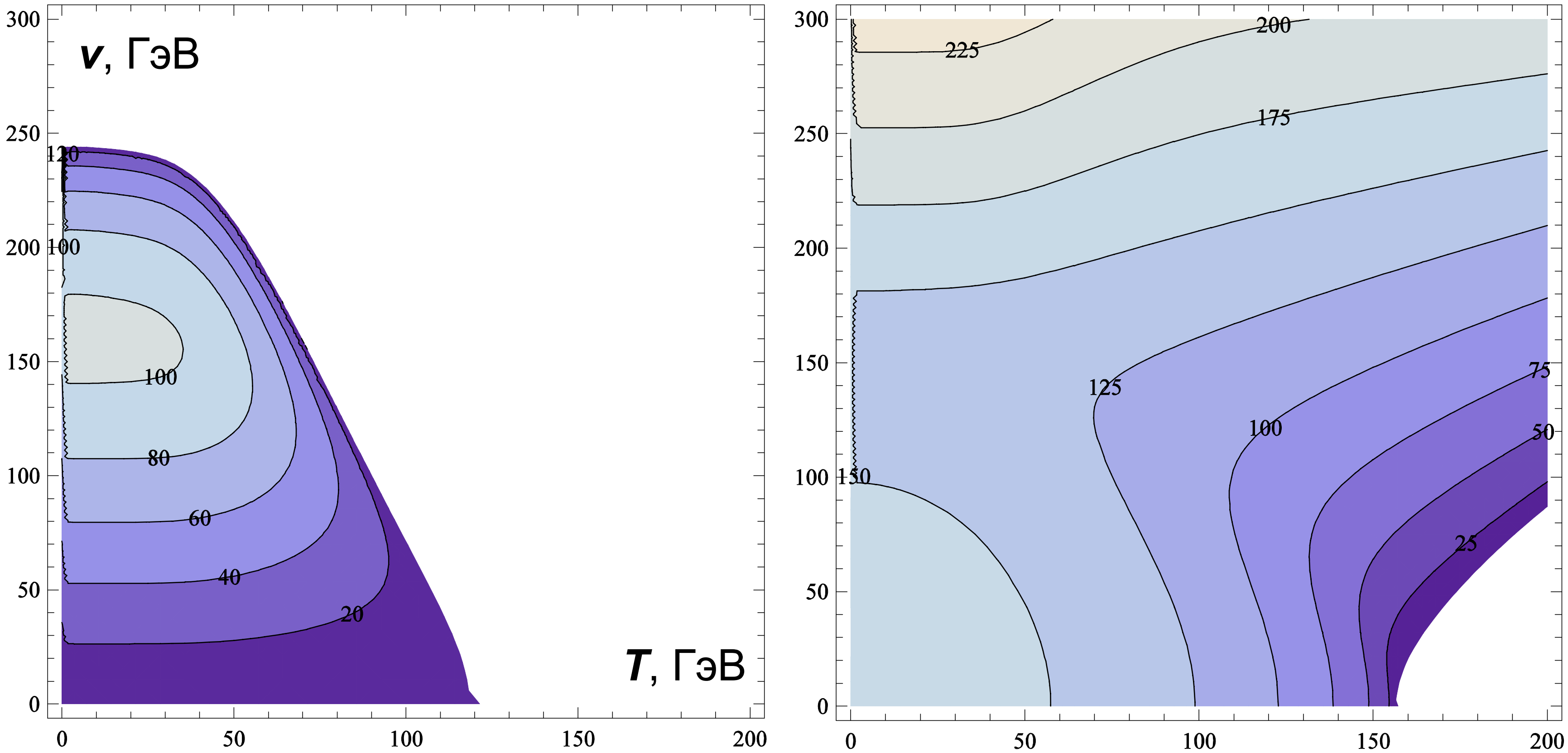
The thermal evolution of the CP-even Higgs bosons h and H is expressed by

$$m_h^2 = c_{\alpha-\beta}^2 m_A^2 + v^2(2\lambda_1 s_\alpha^2 c_\beta^2 + 2\lambda_2 c_\alpha^2 s_\beta^2 - 2(\lambda_3+\lambda_4)c_\alpha c_\beta s_\alpha s_\beta + $$
$$+ \mathrm{Re}\lambda_5(s_\alpha^2 s_\beta^2 + c_\alpha^2 c_\beta^2) - 2c_{\alpha+\beta}(\mathrm{Re}\lambda_6 s_\alpha c_\beta - \mathrm{Re}\lambda_7 c_\alpha s_\beta)),$$
$$m_H^2 = s_{\alpha-\beta}^2 m_A^2 + v^2(2\lambda_1 c_\alpha^2 c_\beta^2 + 2\lambda_2 s_\alpha^2 s_\beta^2 + 2(\lambda_3+\lambda_4)c_\alpha c_\beta s_\alpha s_\beta + $$
$$+ \mathrm{Re}\lambda_5(c_\alpha^2 s_\beta^2 + s_\alpha^2 c_\beta^2) + 2s_{\alpha+\beta}(\mathrm{Re}\lambda_6 c_\alpha c_\beta + \mathrm{Re}\lambda_7 s_\alpha s_\beta)),$$

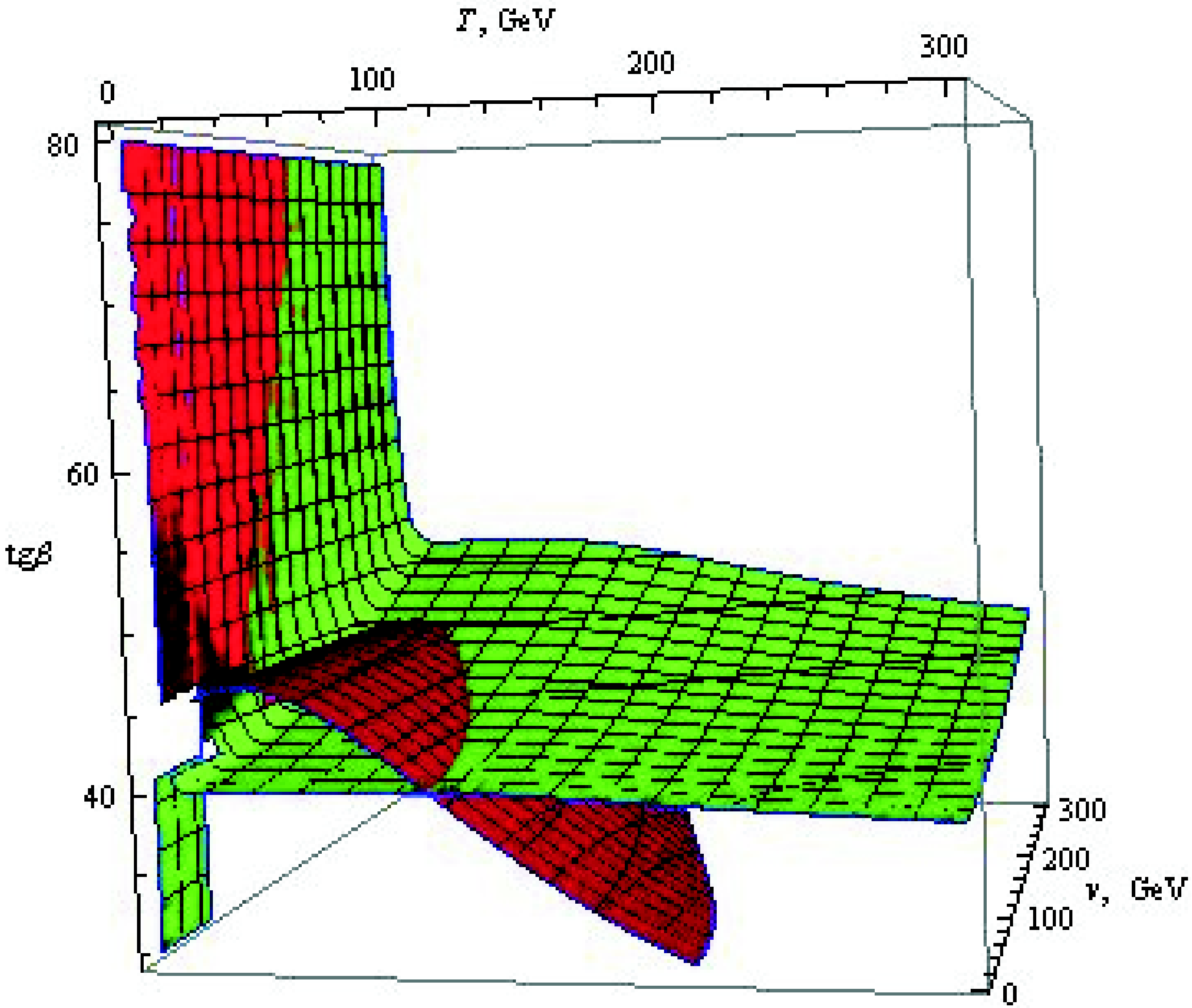
where *α is* the mixing angle of the CP-even states h and H.

[Akhmetzyanova E.N., Dolgopolov M.V., Dubinin M.N. Higgs Bosons in the Two-Doublet Model with CP Violation // Phys.Rev.D. V.71. N7. 2005. P.075008. (hep-ph/0405264)],
and [QUARKS-2008 report]

