A Static Black Hole in $4d$ Higher Spin Gauge Theory

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Plan

- Introduction.

  Motivation. Higher spin (HS) field theory. Black holes.

- HS equations in $d = 4$.

- Unfolded formulation for AdS_4 black hole.

- HS black hole.

- Conclusion.

Motivation

Gravity $s=2$	$\Longrightarrow$	Super-gravity	$\Longrightarrow$	Higher spin gauge theory (HS) $s = 0, 1/2, 1, \ldots, \infty$ **(Vasiliev)**

HS gauge theory:

Consistent theory of interacting massless fields $s = 0, 1/2, \ldots \infty$ in AdS space-time

Current status:

Formulated at the level of equations of motion for all spins in $d = 4$ Bosonic version is known in any d

String theory $\rightarrow$ **unbroken phase** $\rightarrow$ **HS theory** **(???)**

Lack of:

Action principle, quantization.

HS summary

- Field content: **All massless unitary representations**

- Nonlocal→ **all order space-time derivatives in interactions. requires dimensional parameter** $\lambda \to AdS$ **space-time. No flat limit with unbroken HS gauge symmetry.**

- Symmetries: **Infinite dimensional nonabelian HS algebra** $\to$ **mixes all spins. Maximal finite dimensional subalgebra leads to** $s \leq 2$

What we have

$d = 3$→ no degrees of freedom for HS fields $(s = 0\,, \quad s = 1/2)$

$d = 4$→ $\mathcal{N} = 2$ SUSY gauge theory of $s = 0, 1/2, 1, \ldots, \infty$

Truncations→ bosonic $s = 0, 1, \ldots, \infty$ or $s = 0, 2, 4, \ldots, \infty$

$d = d$→ only bosonic theory is available so far $(s = 0, 1, \ldots)$

(M.A. Vasiliev):

Full nonlinear field equations

— **background independence**

— **have only one dimensional constant Λ**

What has to be done:

— Action principle for quantization

— In d dimensions there are mixed symmetry fields to take into account

— Explicit relation to String Theory

— Mechanism of spontaneous symmetry breakdown

— Interpretation beyond AdS background

GR black holes → SUGRA black holes → HS black holes ???

Obstacles:

1. HS does not have decoupled spin-2 sector → all higher spins involved in the equations of motion.

2. The interval $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$ is not gauge invariant quantity in higher spin algebra.

3. Considerable technical difficulties – the equations are essentially non-local involving space-time derivatives of all orders.

Perturbative analysis available

HS theory → 0-th order vacuum AdS → 1-st order free field Fronsdal equations → 2-nd order interactions → . . .

Unfolded formulation

- First order coordinate independent differential equations (differential forms formalism)

- Assumes additional fields (generally infinitely many) that parameterize all on-shell derivatives of physical fields

Example: free massless scalar in Minkowski space-time $\Box\phi(x) = 0$

Unfolding $\rightarrow \varphi(x), \quad \varphi_\mu = \partial_\mu \varphi, \quad \varphi_{\mu\nu} = \partial_\mu \varphi_\nu, \ldots \quad \varphi_{\mu_1 \ldots \mu_n} = \partial_{\mu_1} \varphi_{\mu_2 \ldots \mu_n}$

Set of fields: $\phi, \quad \phi_\mu, \ldots \quad \varphi_{\mu_1 \ldots \mu_n}, \ldots$

Consistency condition: $\varphi_{\mu_1 \ldots \mu_n}$ – symmetric

Equations of motion: ${\varphi^\mu}_{\mu\mu_3 \ldots \mu_n} = 0 \Rightarrow \quad C(y|x) = \sum\limits_{n=0}^{\infty} \frac{1}{n!} \phi_{\mu_1 \ldots \mu_n} y^{\mu_1} \ldots y^{\mu_n} \Rightarrow$

$$dC = h^\mu \frac{\partial}{\partial y^\mu} C \,, \qquad h_\mu - \textbf{vierbein 1-form}$$

