Back reaction of accretion onto black hole

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Statement of contents

- We find corrections to the the Reissner-Nordström metric near the black hole horizon due to the back reaction of accreting fluid

- Metric of the accreting extreme black hole is radically influenced by back reaction

- Test fluid approximation is *violated* for extreme black holes: corrections to the metric are diverging near the event horizon in the limit $\mathbf{e^2 \to m^2}$

- Relation to the third law of black hole thermodynamics: accreting black hole withstands to the transformation into the naked singularity

Perfect fluid in the Reissner-Nordsröm metric

- Metric

$$\mathbf{ds^2 = f\,dt^2 - \frac{1}{f}\,dr^2 - r^2(d\theta^2 + \sin^2\theta\,d\phi^2)}$$

$$\mathbf{f \equiv 1 - \frac{2m}{r} + \frac{e^2}{r^2} = 0, \qquad r_\pm = m \pm \sqrt{m^2 - e^2}}$$

- Perfect fluid

$$\mathbf{T_{\mu\nu} = (\rho + p)u_\mu u_\nu - p g_{\mu\nu}, \quad u_\mu u^\mu = 1, \quad p = p(\rho)}$$

- $\mathbf{p = p(\rho)}$ – arbitrary function

Stationary spherically symmetric accretion

Integrals of motion for test fluid in the background metric

- (I) Energy conservation (Bernoulli equation)

$$T^{\mu\nu}{}_{;\nu} = 0 \;\Rightarrow\; (\rho + p)(f + u^2)^{1/2} r^2 u = C_1, \quad u = \frac{dr}{ds} < 0$$

- (II) Energy flux conservation

$$u_\mu T^{\mu\nu}{}_{;\nu} = 0 \;\Rightarrow\; u^\mu \rho_{,\mu} + (\rho + p) u^\mu{}_{;\mu} = 0$$

$$u r^2 \exp\left[\int_{\rho_\infty}^{\rho} \frac{d\rho'}{\rho' + p(\rho')}\right] = -A m^2, \quad A > 0$$

- (I/II)

$$(\rho + p)(f + u^2)^{1/2} \exp\left[-\int_{\rho_\infty}^{\rho} \frac{d\rho'}{\rho' + p(\rho')}\right] = -\frac{C_1}{A m^2} = \rho_\infty + p(\rho_\infty)$$

- Energy flux (rate of black hole mass changing)

$$\dot{m} = -4\pi r^2 T_0^r = 4\pi A m^2 [\rho_\infty + p(\rho_\infty)] = \text{const}$$

Adjustment of integration constants C_1 and A

Following to Michel 1972

- At infinity $r = \infty$

$$-\frac{C_1}{Am^2} = \rho_\infty + p(\rho_\infty)$$

- Critical points for transonic solution $r = r_*,\ \ c_{s*} = c_s(r_*)$

$$u_*^2 = \frac{mr_* - e^2}{2r_*^2}, \quad c_{s*}^2 = \frac{mr_* - e^2}{2r_*^2 - 3mr_* + e^2}, \quad c_s(\rho)^2 = \frac{\partial p}{\partial \rho}$$

$$\frac{r_*}{m} = \frac{1 + 3c_{s*}^2}{4c_{s*}^2} \left\{ 1 \pm \left[1 - \frac{8c_{s*}^2(1 + c_{s*}^2)}{(1 + 3c_{s*}^2)^2} \frac{e^2}{m^2} \right]^{1/2} \right\}$$

Analytic solution for linear equation of state

$$\boxed{p = \alpha(\rho - \rho_0)}$$

$$\alpha = \text{const} > 0, \quad \rho_0 = \text{const}, \quad \rho \geq 0, \quad \frac{p}{\rho} \equiv w \neq \text{const}$$