Higgs bosons masses

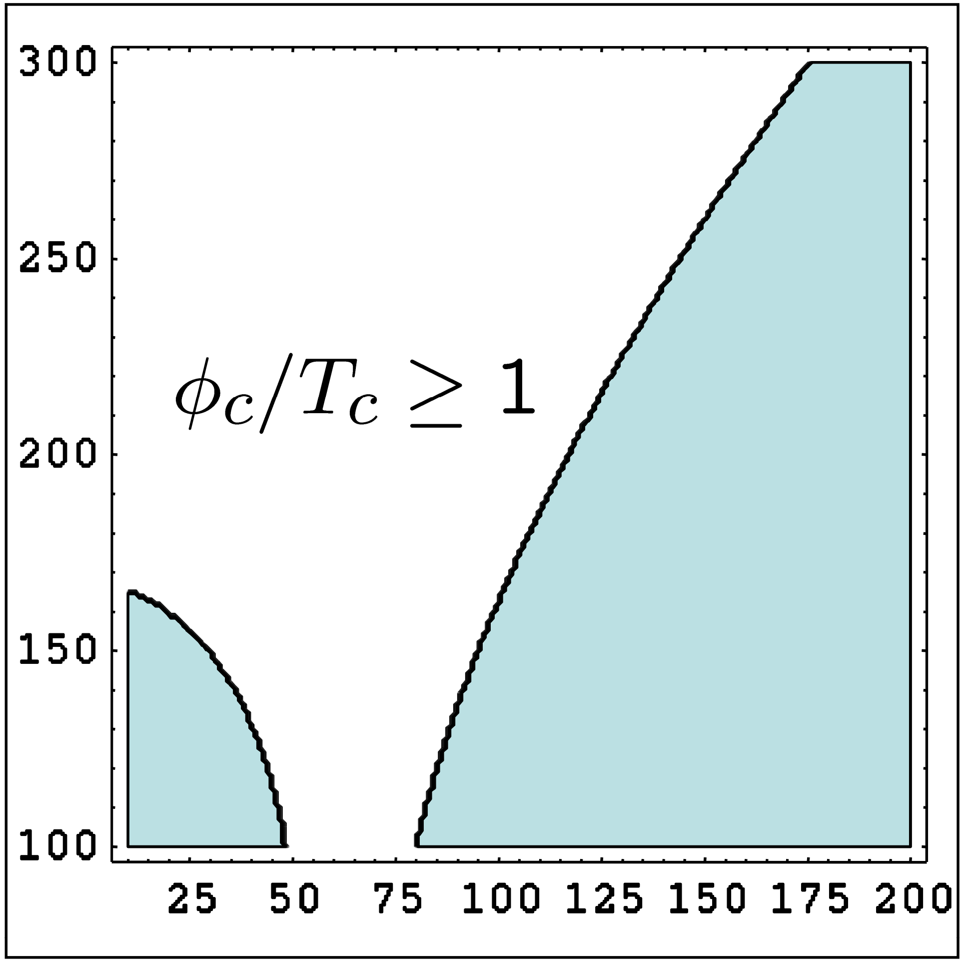


$\text{tg}\beta = 5$, $m_{H^\pm}$ =180 GeV, $A_{t,b} = 1200$ GeV, $\mu = 500$ GeV.


T, GeV
0
100
200
300
80
60
40
tgβ
300
200
100
0
v, GeV

m_h and m_H in the THDM

m_H, GeV



m_h, GeV

Conclusions

1. In the MSSM we calculate the 1-loop finite-temperature corrections from the squarks-Higgs bosons sector, reconstruct the effective two-Higgs-doublet potential and study possibilities of the electroweak phase transition in the full MSSM parameter space ($m_{H\pm}$, tgβ, $A_{t,b}$, μ, m_Q, m_U, m_D).

2. At large values of *A* and *μ* of around 1 TeV, favored indirectly by LEP2 and Tevatron data, the threshold finite-temperature corrections from triangle and box diagrams with intermediate third generation squarks are very substantial.

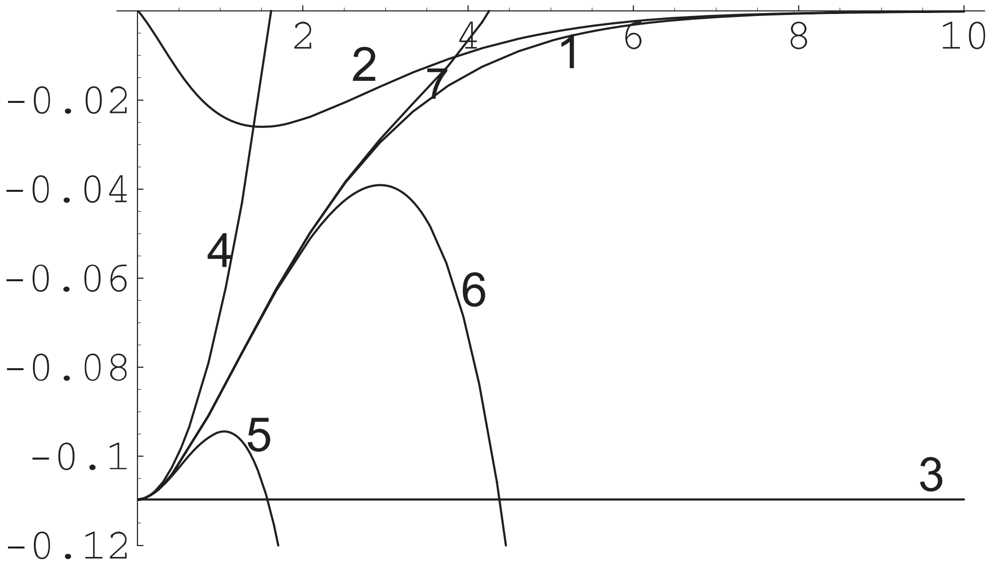
3. High sensitivity of the low-temperature evolution to the effective two-doublet and the MSSM squark sector parameters is observed, but rather extensive regions of the full MSSM parameter space allow the first-order electroweak phase transition respecting the phenomenological constraints at zero temperature.

Perspectives

- **Electroweak baryogenesis is still viable in extended Higgs sectors**
- **It would offer the possibiliy to compute the baryon asymmetry from parameters measured in collider experiments**
- **If the result would match the observations, we could claim to understand the early universe up to electroweak temperatures**
- **viable models:**

  THDM, MSSM,
  Singlet models: many possiblities
- **Strong constraints on CP phases from EDM's**

Finite temperature effective potential in the SM



Поведение функции $V_1^{T\neq0}/T^4$

1 – полный результат, полученный численно,

2 – низкотемпературное приближение, большие значения y,

3 – постоянная составляющая высокотемпературном приближении, $y<<1$,

4 – дополнительно учет члена $\sim y^2$,

5 – учет члена $\sim y^3$,

6 – учет члена $\sim y^4$, в том числе с логарифмом,

7 – учет члена $\sim y^6$.

Мотивация исследования *CP*-нарушения в суперсимметричных теориях

Одно из самых важных следствий *CP*-нарушения – возможность объяснения асимметрии материя-антиматерия.

Электрослабый бариогенезис может быть реализован в минимальном суперсимметричном расширении СМ, но его рассмотрение требует введения новых источников *CP*-нарушения в секторе третьего поколения скалярных кварков или в секторе калибрино-хиггсино.

В минимальной суперсимметричной модели необходимо рассматривать легкий и тяжелый скалярные топ-кварки, для того чтобы имели место сильные фазовые переходы первого рода.

В модели Next-to-MSSM (следующей за минимальной, НМССМ) отсутствуют ограничения на сектор третьего поколения. и возможно *CP*-нарушение в древесном потенциале.

Заключение

1. ЭСФП первого рода ⇔ легкий бозон
2. *CP*-нарушение

СМ Легкий бозон, m_h<50 ГэВ, LEP: m_{hSM}>114ГэВ

CP-нарушение в матрице СКМ слишком мало для генерирования достаточного барионного числа

МССМ Легкий скалярный t-скварк

Ограничение на легчайший бозон Хиггса сужает возможное пространство параметров

CP-нарушение в членах мягкого нарушения суперсимметрии Если *CP*-нарушение в скалярном секторе большое, то ЭСФП I рода подавлен.

ДДМ Сильный ЭСФП I рода $V_{эфф}(\varphi,T)$

Большие петлевые поправки к константам самодействия (в зависимости от *CP*-фазы)

НМССМ Легкий бозон Хиггса за счет малой константы g_{h1ZZ}

Неисчезающая *CP*-фаза даже в древесном хиггсовском потенциале

Поправки к параметрам потенциала Хиггса МССМ (диаграммы «рыбы»), разные массовые параметры скалярных кварков

$$-\Delta\lambda_1^f = \left[h_b^2 - \frac{g_1^2}{6}\right]^2 (I(m_Q) + I(m_D)) + \frac{g_1^4}{9} I(m_U),$$

$$-\Delta\lambda_2^f = \left[h_t^2 + \frac{g_1^2}{6}\right]^2 I(m_Q) + [h_t^2 - \frac{g_1^2}{3}]^2 I(m_U) + \frac{g_1^4}{36} I(m_D),$$

$$-(\Delta\lambda_3 + \Delta\lambda_4)^f = \frac{1}{72}\left(-g_1^4 + 6(h_b^2 - h_t^2)g_1^2 - 9(g_2^4 - 2(h_b^2 + h_t^2)g_2^2)\right) I(m_Q) +$$

$$+\frac{g_1^2}{3}(h_t^2 - \frac{g_1^2}{3}) I(m_U) + \frac{g_1^2}{6}(h_b^2 - \frac{g_1^2}{6}) I(m_D),$$

$$-\Delta\lambda_3^f = \frac{1}{72}\left(-g_1^4 + 6(h_b^2 - h_t^2)g_1^2 + 9\left(g_2^4 - 2\left(h_b^2 + h_t^2\right)g_2^2 + 8h_b^2h_t^2\right)\right) I(m_Q) +$$

$$+\frac{g_1^2}{3}(h_t^2 - \frac{g_1^2}{3}) I(m_U) + \frac{g_1^2}{6}(h_b^2 - \frac{g_1^2}{6}) I(m_D) + h_t^2 h_b^2 I(m_U, m_D).$$

$$-\Delta\lambda_4^f = (h_b^2 - \frac{g_2^2}{2})(\frac{g_2^2}{2} - h_t^2) I(m_Q) - h_t^2 h_b^2 I(m_U, m_D).$$

$$J(m_I) = \frac{1}{8\pi m_I}, \qquad J(m_U, m_D) = \frac{1}{4\pi(m_U + m_D)}.$$

Поправки к параметрам потенциала Хиггса МССМ (логарифмические), разные массовые параметры скалярных кварков

$$\Delta\lambda_1^{log} = -\frac{1}{384\pi^2}\left(11g_1^4 - 36h_b^2g_1^2 + 9\left(g_2^4 - 4h_b^2g_2^2 + 16h_b^4\right)\right)\ln\left(\frac{m_Q m_U}{m_t^2}\right),$$

$$\Delta\lambda_2^{log} = -\frac{1}{1536\pi^2}\left(44g_1^4 - 144h_t^2g_1^2 + 36g_2^4 + 576h_t^4 - 144g_2^2h_t^2\right)\ln\left(\frac{m_Q m_U}{m_t^2}\right),$$

$$\Delta\lambda_3^{log} = -\frac{1}{384\pi^2}\left(-11g_1^4 + 18\left(h_b^2 + h_t^2\right)g_1^2 +\right.$$

$$\left. +9\left(g_2^4 - 2\left(h_b^2 + h_t^2\right)g_2^2 + 16h_b^2h_t^2\right)\right)\ln\left(\frac{m_Q m_U}{m_t^2}\right),$$

$$\Delta\lambda_4^{log} = \frac{3}{64\pi^2}\left(g_2^4 - 2\left(h_b^2 + h_t^2\right)g_2^2 + 8h_b^2h_t^2\right)\ln\left(\frac{m_Q m_U}{m_t^2}\right).$$