Structure of nonlinear higher spin equations in $d = 4$

$$\hat{d}A + A \star A = B \cdot d^2 Z \,,$$

$$\hat{d} = d_x + d_Z \,, \qquad A = W_\mu(Y,Z|x)dx^\mu + S_A(Y,Z|x)dZ^A \,,$$

$W_\mu(Y,Z|x)$ – **higher spin potentials**

$S_A(Y,Z|x)$ – **auxiliary field encoding interactions**

$B(Y,Z|x)$ – **higher spin curvature**

$$\Phi(Y,Z|x) = \sum_{m,n=0}^{\infty} \frac{1}{m!n!} \Phi_{A_1 \dots A_n, B_1 \dots B_m} Y^{A_1} \dots Y^{A_n} Z^{B_1} \dots Z^{B_m} \,.$$

Vacuum:

$$S_A = 0 \,, \qquad B = 0 \,, \qquad W_\mu = \omega_\mu^{AB} Y_A Y_B \,.$$

HS ingredients

- **Star-product operation**

Let $Y_A = (y_\alpha, \bar{y}_{\dot{\alpha}})$ and $Z_A = (z_\alpha, \bar{z}_{\dot{\alpha}})$ be commuting variables.

$$(f \star g)(Y,Z) = \int f(Y+s, Z+s) g(Y+t, Z-t) e^{s_A t^A} \longrightarrow$$

associative algebra with

$$[Z_A, Z_B]_\star = -[Y_A, Y_B]_\star = 2\epsilon_{AB}\,, \qquad [Y_A, Z_B]_\star = 0$$

Klein operators $\longrightarrow$

$v = \exp\left(z_\alpha y^\alpha\right)$, $\quad \bar{v} = \exp\left(\bar{z}_{\dot{\alpha}} \bar{y}^{\dot{\alpha}}\right)$ with the following properties

$$v \star v = \bar{v} \star \bar{v} = 1\,, \qquad v \star f(y,z) = f(-y,-z) \star v\,, \qquad \bar{v} \star f(\bar{y},\bar{z}) = f(-\bar{y},-\bar{z}) \star \bar{v}$$

• AdS_4 vacuum

Introduce 1-form $w_0 \in o(3,2) \sim sp(4)$

$$w_0 = -\frac{1}{8}(\omega_{\alpha\alpha} y^\alpha y^\alpha + \bar{\omega}_{\dot{\alpha}\dot{\alpha}} \bar{y}^{\dot{\alpha}} \bar{y}^{\dot{\alpha}} - 2\lambda h_{\alpha\dot{\alpha}} y^\alpha \bar{y}^{\dot{\alpha}}) \,, \qquad dw_0 - w_0 \star \wedge w_0 = 0$$

Equiv. to

$$d\omega_{\alpha\alpha} + \frac{1}{2}\omega_\alpha{}^\gamma \wedge \omega_{\gamma\alpha} = \frac{\lambda^2}{2} h_{\alpha\dot{\gamma}} \wedge h_\alpha{}^{\dot{\gamma}} \qquad \rightarrow \qquad AdS_4 \text{ Riemann tensor}$$

$$dh_{\alpha\dot{\alpha}} + \frac{1}{2}\omega_\alpha{}^\gamma \wedge h_{\gamma\dot{\alpha}} + \frac{1}{2}\bar{\omega}_{\dot{\alpha}}{}^{\dot{\gamma}} \wedge h_{\alpha\dot{\gamma}} = 0 \qquad \rightarrow \qquad \text{zero torsion}$$

- **Free HS equations**

HS field strengths $B(Y,Z|x) = C(y,\bar{y}|x) = \sum_{n,m=0}^{\infty} \frac{1}{n!m!} C_{\alpha(n),\dot{\alpha}(m)} y^{\alpha} \dots y^{\alpha} \bar{y}^{\dot{\alpha}} \dots \bar{y}^{\dot{\alpha}}$

HS potentials $W(Y,Z|x)_{|Z=0} \to w(y,\bar{y}|x) = \sum_{n,m=0}^{\infty} \frac{1}{n!m!} w_{\alpha(n),\dot{\alpha}(m)} y^{\alpha} \dots y^{\alpha} \bar{y}^{\dot{\alpha}} \dots \bar{y}^{\dot{\alpha}}$