Advantage for $p < 0$: $c_s^2 = \frac{\partial p}{\partial \rho} = \alpha > 0$ – hydrodynamical stability

$$A = \alpha^{1/2} \frac{r_*^2}{m^2} \left(\frac{2\alpha r_*^2}{m r_* - e^2} \right)^{\frac{1-\alpha}{2\alpha}}$$

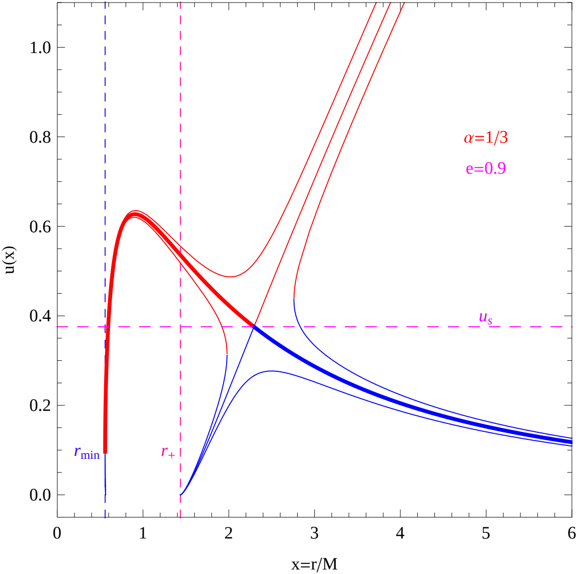
$$f + u^2 = \left(-\frac{u x^2}{A} \right)^{2\alpha}, \quad \frac{\rho + p}{\rho_\infty + p(\rho_\infty)} = \left(-\frac{A}{u x^2} \right)^{1+\alpha}$$

Analytical solutions $u = u(r) < 0$, $\rho = \rho(r)$ and $p = p(r)$ at $\alpha = 1/4,\ 1/3,\ 1/2,\ 2/3,\ 1$ and 2

α	1	$1/3$	$\alpha \to 0$
$A(e = 0, \alpha)$	4	$6\sqrt{3}$	$\frac{e^{3/2}}{4\alpha^{3/2}}$
$A(e = m, \alpha)$	1	$\frac{32}{3\sqrt{3}}$	$A \to \infty$

Accretion of thermal photon gas, $c_s^2 = \alpha = 1/3$

u(r) — radial 4-velocity



Self-consistent stationary accretion of test fluid

in the background metric

m — black hole masss
ρ_∞ — fluid energy density far from black hole

Small parameter $\rho_\infty m^2 \ll 1$
Test fluid approximation at $r \ll R_{max} = m(\rho_\infty m^2)^{-1/3}$
Back reaction is unimportant at $\rho_\infty m^2 \ll 1$, $r \ll R_{max}$

Accretion rate

$$\dot{m} = 4\pi m^2 A[\rho_\infty + p(\rho_\infty)]$$

$A = \text{const} > 0$, $\quad \dot{m} = \mathcal{O}(\rho_\infty m^2) \ll 1$

- $\dot{m} \ll 1$ is a small parameter
- Black hole mass is diminishing by phantom accretion (if $\rho + p < 0$)

Babichev Dokuchaev Eroshenko 2003

Approaching to extreme state $a^2 + e^2 \to m^2$ in test fluid approximation

$$e = \text{const}, \quad J = ma = \text{const}$$

Extreme state is reached in finite time t_{NS}

Reissner-Nordström case

$$\int_0^{t_{NS}} \dot{m}\, dt + m(0) = e, \quad t_{NS} = \frac{q^3 - 3q^2 + 2 - 2(1-q^2)^{3/2}}{3q^4}\, \tau$$

$$q = e/m(0), \quad \tau = -\{4\pi[\rho_\infty + p(\rho_\infty)]m(0)\}^{-1}$$

In the test fluid approximation a charged black hole is transformed by phantom accretion into the naked singularity in a finite time!