Поправки к параметрам потенциала Хиггса МССМ (перенормировка поля), разные массовые параметры скалярных кварков

$$\Delta\lambda_1^{\rm wfr} = \frac{1}{2}(g_1^2+g_2^2)A'_{11}, \qquad \Delta\lambda_2^{\rm wfr} = \frac{1}{2}(g_1^2+g_2^2)A'_{22},$$

$$\Delta\lambda_3^{\rm wfr} = -\frac{1}{4}(g_1^2-g_2^2)(A'_{11}+A'_{22}), \quad \Delta\lambda_4^{\rm wfr} = -\frac{1}{2}g_2^2(A'_{11}+A'_{22}), \quad \Delta\lambda_5^{\rm wfr} = 0,$$

$$\Delta\lambda_6^{\rm wfr} = \frac{1}{8}(g_1^2+g_2^2)(A'_{12}-A'^{\,*}_{21}) = 0, \quad \Delta\lambda_7^{\rm wfr} = \frac{1}{8}(g_1^2+g_2^2)(A'_{21}-A'^{\,*}_{12}) = 0.$$

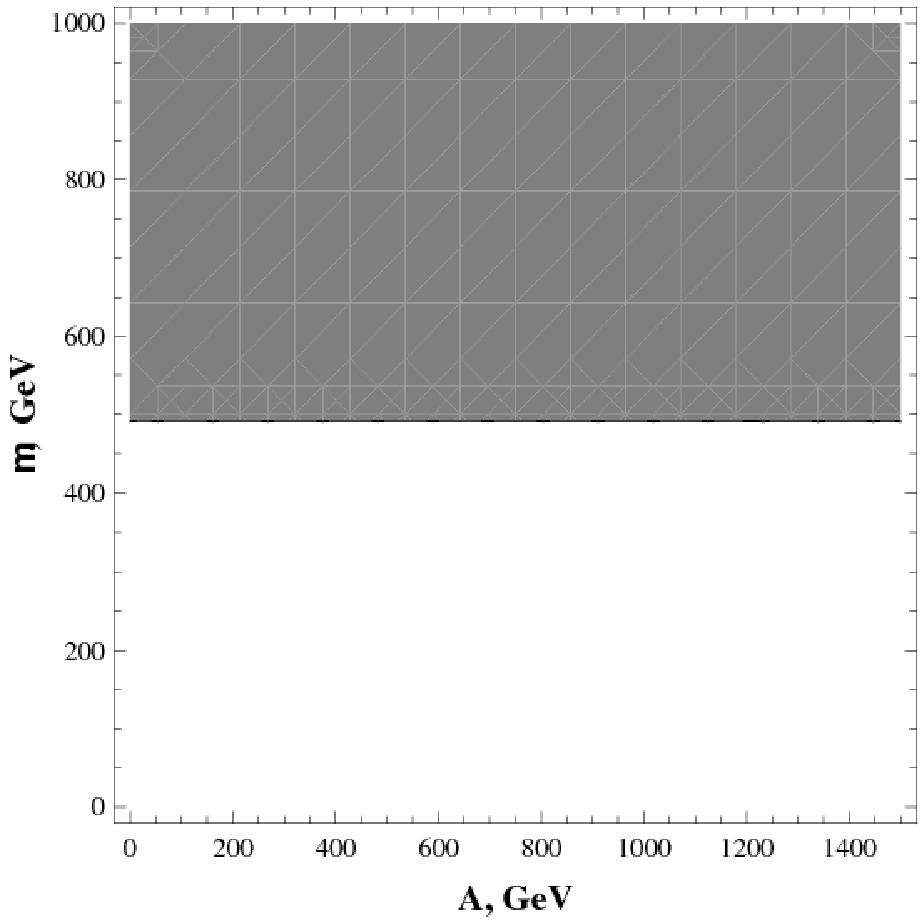
$$A'_{ij} = \{\frac{2\cdot 3h_U^2}{24\,\pi}F(m_Q^2, m_U^2, T)\begin{bmatrix} |\mu|^2 & -\mu^* A_U^* \\ -\mu A_U & |A_U|^2 \end{bmatrix} + \\ + (U \longrightarrow D, A \longleftrightarrow \mu)\}(1-\frac{1}{2}l)$$

$$F(m_1^2, m_2^2, T) = T\sum_{n=-\infty}^{+\infty}\frac{1}{(\sqrt{m_1^2+(2\pi nT)^2}+\sqrt{m_2^2+(2\pi nT)^2})^3} =$$

$$= \frac{T}{(m_1+m_2)^3} + 2T\sum_{n=1}^{+\infty}\frac{1}{(\sqrt{m_1^2+(2\pi nT)^2}+\sqrt{m_2^2+(2\pi nT)^2})^3}.$$