Equations of motion:

$$\tilde{\mathcal{D}}_0 C \equiv dC - w_0 \star C + C \star \tilde{w}_0 = 0 \qquad \leftarrow \textbf{twisted-adjoint}$$

$$\mathcal{D}_0 w \equiv dw - [w_0, w] = h \wedge h\, C \qquad \leftarrow \textbf{adjoint}$$

where $\tilde{f}(y,\bar{y}) = f(-y,\bar{y})$ ← **twist operator**

matter fields: scalar $s=0 \to C(x)$, fermion $s=1/2 \to C_{\alpha}(x) \oplus \bar{C}_{\dot{\alpha}}(x)$

HS fields: potentials $\to \omega_{\alpha(s-1),\dot{\alpha}(s-1)}$, strengths$\to$ $C_{\alpha(2s)} \oplus \bar{C}_{\dot{\alpha}(2s)}$

Classical black hole properties

Ex. $d=4$ Kerr Solution

1. $g_{\mu\nu} = \eta_{\mu\nu}(x) + M h_{\mu\nu}(x)$ – no $O(M^2)$ terms $\Longrightarrow$ Einstein equations reduce to **free** $s=2$ Pauli-Fierz eqs.

$$\Box h_{\mu\nu} - \partial_\mu \partial_\lambda h^\lambda{}_\nu - \partial_\nu \partial_\lambda h^\lambda{}_\mu = 0 \qquad (h_\mu{}^\mu = 0)$$

2. $h_{\mu\nu} = \frac{1}{U(x)} k_\mu(x) k_\nu(x)$ – factorized form. k^μ – Kerr-Schild vector

$$k_\mu k^\mu = 0\,, \qquad k^\mu D_\mu k_\nu = k^\mu \partial_\mu k_\nu = 0$$

3. BH provides Fronsdal fields $\phi_{\mu_1 \dots \mu_s} = \frac{M}{U} k_{\mu_1} \dots k_{\mu_s}$

$\mathrm{s}=0 \Longrightarrow$	Klein-Gordon	$\Box\phi = 0$
$\mathrm{s}=1 \Longrightarrow$	Maxwell	$\Box\phi_\mu - \partial_\lambda \partial_\mu \phi^\lambda = 0$
$\mathrm{s}=2 \Longrightarrow$	Pauli-Fierz	$\Box\phi_{\mu\nu} - 2\partial_\lambda \partial_{(\mu} \phi_{\nu)}{}^\lambda = 0$
$\mathrm{s}=\mathrm{s} \Longrightarrow$	Fronsdal	$\Box\phi_{\mu_1 \dots \phi_{\mu_s}} - s\partial_\lambda \partial_{(\mu_1} \phi^\lambda{}_{\mu_2 \dots \mu_s)} = 0$

4. Kerr-Schild presentation is also valid in AdS

$$g_{\mu\nu} = \eta^{AdS}_{\mu\nu}(x) + \frac{M}{U} k_\mu k_\nu \,, \quad k^\mu k_\mu = 0 \,, \quad k^\mu \mathcal{D}_\mu k_\nu = k^\mu D_\mu k_\nu = 0$$

Just as well,

$$\phi_{\mu_1 \ldots \mu_s} = \frac{M}{U} k_{\mu_1} \ldots k_{\mu_s}$$

satisfies free massless spin-s equations (**Metsaev**) in AdS_4

$$\Box \phi_{\mu_1 \ldots \mu_s} - s D_\lambda D_{(\mu_1} \phi^\lambda{}_{\mu_2 \ldots \mu_s)} = -2(s-1)(s+1)\lambda^2 \phi_{\mu_1 \ldots \mu_s}$$

$\phi_{\mu_1 \ldots \mu_s}(x)$ – **Black hole massless fields**

Program for HS black holes

vacuum AdS_4	$\longrightarrow$	black hole massless fields $\phi_{\mu_1 \ldots \mu_s}(x)$	$\longrightarrow$	HS corrections	$\longrightarrow$	$\ldots$

Difficulty: To perform perturbative analysis of black holes in HS theory one has to have explicit expressions for all AdS_4 derivatives $D \ldots D(\phi_{\mu_1 \ldots \mu_n})$.