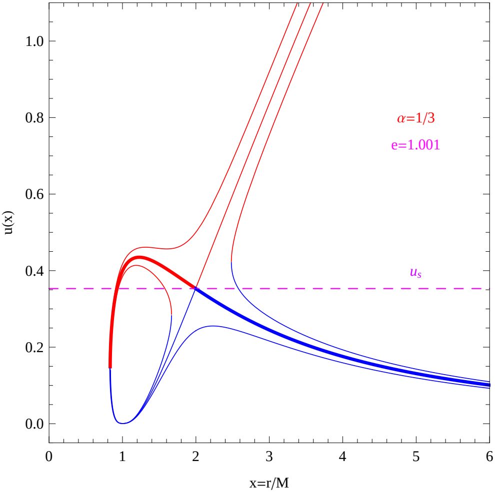
Violation of the Cosmic Censorship conjecture (R. Penrose) and the third law of black hole thermodynamics

What about the back reaction?

Hint for resolving the problem:

there are no solutions for stationary accretion onto the naked singularity

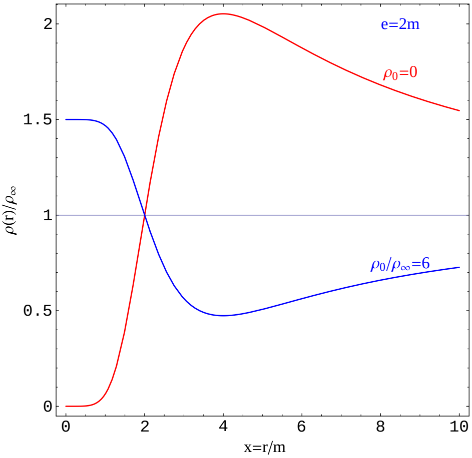
All possible hydrodynamical branches have bad behavior:
they do not reach the central singularity or "terminate" at finite radius



Instead of accretion, there are solutions with a zero influx:

Static atmosphere around Reissner-Nordström naked singularity

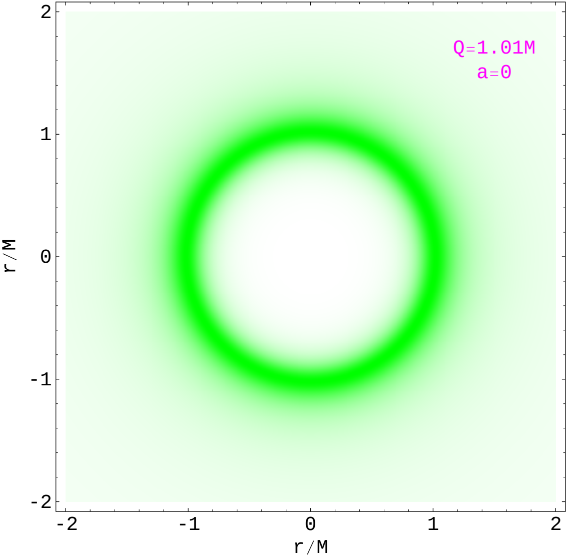
Thermal photon gas: $\rho_0 = 0$ **Phantom dark energy: $\rho_0/\rho_\infty = 6$**



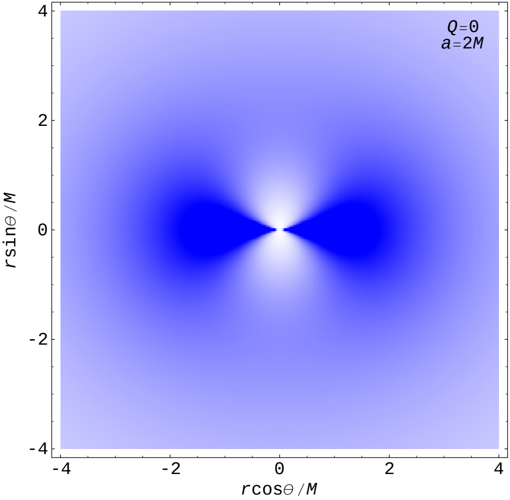
Energy density distribution of fluids with $c_s^2 = \alpha = 1/3$