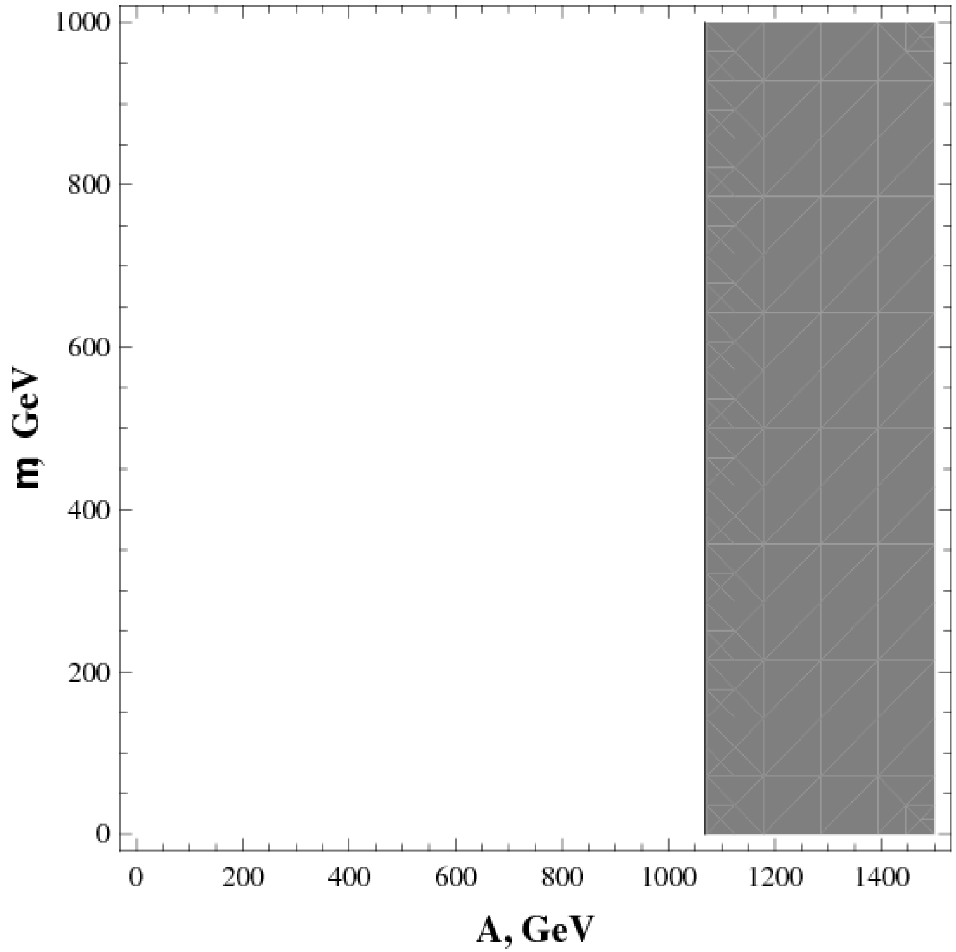
Ограничения на параметры модели

$\lambda_1 < 0$



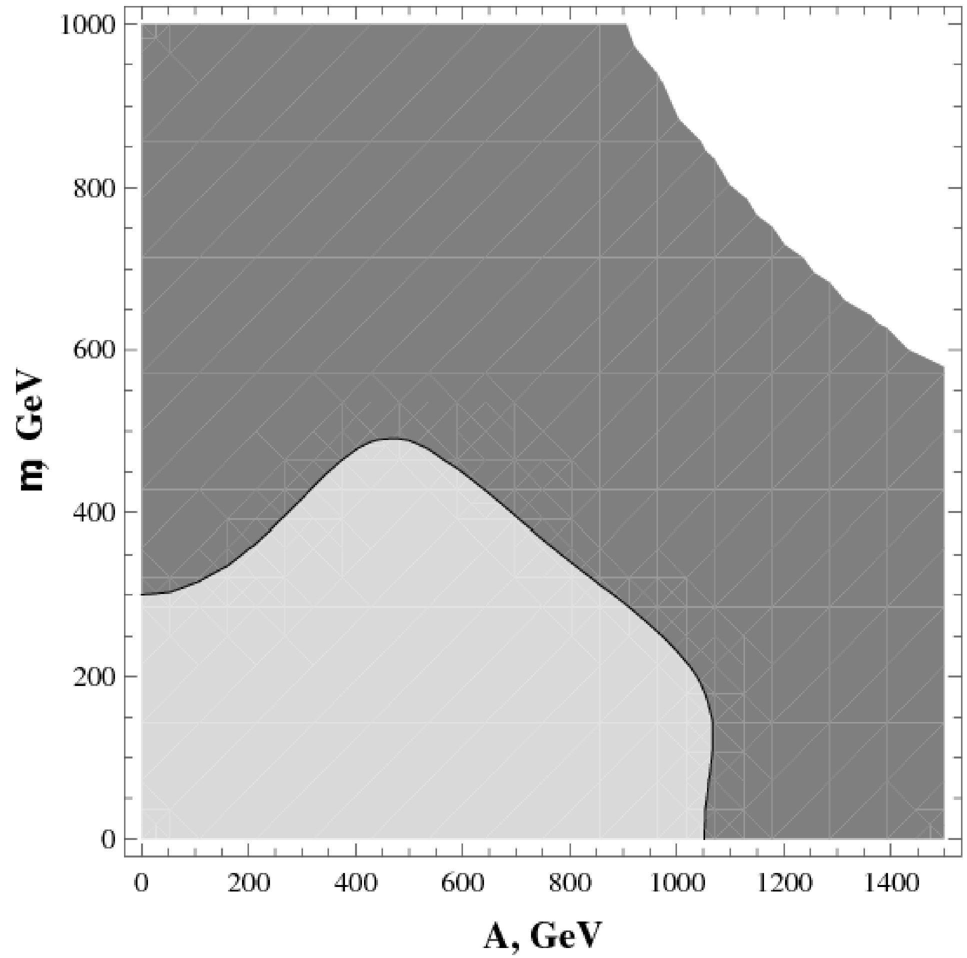
Ограничения на параметры модели

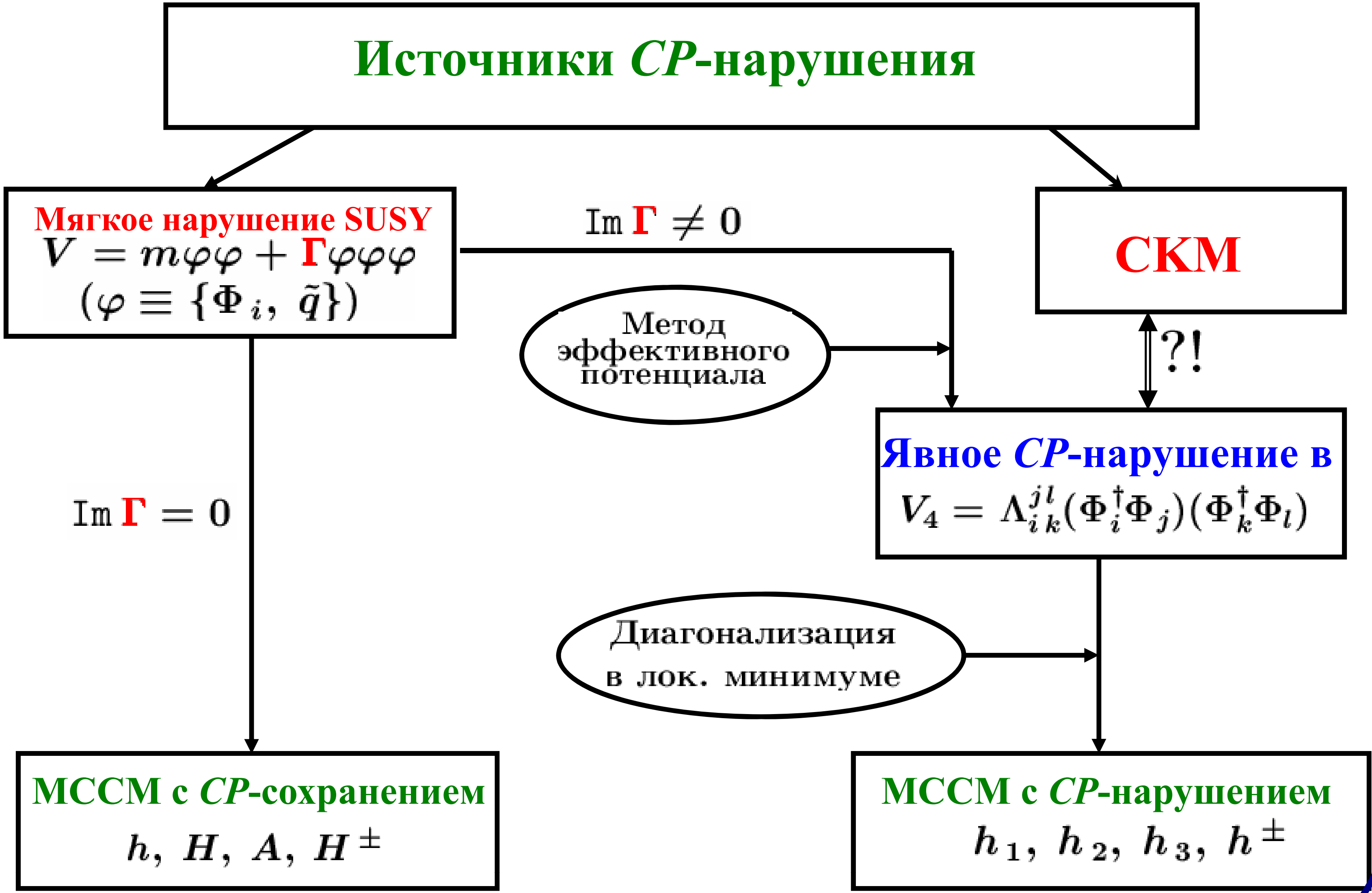
$\lambda_2 < 0$



Ограничения на параметры модели

$$\lambda_1\lambda_2 - \frac{\lambda_{345}^2}{4} < 0$$





Источники CP-нарушения
Мягкое нарушение SUSY
$V = m\varphi\varphi + \Gamma\varphi\varphi\varphi$
$(\varphi \equiv \{\Phi_i, \tilde{q}\})$
СКМ
$\mathrm{Im}\,\Gamma \neq 0$
Метод эффективного потенциала
?!
Явное CP-нарушение в
$V_4 = \Lambda_{i\,k}^{j\,l}(\Phi_i^\dagger \Phi_j)(\Phi_k^\dagger \Phi_l)$
$\mathrm{Im}\,\Gamma = 0$
Диагонализация в лок. минимуме
МССМ с CP-сохранением
$h,\ H,\ A,\ H^\pm$
МССМ с CP-нарушением
$h_1,\ h_2,\ h_3,\ h^\pm$

CP-нарушение в СМ

$$J^{\mu +} = \frac{1}{\sqrt{2}} \bar{U}^i_L \gamma^\mu D^i_L \longrightarrow \frac{1}{\sqrt{2}} \bar{U}^i_L \gamma^\mu (U^+_U U_D)^{ij} D^j_L$$

переход в массовый базис

$$U_L \equiv (u_L, c_L, t_L)^T$$ верхние левые кварки,

$$D_L \equiv (d_L, s_L, b_L)^T$$ нижние левые кварки

$$V_{U,D} = U^+_U U_D$$ 3×3 матрица смешивания СКМ

9 параметров - 3 угла и 6 фаз

↓ (3 поколения) ↓

3 угла, $\delta_{СКМ}$ – источник *CP*-нарушения

$$\frac{m_i^{u(d)}}{v}(a + i\gamma_5 b) \rightarrow \frac{m_i^{u(d)}}{v}(a - i\gamma_5 b)$$

$$\text{СМ:} \quad a = 1, \quad b = 0$$

Диагонализация в локальном минимуме

$$U_{\rm mass} = c_1 hA + c_2 HA + \frac{m_h^2}{2}h^2 + \frac{m_H^2}{2}H^2 + \frac{m_A^2}{2}A^2 + m_{H^\pm}^2 H^+H^-$$

$$c_1 = \frac{v^2}{2}(s_\alpha s_\beta - c_\alpha c_\beta)\,\mathrm{Im}\lambda_5 + v^2\,(s_\alpha c_\beta\,\mathrm{Im}\lambda_6 - c_\alpha s_\beta\,\mathrm{Im}\lambda_7),$$

$$c_2 = -\frac{v^2}{2}(s_\alpha c_\beta + c_\alpha s_\beta)\,\mathrm{Im}\lambda_5 - v^2\,(c_\alpha c_\beta\,\mathrm{Im}\lambda_6 + s_\alpha s_\beta\,\mathrm{Im}\lambda_7).$$

Для устранения недиагональных членов *hA* и *HA* проводится ортогональное преобразование в секторе $(h, H, A)=a_{ij}h_j$ **:**

$$(\mathrm{h}, \mathrm{H}, \mathrm{A})\mathrm{M}^2\begin{pmatrix}\mathrm{h}\\ \mathrm{H}\\ \mathrm{A}\end{pmatrix} = (\mathrm{h}_1, \mathrm{h}_2, \mathrm{h}_3)\mathrm{a}^T_{ik}\mathrm{M}^2_{kl}\mathrm{a}_{lj}\begin{pmatrix}\mathrm{h}_1\\ \mathrm{h}_2\\ \mathrm{h}_3\end{pmatrix}$$

где массовая матрица имеет вид

$$\mathrm{M}^2 = \frac{1}{2}\begin{pmatrix}\mathrm{m}_\mathrm{h}^2 & 0 & c_1\\ 0 & \mathrm{m}_\mathrm{H}^2 & c_2\\ c_1 & c_2 & \mathrm{m}_\mathrm{A}^2\end{pmatrix} \quad \Rightarrow \quad \frac{1}{2}\begin{pmatrix}\mathrm{m}_{\mathrm{h}_1}^2 & 0 & 0\\ 0 & \mathrm{m}_{\mathrm{h}_2}^2 & 0\\ 0 & 0 & \mathrm{m}_{\mathrm{h}_3}^2\end{pmatrix}$$

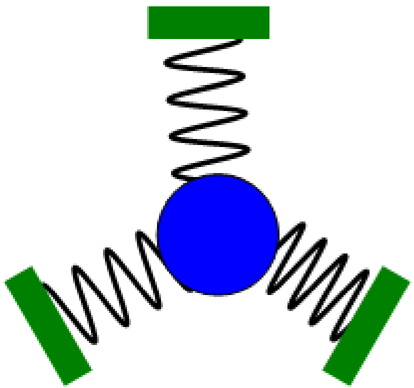
Аналогия с колебаниями систем со многими степенями свободы

Классическим аналогом рассматриваемой задачи об определении массовых состояний в минимуме потенциала является задача о нахождении собственных частот малых колебаний системы с несколькими степенями свободы (диагонализация квадратичной формы), причем параметры, определяющие интенсивность взаимодействия являются комплексными.