Strategy

1. **Find unfolded, AdS_4 covariant, coordinate free description of a black hole (arxiv: 0801.2213, 0901.2172 [hep-th])**

2. **Choose AdS_4 black hole with its generalization for the rest of spins as the first order solution in HS perturbative expansion**

3. **Calculate HS nonlinear corrections**

AdS_4 space-time (A)

$ds^2 = -du^2 - dv^2 + dx^2 + dy^2 + dz^2 \,, \qquad -u^2 - v^2 + x^2 + y^2 + z^2 = \lambda^{-2}$

Isometries: $o(3,2) \Longrightarrow$ 10 Killing vectors. Let V^a be an AdS_4 Killing vector:

$$D_a V_b + D_b V_a = 0 \,, \qquad \kappa_{ab} = D_a V_b = -\kappa_{ba}$$

(Ricci identity: $D_a D_b V_c = R^d{}_{abc} V_d$**)**

$$DV_a = \kappa_{ab} h^b \,, \quad D\kappa_{ab} = \lambda^2 (V_a h_b - V_b h_a) \longleftarrow \textbf{unfolded equations}$$

$$d\Omega_{ab} + \Omega_a{}^c \wedge \Omega_{cb} = \lambda^2 h_a \wedge h_b \,, \quad dh_a + \Omega_a{}^b \wedge h_b = 0 \longleftarrow \textbf{consistency, } AdS_4$$

Spinor form for AdS_4 equations:

$$DV_{\alpha\dot\alpha} = \frac{1}{2} h^{\gamma}{}_{\dot\alpha} \kappa_{\gamma\alpha} + \frac{1}{2} h_{\alpha}{}^{\dot\gamma} \bar\kappa_{\dot\alpha\dot\gamma}$$

$$D\kappa_{\alpha\alpha} = \lambda^2 h_{\alpha}{}^{\dot\gamma} V_{\alpha\dot\gamma}\,, \qquad D\bar\kappa_{\dot\alpha\dot\alpha} = \lambda^2 h^{\gamma}{}_{\dot\alpha} V_{\gamma\dot\alpha}\,.$$

Properties of the system (A)

AdS_4 **covariant form**

$$K_{AB} = K_{BA} = \begin{pmatrix} \lambda^{-1}\kappa_{\alpha\beta} & V_{\alpha\dot\beta} \\ V_{\beta\dot\alpha} & \lambda^{-1}\bar\kappa_{\dot\alpha\dot\beta} \end{pmatrix}, \qquad \Omega_{AB} = \Omega_{BA} = \begin{pmatrix} \omega_{\alpha\beta} & -\lambda h_{\alpha\dot\beta} \\ -\lambda h_{\beta\dot\alpha} & \bar\omega_{\dot\alpha\dot\beta} \end{pmatrix} =$$

$$D_0 K_{AB} = 0\,, \qquad D_0^2 \sim R_{0AB} = d\Omega_{AB} + \frac{1}{2}\Omega_A{}^C \wedge \Omega_{CB} = 0\,.$$

K_{AB} – AdS_4 **global symmetry parameter**

Deformation of $AdS_4 \to$ black hole unfolded system (B)

$$D_0 K_{AB} = 0 \quad \to \quad DK_{AB} = f(K_{AB}|\mathcal{M}, \mathbf{q})$$

It leads to

$$g_{\mu\nu} = \eta^{AdS}_{\mu\nu} + f_{\mu\nu}(K_{AB}|M, \bar{M}, e, g)\,.$$

Curvature 2-form is given by

$$\mathcal{R}_{\alpha\alpha} = \frac{\lambda^2}{2}\mathbf{H}_{\alpha\alpha} - \frac{3(\mathcal{M} - \mathbf{q}\,\bar{\mathcal{G}})}{4\mathcal{G}}\mathbf{H}^{\beta\beta}\mathcal{F}_{(\beta\beta}\mathcal{F}_{\alpha\alpha)} + \frac{\mathbf{q}}{4}\,\bar{\mathbf{H}}^{\dot\beta\dot\beta}\bar{\mathcal{F}}_{\dot\beta\dot\beta}\mathcal{F}_{\alpha\alpha}, \qquad \mathbf{H}_{\alpha\alpha} = h_\alpha{}^{\dot\alpha}\wedge h_{\alpha\dot\alpha}$$

AdS_4-Kerr-Newman-Taub-NUT black hole (rotated, EM and NUT-charged)

M=Re$\mathcal{M}$ – black hole mass

N=Im$\mathcal{M}$– NUT charge

$\mathbf{q} = e^2 + \mathrm{g}^2$ – sum of squared electric and magnetic charges

Integrating flow (A)⇔(B)