Static atmosphere around Reissner-Nordström naked singularity

Density plot for ultra-hard fluid, $c_s^2 = \alpha = 1$

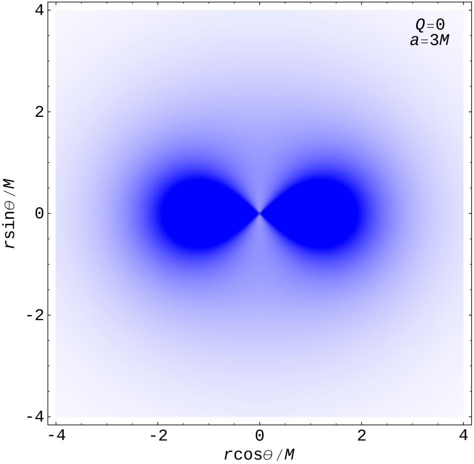


Ultra-hard fluid with a zero influx around the Kerr naked singularity



Density plot for energy density distribution $\rho = \rho(\mathbf{r}, \theta)$

Massless scalar field with a zero influx around the Kerr naked singularity



No perfect fluid analogue! Total mass of scalar field is finite:

$$\mathbf{M_f = \pi B^2(\pi + 2\,\mathrm{arccot}\,\epsilon)\,\epsilon m \ll m, \quad \epsilon = \sqrt{a^2 + e^2 - m^2}/m}$$

Back reaction of accreting fluid on the metric

Corrections to the background Reissner-Nodström metric

General spherically symmetric gravitational field

$$ds^2 = e^{\nu(r,t)}dt^2 - e^{\lambda(r,t)}dr^2 - r^2(d\theta^2 + \sin^2\theta\, d\phi^2)$$

Zero approximation at $\rho_\infty = 0$ $(\dot{m} = 0)$: $e^\nu = e^{-\lambda} = f$

We find corrections to the background in the form

$$e^\nu = g_{00} = f_0(r,t), \quad e^\lambda = g_{11} = f_1(r,t)^{-1}$$

$$f_{0,1} = 1 - \frac{2\mu_{0,1}(x,t)}{x} + \frac{e^2}{m^2(t)x^2}, \; x = \frac{r}{m(t)}, \; m(t) = m(0) + \int_0^t \dot{m}(t')dt'$$

$$\mu_{0,1}(x) = \frac{m_{0,1}(x,t)}{m(t)} = 1 + \mathcal{O}(\dot{m}), \quad \dot{m} \ll 1$$

Einstein Equations

$$8\pi T_0^1 = -e^{-\lambda}\frac{\dot\lambda}{r}$$

$$8\pi T_0^0 = -e^{-\lambda}\left(\frac{1}{r^2}-\frac{\lambda'}{r}\right)+\frac{1}{r^2}$$

$$8\pi T_1^1 = -e^{-\lambda}\left(\frac{1}{r^2}+\frac{\nu'}{r}\right)+\frac{1}{r^2}$$

$$8\pi T_2^2 = \frac{e^{-\nu}}{2}\left[\ddot\lambda+\frac{\dot\lambda}{2}\left(\dot\lambda-\dot\nu\right)\right]-\frac{e^{-\lambda}}{2}\left[\nu''+(\nu'-\lambda')\left(\frac{\nu'}{2}+\frac{1}{x}\right)\right]$$

$$T_0^1 = (\rho+p)u\sqrt{\frac{f_0}{f_1}(f_1+u^2)}, \qquad T_0^0 = \rho+(\rho+p)\frac{u^2}{f_1}+\frac{e^2}{8\pi r^4},$$

$$T_1^1 = -\left[(\rho+p)\frac{u^2}{f_1}+p\right]+\frac{e^2}{8\pi r^4}, \qquad T_2^2 = -p-\frac{e^2}{8\pi r^4}$$