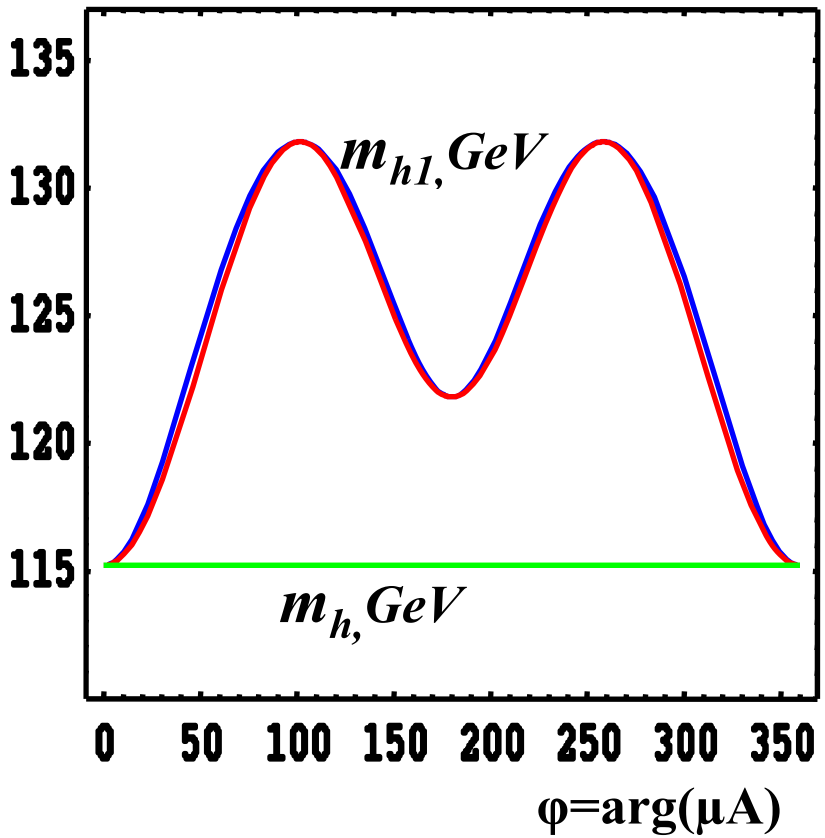
$$E = \frac{1}{2}(m_{ij}\dot{x}_i\dot{x}_j + k_{ij}x_ix_j) + l_{ijk}x_ix_jx_k + \ldots$$

$$\Theta = \|A\|x$$

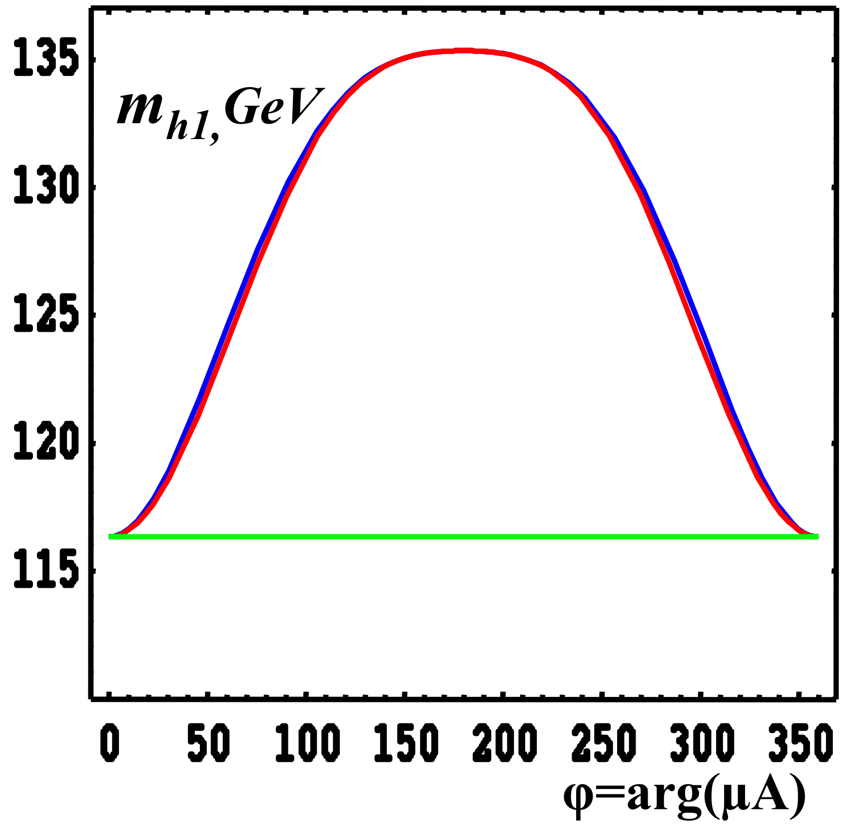
$$E = \frac{1}{2}(M_i\dot{\Theta}_i^2 + K_i\Theta_i^2) + L_{ijk}\Theta_i\Theta_j\Theta_k + \ldots$$