Let the deformation parameters $\chi = (\mathcal{M}, \bar{\mathcal{M}}, \mathbf{q})$ run $\Rightarrow$ one has corresponding flows $\frac{\partial}{\partial\chi}$. **Applying the integrability conditions to (B):**

$$[d, \frac{\partial}{\partial\chi}] = [\frac{\partial}{\partial\chi}, \frac{\partial}{\partial\chi'}] = 0$$

Unfolded formulation of AdS_4 black hole

Let $K_{AB}(x) = K_{BA}(x)\,, \quad A = (\alpha, \dot\alpha) = 1 \ldots 4$ be an AdS_4 global symmetry parameter

$$D_0 K_{AB} = 0\,, \qquad D_0^2 = 0$$

Lorentz components $\longrightarrow$ $K_{AB} = K_{BA} = \begin{pmatrix} \lambda^{-1}\kappa_{\alpha\beta} & V_{\alpha\dot\beta} \\ V_{\beta\dot\alpha} & \lambda^{-1}\bar\kappa_{\dot\alpha\dot\beta} \end{pmatrix}$

$4d$ Kerr black hole: $\quad g_{\mu\nu} = \eta^{AdS}_{\mu\nu} + \frac{2M}{U} k_\mu k_\nu$

$$\frac{2}{U} = \frac{\lambda^2}{\sqrt{-\kappa^2}} + \frac{\lambda^2}{\sqrt{-\bar\kappa^2}}\,, \qquad k_{\alpha\dot\alpha} = \frac{1}{(V^- V^+)} V^-_{\alpha\dot\alpha}\,, \quad \text{where} \quad V^{\pm}_{\alpha\dot\alpha} = \pi^{\pm}_{\alpha}{}^{\gamma} \bar\pi^{\pm}_{\dot\alpha}{}^{\dot\gamma} V_{\gamma\dot\gamma}$$

and

$$\pi^{\pm}_{\alpha\beta} = \frac{1}{2}\Big(\epsilon_{\alpha\beta} \pm \frac{\kappa_{\alpha\beta}}{\sqrt{-\kappa^2}}\Big)\,, \qquad \kappa^2 = \frac{1}{2}\kappa_{\alpha\beta}\kappa^{\alpha\beta}$$

Einstein equations satisfied for any $K_{AB}(x)$

Type of a black hole (M – real)

1. $K_A{}^C K_C{}^B \neq C\delta_A{}^B$ – **stationary (Kerr)**

2. $K_A{}^C K_C{}^B = C\delta_A{}^B$ – **static (Schwarzschild)** $\Longrightarrow$

Black holes from the free HS theory

$K_{AB}(x) \to AdS_4$ **global symmetry parameter** $(D_0 K_{AB} = 0) \Longrightarrow$

$$\mathcal{D}_0 f(K_{AB} Y^A Y^B) = 0$$

generates solution of the twisted-adjoint module

$$C(y, \bar{y}|x) = M f(K_{AB} Y^A Y^B) \star \delta(y) \qquad \leftarrow \qquad \textbf{complex solution}$$

$C(y, \bar{y}|x)$ **describes two** AdS_4**-Kerr-Taub-NUT black holes in** $s = 2$ **sector and HS generalization for the rest of integer spins** **(Didenko, Matveev, Vasiliev)**

$$m \sim \mathbf{Re} M\,, \qquad n \sim \mathbf{Im} M$$

Schwarzschild case $\to M$ - real, $K_A{}^C K_C{}^B = \delta_A{}^B$

Solving nonlinear HS equations

Main idea: The function $f = F_K = \exp\left(\frac{1}{2}K_{AB}Y^A Y^B\right)$ generates invariant subspace in the star-product algebra and provides suitable ansatz for solving nonlinear HS equations

Properties of F_K

1. $F_K \star F_K = F_K\,, \quad Y_{-A} \star F_K = F_K \star Y_{+A} = 0\,,\ \ Y_{\pm A} = \Pi_{\pm A}{}^C Y_C = \frac{1}{2}(\delta_A{}^B \pm K_A{}^B)Y_B \quad \leftarrow$ **Fock vacuum projector**
2. $\mathcal{D}_0 F_K = 0 \quad \leftarrow$ **by definition**
3. **Generates subalgebra of the form** $F_K\phi(a|x)$, where $a_A = Z_A + K_A{}^B Y_B$

$$(F_K\phi_1(a|x)) \star (F_K\phi_2(a|z)) = F_K(\phi_1(a|x) * \phi_2(a|x))$$

$*$ - is Fock induced associative star-product operation on the space of a_A - oscillators