We solve these equations near the event horizon with a linear to $\dot m \ll 1$ accuracy
by substituting perturbations $u(r)$, $\rho(r)$ and $p(\rho)$ from the stationary accretion solution

Back reaction in the Schwarzschild metric

Analytic solution near the event horizon, $x - x_+ \ll 1$

$$\mu_1(x) = \mu_+ + 2\dot{m}\log|1 - \frac{x - x_+}{4\dot{m}}|, \quad \mu_+ = \frac{x_+}{2}, \quad \dot{m} \ll \dot{m}\log|\dot{m}| \ll 1$$

or

$$x = x_+ + 4\dot{m}\left(1 + \frac{\mu_1 - \mu_+}{2\dot{m}} - \exp\frac{\mu_1 - \mu_+}{2\dot{m}}\right)$$

Correction to the event horizon radius: $x_+ - x_+^0 \ll 1, \quad x_+^0 = 2$

Radius of the modified horizon

(by using boundary conditions)

$$\boxed{x_+ = 2 + 4\dot{m}\log|\dot{m}|}$$

Corrections to metric far from horizon $\sim \dot{m} \ll 1$

(everywhere, except $x - x_+ \ll 1$)

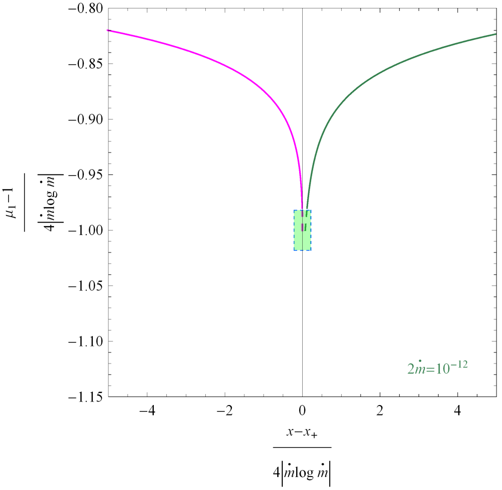
Corrections to metric near horizon $\sim \dot{m}\log\dot{m} \gg \dot{m}$

(in the narrow region near horizon $x - x_+ \ll 1$)

Correction to Schwarzschild metric near horizon

Analytical solution with back reaction

$$\mu_1(x) = \frac{x_+}{2} + 2\dot{m}\log\left|1 - \frac{x - x_+}{4\dot{m}}\right|, \qquad \dot{m} \ll \dot{m}\log|\dot{m}| \ll 1$$



Back reaction of accretion on the metric

Reissner-Nordström case

$x_+^0 = 1 + \epsilon$ — event horizon of the background metric
$\epsilon \equiv \sqrt{m^2 - e^2}/m$ — extremal parameter

Correction to the horizon radius $x_+ - x_+^0 \ll 1$:

$$x_+ = (1+\epsilon)\left[1 + \frac{1}{2}\left(\frac{1+\epsilon}{\epsilon}\right)^2 \dot{m} \log\left|\frac{\dot{m}}{\epsilon^2}\right|\right]$$

Divergence at $\epsilon \to 0$!

Main result: The test fluid approximation is violated for a near extreme black hole $(\epsilon \to 0)$ irrespective of e.o.s.!

Results and conclusions

- We find corrections to the Reissner-Nordström metric near the black hole horizon due to the back reaction of accreting fluid
- Universality: Back reaction near the event horizon depends only on the value of influx $\dot{m}$, not on the e.o.s.
- Test fluid approximation is *violated* for a near extreme black hole: Metric corrections are diverging near the event horizon in the limit $e^2 \to m^2$
- Extreme black hole anomaly:
  Back reaction radically transforms the metric of the accreting extreme black hole
- Hypothesis:
  The back reaction of accreting fluid on the geometry of near extreme black hole prevents its transformation to the naked singularity in accordance with the Cosmic Censorship conjecture $a^2 + e^2 \leq m^2$