Mass of the lightest Higgs boson

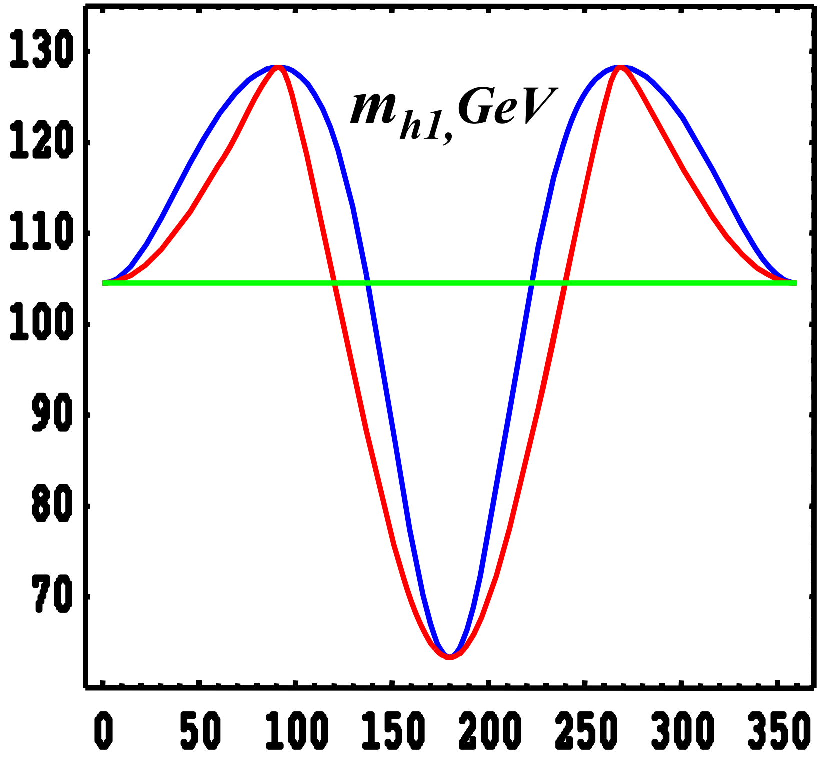


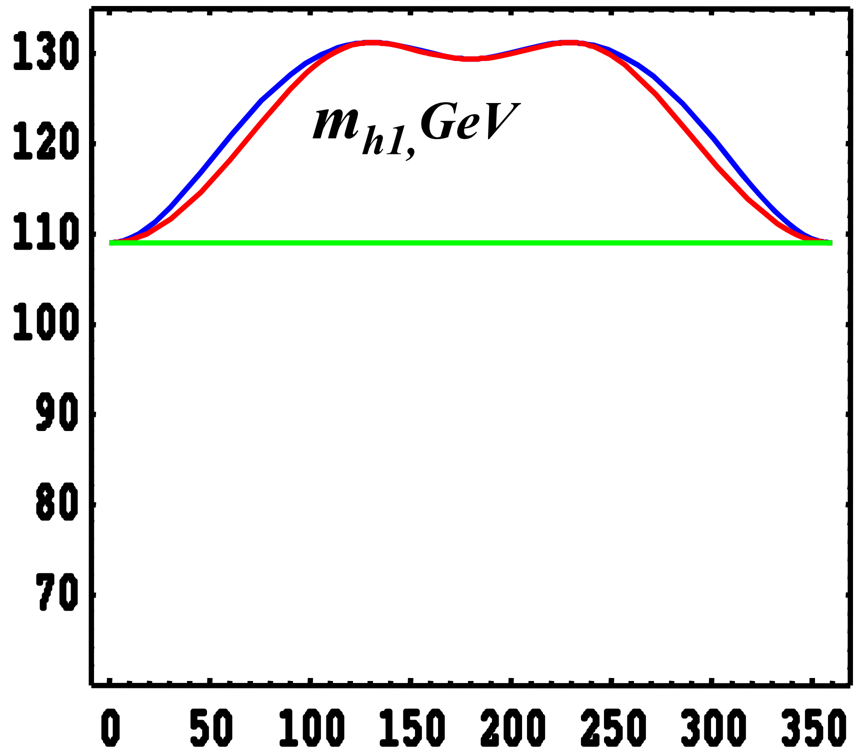
Msusy=500GeV
$m_{H^\pm}$=300GeV



mQ=500GeV, mU=800GeV,
mD=200GeV

Mass of the lightest Higgs boson

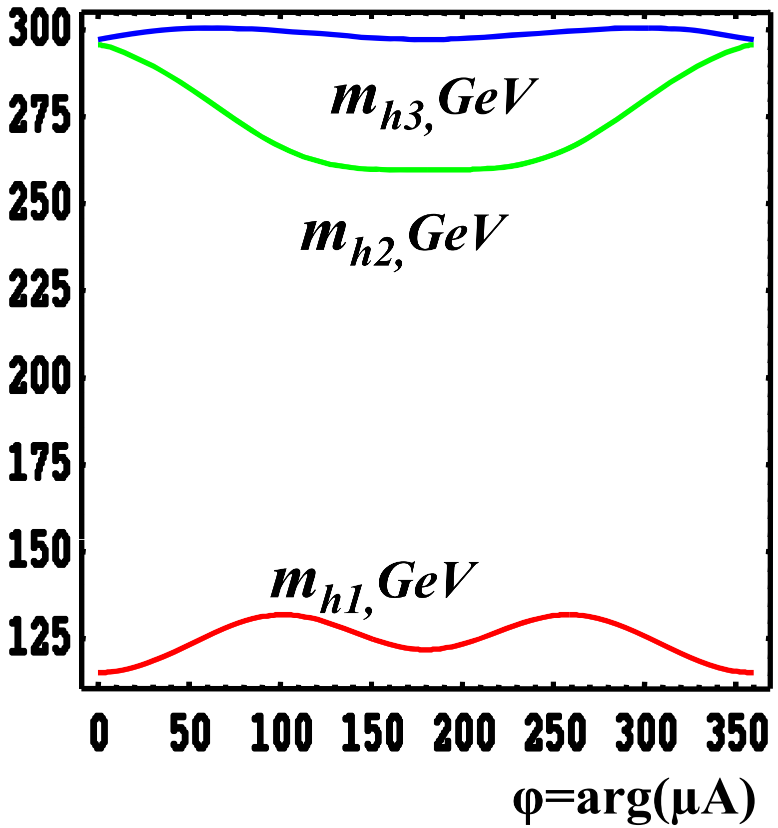


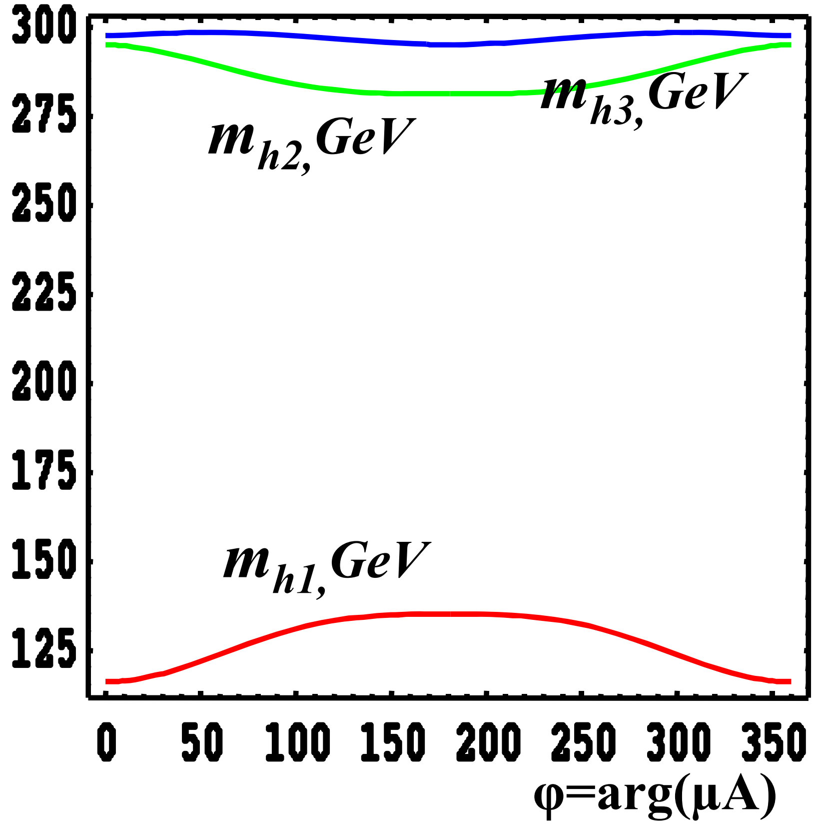


Msusy=500GeV
$m_{H^\pm}$=190GeV

mQ=500GeV, mU=800GeV, mD=200GeV

Masses of the Higgs bosons

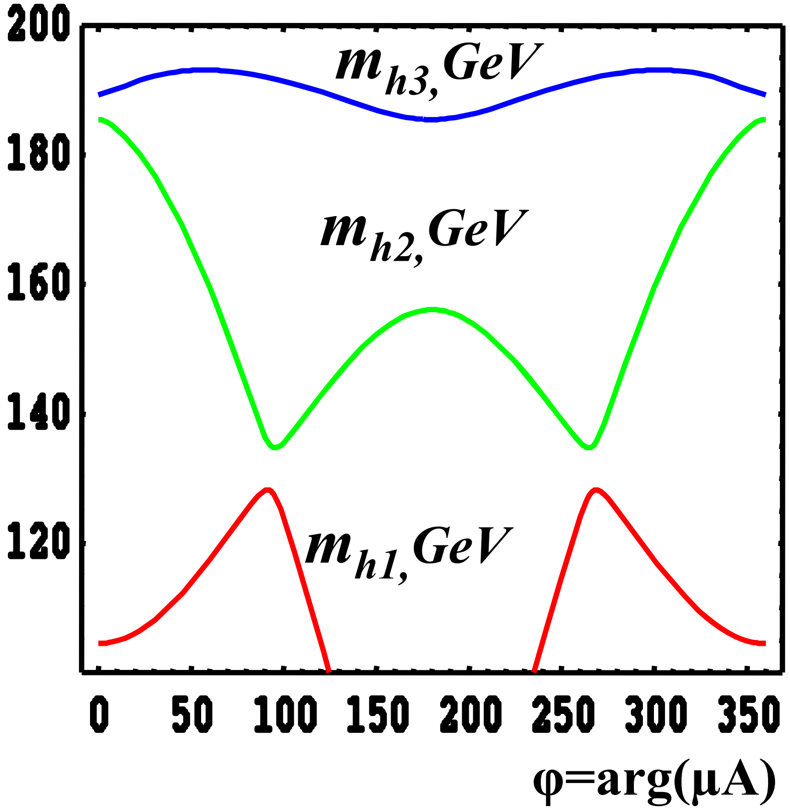




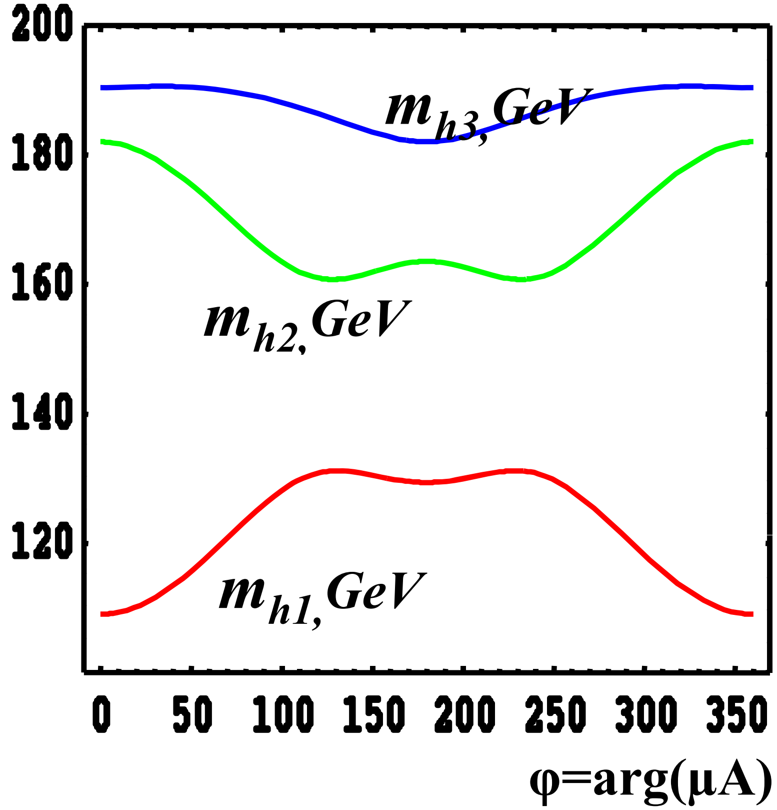
Msusy=500GeV
m_{H}±=300GeV

mQ=500GeV, mU=800GeV,
mD=200GeV

Masses of the Higgs bosons

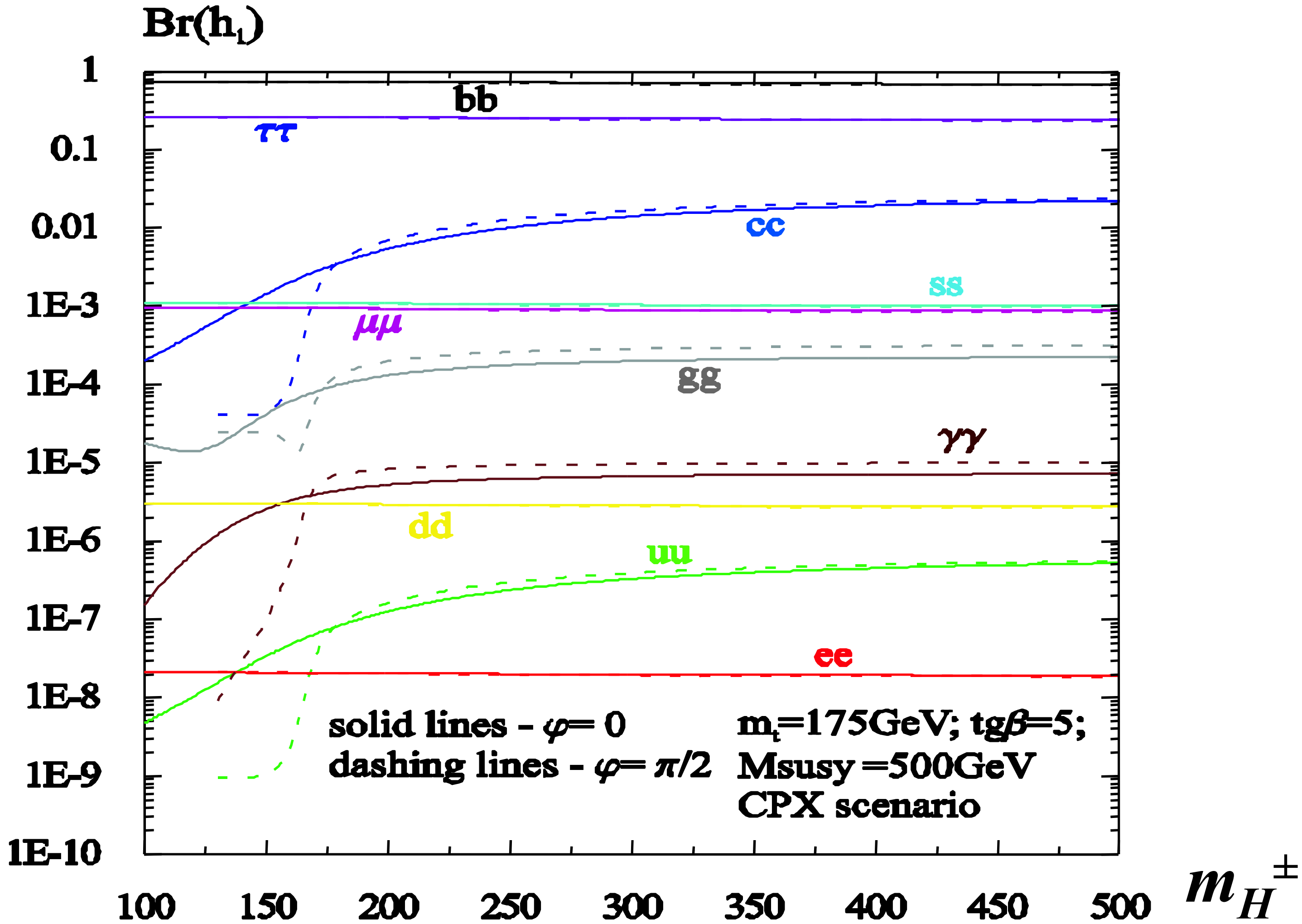


Msusy=500GeV
$m_H\pm$=190GeV

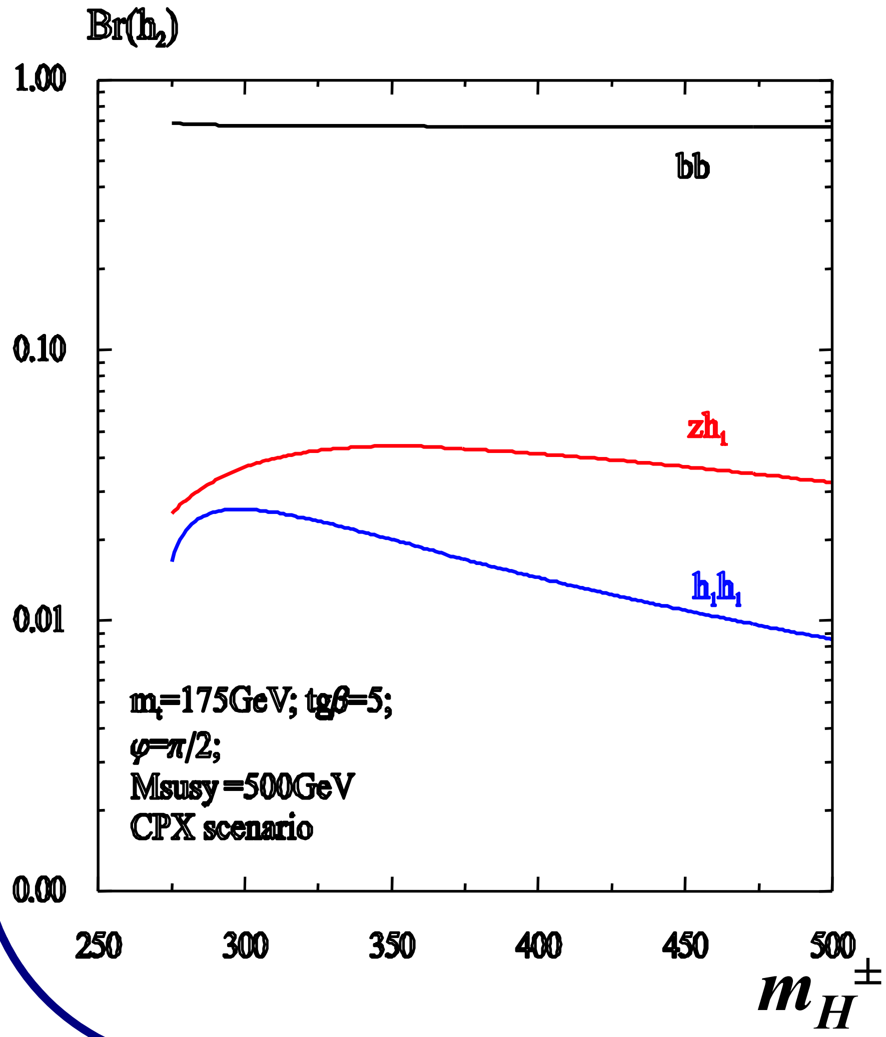


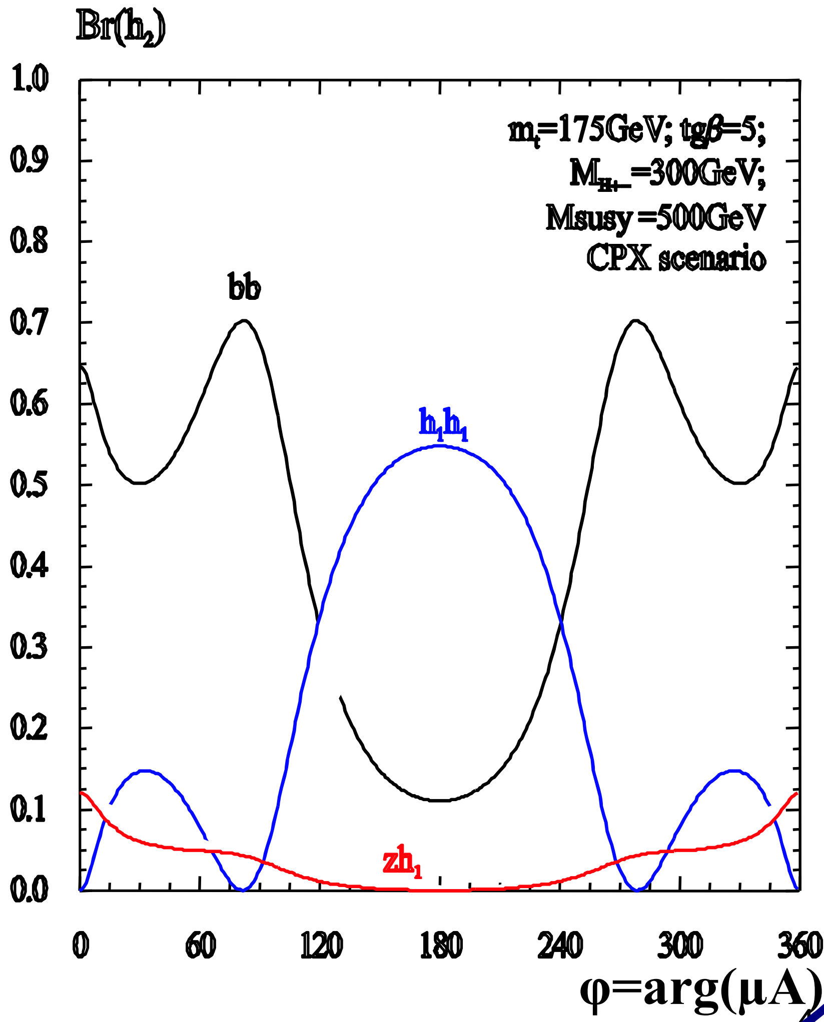
mQ=500GeV, mU=800GeV,
mD=200GeV

Branching ratios