4. $\mathcal{D}_0(F_K\phi(a|x)) = F_K(\hat{d} - \frac{1}{2}dK^{AB}\frac{\partial^2}{\partial a^A \partial a^B})\phi(a|x)\,, \qquad \hat{d}a_A = 0$

Final solution

$$S_\alpha = z_\alpha + M F_K \frac{a^{+}_{\alpha}}{r} \int_0^1 \mathrm{dt} \exp\left(\frac{t}{2} \kappa^{-1}_{\beta\beta} a^{\beta} a^{\beta}\right),$$

$$\bar{S}_{\dot\alpha} = \bar{z}_{\dot\alpha} + M F_K \frac{\bar{a}^{+}_{\dot\alpha}}{r} \int_0^1 \mathrm{dt} \exp\left(\frac{t}{2} \bar\kappa^{-1}_{\dot\beta\dot\beta} \bar{a}^{\dot\beta} \bar{a}^{\dot\beta}\right),$$

$$B = \frac{M}{r} \exp\left(\frac{1}{2} \kappa^{-1}_{\alpha\beta} y^{\alpha} y^{\beta} + \frac{1}{2} \bar\kappa^{-1}_{\dot\alpha\dot\beta} \bar{y}^{\dot\alpha} \bar{y}^{\dot\beta} - \kappa^{-1}_{\alpha\gamma} v^{\gamma}{}_{\dot\alpha} y^{\alpha} \bar{y}^{\dot\alpha}\right),$$

$$W = W_0 + \left(\frac{M}{8r} F_K d\tau^{\alpha\beta} \pi^{+}_{\beta}{}^{\alpha} a_\alpha a_\alpha \int_0^1 dt (1-t) \exp\left(\frac{t}{2} \kappa^{-1}_{\beta\beta} a^{\beta} a^{\beta}\right) + F_K f_0 + c.c.\right),$$

Symmetries

Let $\epsilon(Z,Y|x)$ be a global symmetry parameter $\rightarrow$

$$B\star\epsilon-\tilde{\epsilon}\star B=0\,,\qquad [S,\epsilon]_\star=0\,,\qquad d\epsilon-[W,\epsilon]_\star=0\qquad\Rightarrow$$

$$[F_K,\epsilon]_\star=0$$

$$\epsilon(Y|x)=\sum_{m,n=1}^{\infty} f_{0A(m),B(n)}(x)\,\underbrace{Y_+^A\star\ldots\star Y_+^A}_{m}\star\underbrace{Y_-^B\star\ldots\star Y_-^B}_{n}+c_0(x)=:f(Y_-,Y_+):+c_0,\qquad \mathcal{D}_0 f=0$$

Max. finite dimensional subalgebra: $T^{AB}=Y_+^{(A}Y_-^{B)}$, $T=Y_{-A}Y_+^A$ $\quad\rightarrow su(2)\oplus u(1)$

More (Supersymmetry)! Global SUSY is a quarter of the $\mathcal{N}=2$ SUSY with two supergenerators $\mathcal{Q}_A^\alpha$ of the AdS_4 vacuum.

Vacuum AdS_4 symmetry algebra $osp(2,4)$ is broken giving BPS HS black hole

Conclusion

- The new exact solution of $4d$ bosonic HS theory is presented. It is constructed to correspond to Schwarzschild black hole in spin two sector and to natural generalization for the rest of Fronsdal fields at free level and is called HS black hole. The HS corrections cancel each other (in a specific gauge) for this solution reducing HS nonlinear equations to free ones.

- The crucial element of the proposed construction is a Fock vacuum in the star–product algebra which is available for static black hole. It allows us to project $4d$ equations to $3d$ ones upon inducing the inner star–product operation. This reduction suggests an interesting duality between AdS_3 massive HS vacuum of a mass scale ν and $4d$ HS black hole of mass M

$$\nu = \lambda G M$$

- The construction admits natural generalization for the Kerr and supersymmetric cases as well as for higher dimensions (black rings??)