Branching ratios





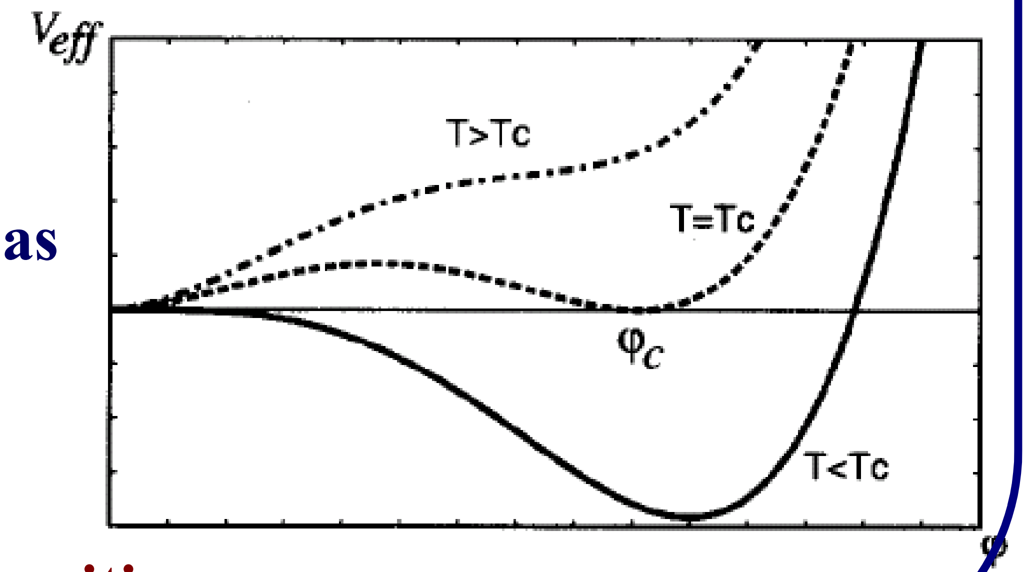
Sakharov's Conditions for Baryogenesis

- ***Necessary* requirements for baryogenesis:**
  - **Baryon number violation**
  - **CP violation**
  - **Non-equilibrium**
  - $\Rightarrow \Gamma\,(\Delta B>0) > \Gamma(\Delta\, B<0)$
- **Possible new consequences in**
  - **Proton decay**
  - **CP violation**

Condition of the strong first order transition

The first order phase transition is needed for a bubble nucleation.

The sphaleron transition rate should be suppressed in the broken phase at the critical temperature, in order not to erase the created baryon number.

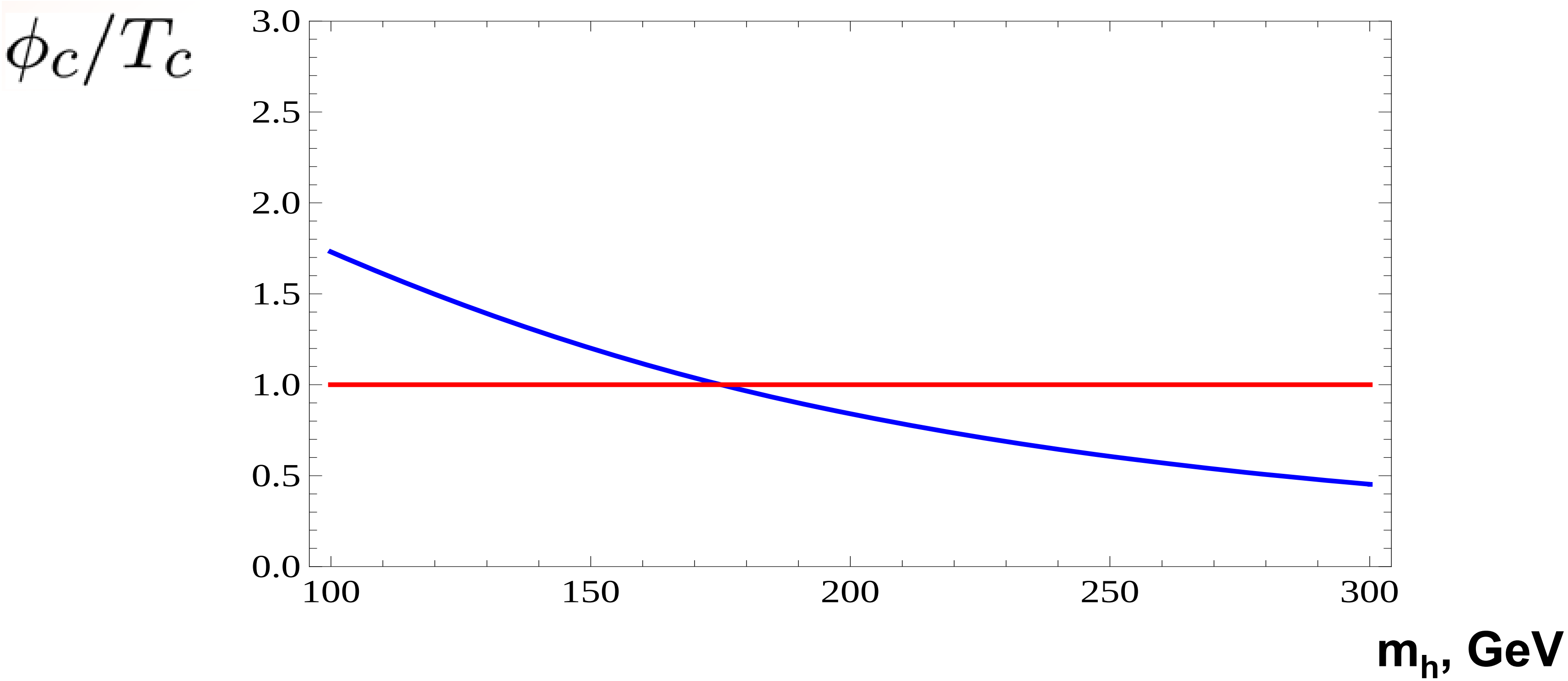


This condition is expressed as

$$\phi_C/T_C \geq 1$$

Strong first order phase transition.

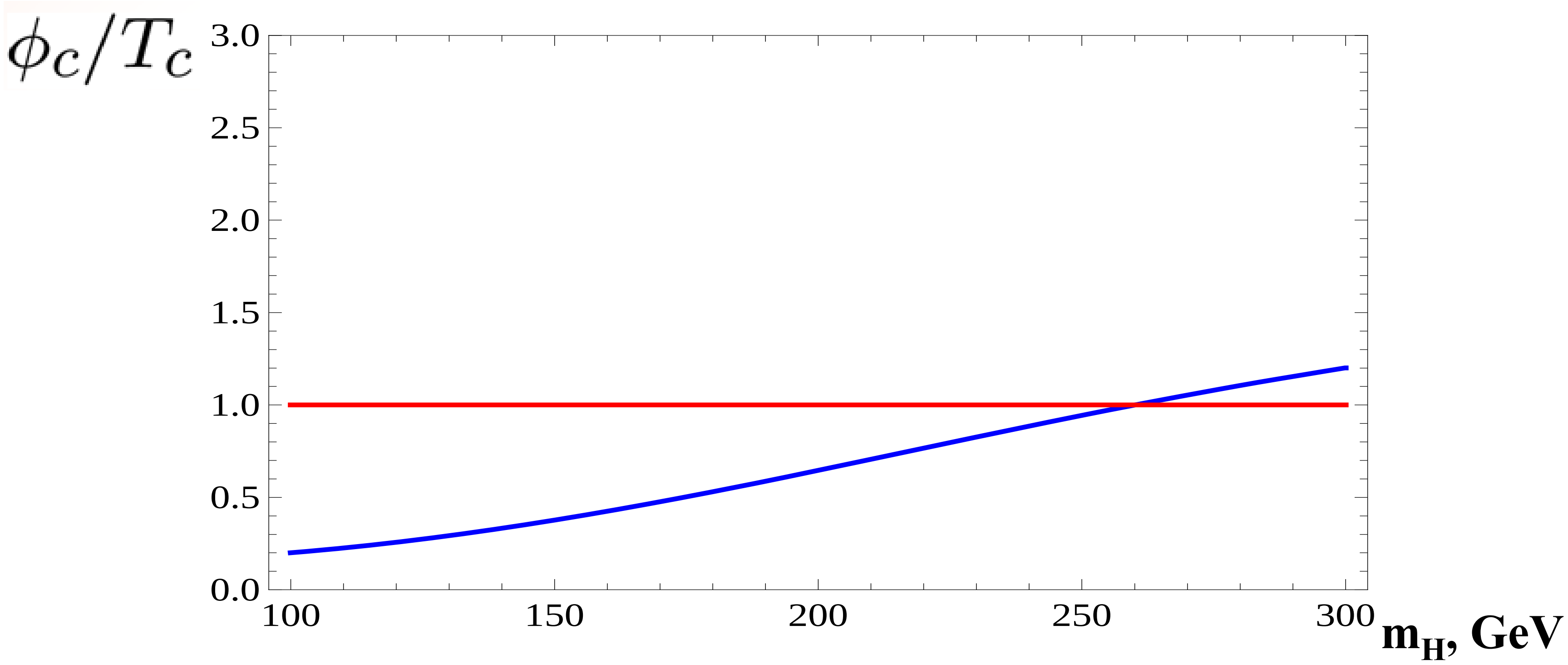
m_h in the THDM



$\phi_c/T_c \geq 1$ → **m_h<190 GeV**

LEP → **m_h>114.4 GeV**

m_H in the THDM



$\phi_c/T_c \geq 1$ → m_H>200 GeV

Summary

1. The potential of the THDM in general case is not CP-invariant and the parameters $\lambda_{5,6,7}$ of the two-doublet MSSM sector should be taken complex.

2. The deviations of the observable effects in the scenario with nondegenerated masses of the squark sector from the phenomenology of the standard scenario with degenerated scalar quarks masses can be substantial.

3. The deviations are large if the power terms $|\mu A_t|/M_{SUSY}$ are large and the charged Higgs boson mass does not exceed 150-200 GeV, being rather weakly dependent on $\tan\beta$.

4. Such models could lead to a reconsideration of some experimental properties for the signal of the Higgs boson production at the modern and future colliders.

Integration and summation method

A number of integrals can be easily calculated

$$J \equiv \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{1}{(\mathbf{k}^2 + a_1^2)(\mathbf{k}^2 + a_2^2)},$$

taking a residue in the spherical coordinate system:

$$J = \frac{1}{4\pi(a_1 + a_2)}$$

$a_{1;2}^2$ - the sums of squared frequency and squared mass.

Integration and summation method

Derivatives of first integral with respect to a_1 and a_2 can be used for calculation of integrals

$$J_1[a_1, a_2] \equiv \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{1}{(\mathbf{k}^2 + a_1^2)^2(\mathbf{k}^2 + a_2^2)} =$$

$$= -\frac{1}{2a_1}\frac{\partial I}{\partial a_1} = \frac{1}{8\pi a_1(a_1 + a_2)^2},$$

$$J_2[a_1, a_2] \equiv \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{1}{(\mathbf{k}^2 + a_1^2)^2(\mathbf{k}^2 + a_2^2)^2} =$$

$$= \frac{1}{4a_1 a_2}\frac{\partial^2 I}{\partial a_1 \partial a_2} = \frac{1}{8\pi a_1 a_2(a_1 + a_2)^3}.$$

Integration and summation method

and

$$J_3[a_1, a_2, a_3] \equiv \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{1}{(\mathbf{k}^2 + a_1^2)(\mathbf{k}^2 + a_2^2)(\mathbf{k}^2 + a_3^2)} =$$

$$= \frac{1}{4\pi(a_1 + a_2)(a_1 + a_3)(a_2 + a_3)},$$

$$J_4[a_1, a_2, a_3] \equiv \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{1}{(\mathbf{k}^2 + a_1^2)^2(\mathbf{k}^2 + a_2^2)(\mathbf{k}^2 + a_3^2)} =$$

$$= \frac{2a_1 + a_2 + a_3}{8\pi a_1(a_1 + a_2)^2(a_1 + a_3)^2(a_2 + a_3)}.$$

Integration and summation method

Substituting

$$a_1 \to \sqrt{4\pi^2 n^2 T^2 + m_1^2} \;\; и \;\; a_2 \to \sqrt{4\pi^2 n^2 T^2 + m_2^2},$$

for

$$J^n = \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{1}{(\mathbf{k}^2 + \omega_n^2 + m_1^2)(\mathbf{k}^2 + \omega_n^2 + m_2^2)},$$

taking the sum over Matsubara frequencies after the integration we get:

$$\sum_{n=-\infty, n\neq 0}^{\infty} J^n = \sum_{n=-\infty, n\neq 0}^{\infty} \frac{1}{4\pi(\sqrt{4\pi^2 n^2 T^2 + m_1^2} + \sqrt{4\pi^2 n^2 T^2 + m_2^2})}.$$

Thus the temperature corrections to effective potential are expressed by summed integrals,

Integration and summation method

after redefinition of mass parameters

$$m_{1;2} \longrightarrow m'_{1;2} = 2\pi T\sqrt{M_{1;2}^2 + n^2}, \quad \text{где } M_{1;2} = \frac{m_{1;2}}{2\pi T},$$

$$I_1 = \frac{-T}{8\pi}\frac{1}{(2\pi T)^3}\sum_{n=-\infty, n\neq 0}^{\infty} \frac{1}{\sqrt{M_1^2+n^2}(\sqrt{M_1^2+n^2}+\sqrt{M_2^2+n^2})^2},$$

$$I_2 = \frac{T}{8\pi}\frac{1}{(2\pi T)^5}\sum_{n=-\infty, n\neq 0}^{\infty} \frac{1}{\sqrt{M_1^2+n^2}\sqrt{M_2^2+n^2}(\sqrt{M_1^2+n^2}+\sqrt{M_2^2+n^2})^3}.$$

The sum of integrals can be expressed by means of the generalized zeta-function

Such forms can be derived if weintroduce Feynman parameters in the integrand of

$$\frac{1}{[\mathbf{k}^2+m_a^2][\mathbf{k}^2+m_b^2]}=\int_0^1\frac{dx}{([\mathbf{k}^2+m_a^2]x+[\mathbf{k}^2+m_b^2](1-x))^2},$$

and redefine

$$\mathbf{k}\longrightarrow\mathbf{p}=\frac{\mathbf{k}}{2\pi T}\quad \text{и}\quad M^2=(M_a^2-M_b^2)x+M_b^2,$$

then we get

$$\frac{1}{[\mathbf{k}^2+m_a^2][\mathbf{k}^2+m_b^2]}=\frac{1}{(2\pi T)^4}\int_0^1\frac{dx}{[\mathbf{p}^2+n^2+M^2]^2}.$$

The sum of integrals can be expressed by means of the generalized Hurwitz zeta-function

$$d\mathbf{k} = (2\pi T)^3 d\mathbf{p},$$

$$I' = \sum_{n=-\infty, n\neq 0}^{\infty} J = \frac{1}{2\pi T}\int_0^1 dx \sum_{n=-\infty, n\neq 0}^{\infty} \int \frac{d\mathbf{p}}{(2\pi)^3} \frac{1}{[\mathbf{p}^2 + n^2 + M^2]^2},$$

With the help of dimensional regularization or differentiating the integral

$$I' = \frac{1}{16\pi^2 T}\int_0^1 dx\, \zeta(2, \frac{1}{2}, M^2),$$

Where the generalized Hurwitz zeta-function

$$\zeta(u, s, t) = \sum_{n=1}^{\infty} \frac{1}{[n^u + t]^s}.$$

The sum of integrals can be expressed by means of the generalized Hurwitz zeta-function

$$I_1 = \frac{T}{2m_a}\frac{\partial}{\partial m_a}I' = -\frac{T}{8\pi}\frac{1}{(2\pi T)^3}\int_0^1 dxx\zeta(2,\frac{3}{2},M^2),$$

$$I_2 = \frac{1}{2m_b}\frac{\partial}{\partial m_b}(I_1) = \frac{3T}{8\pi}\frac{1}{(2\pi T)^5}\int_0^1 dxx(1-x)\zeta(2,\frac{5}{2},M^2).$$