Soliton-antisoliton pair production in collisions of high–energy particles

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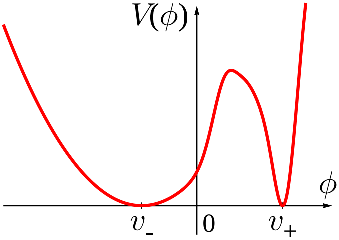
Kink-like solitons in (1+1) scalar field theories

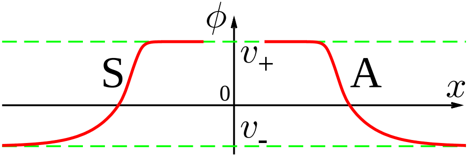
$\boxed{\phi}$ $(1+1)$ $\hbar = c = 1$



$$S = \frac{1}{g^2}\int dxdt \left[(\partial_\mu \phi)^2/2 - V(\phi)\right]$$

g — semiclassical parameter
and coupling constant ($\phi = g\phi'$)

Soliton and Antisoliton

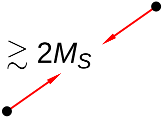


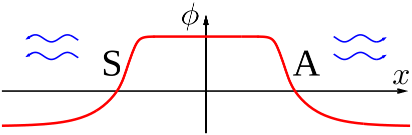
Properties: $L_S \sim m^{-1}$
$M_S \sim m/g^2$

m — mass scale of $V(\phi)$

Production of soliton-antisoliton pair

$E \gtrsim 2M_S$





$\lambda \sim 1/E \sim g^2/m$

$L_S \sim 1/m$

Exponential suppression!

$$\mathcal{P}(E) \approx A(E) \cdot \mathrm{e}^{-F(E)/g^2}$$

- Coherent–state “estimate”

  $\bar{n}_S \sim M_S/m \sim 1/g^2$ — *Drukier, Nussinov (1982)*

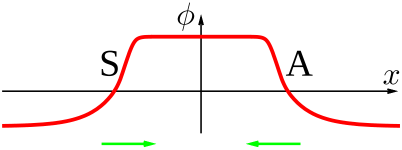
  $\langle 2|SA\rangle \sim \frac{\bar{n}_S^2}{2!}\mathrm{e}^{-\bar{n}_S} \sim \mathrm{e}^{-c/g^2}$ — *Banks at al. (1990)*

- Unitarity arguments for multiparticle production — *Zakharov (1991)*

No reliable estimate of $\mathcal{P}(E)$ so far!

Aim: calculate semiclassically $F(E)$.

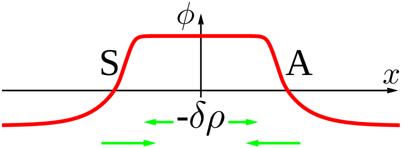
HOWTO calculate: NOT tunneling?!

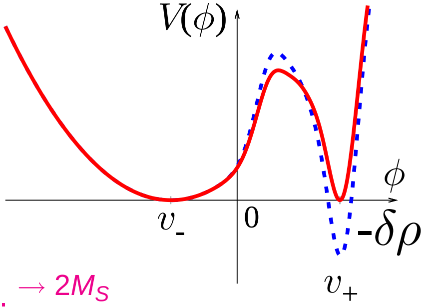


Attraction!

$\Rightarrow$ Many particles

Not a tunneling process? Introduce a potential barrier:





Critical bubble $\Leftrightarrow$ Barrier top

$\delta\rho \longrightarrow 0$: cr. bubble $\longrightarrow$ SA; $E_{\text{cr.b.}} \to 2M_S$

HOWTO calculate: In–state

Rubakov, Son, Tinyakov, 1992

Not semiclassical!

$$E \sim m/g^2$$

RST conjecture: $F(E)$ universal

Does not depend on details of the in–state

$$\mathcal{P}(E,N) = \sum_{i,f} \left| \langle i | \hat{P}_E \hat{P}_N \hat{S} | f \rangle \right|^2$$

Projectors

- $N \gg 1 \Rightarrow$ semiclassical in–states
- $N \ll 1/g^2 \Rightarrow F(E,N) \approx F(E)$

Checks of universality:

- Field theory

  Tinyakov, 1991

  Mueller, 1992

- Toy QM models

  Bonini et al, 1999

  Levkov et al, 2009

$$F(E) = \lim_{g^2 N \to 0} F(E,N)$$

HOWTO calculate: Semiclassical method

Rubakov, Son, Tinyakov, 1992

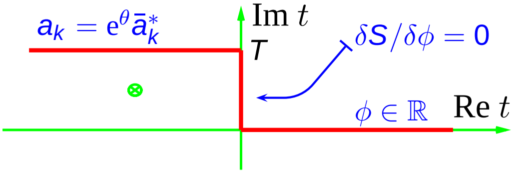
$$\mathcal{P}(E,N) = \sum_{i,f} \left| \langle i | \hat{P}_E \hat{P}_N \hat{S} | f \rangle \right|^2$$

$$\langle i | \hat{P}_E \hat{P}_N \hat{S} | f \rangle = \sum_{i'} \langle i | \hat{P}_E \hat{P}_N | i' \rangle \langle i' | \hat{S} | f \rangle = \int d\phi_{i'} \mathrm{e}^{iB(\phi_i,\phi_{i'})} \int [d\phi] \mathrm{e}^{iS[\phi]}$$

$$\mathcal{P}(E,N) = \int [d\phi][d\phi'] \, \mathrm{e}^{iW}$$

$$W \propto 1/g^2 \Rightarrow \text{Saddle–point method!}$$

Boundary value problem



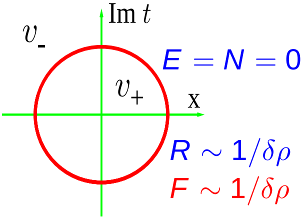
$$\phi(x,t) \in \mathbb{C}$$

$$\mathcal{P} = A \cdot \mathrm{e}^{-F/g^2} \qquad \frac{1}{g^2} F(E,N) = 2\mathrm{Im}S[\phi] - TE - \theta N$$

Outline of the procedure

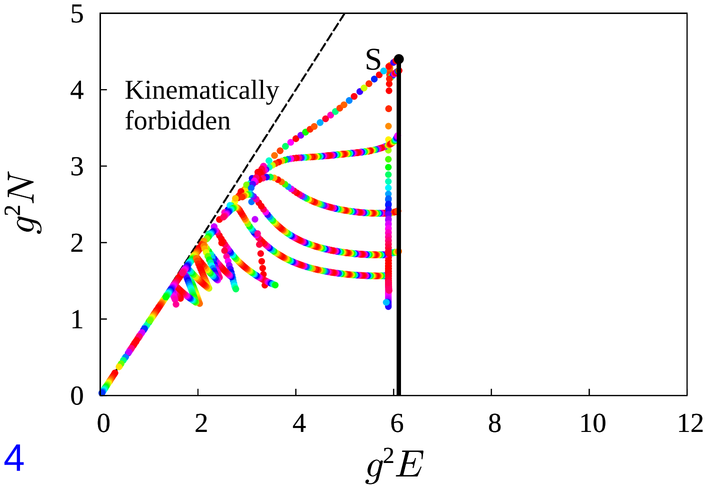
1. Fix small value $\delta\rho$
2. Solve boundary value problem for fixed E and N and find $F(E,N)$
3. Take the limit $\lim_{g^2N\to 0} F(E,N)$ at nonzero $\delta\rho$
4. Take the limit $\delta\rho \to 0$

Starting point, solution at $E = 0$ and $N = 0$ — bounce



Numerical solutions

$$V(\phi) = \frac{1}{2}(\phi+1)^2 \left[1 - v \cdot f\left(\frac{\phi-1}{a}\right)\right], \qquad f(x) = e^{-x^2}\left(1 + x^3 + x^5\right)$$

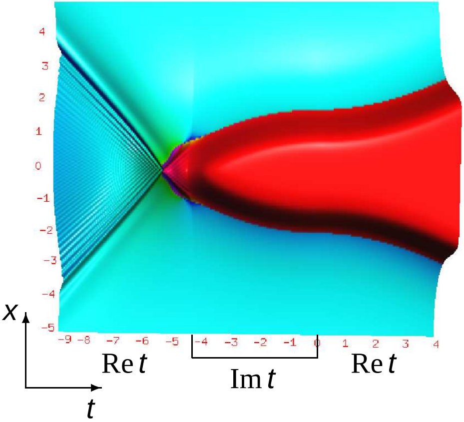


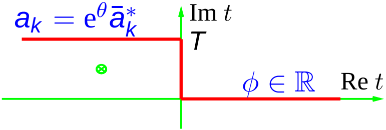
$\delta\rho = 0.4$

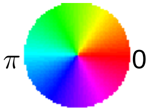
$E < 2M_S$, direct tunneling

$E \approx 5.48$ $(2M_S \approx 6.23)$

$N \approx 2.39$, $\delta\rho = 0.4$







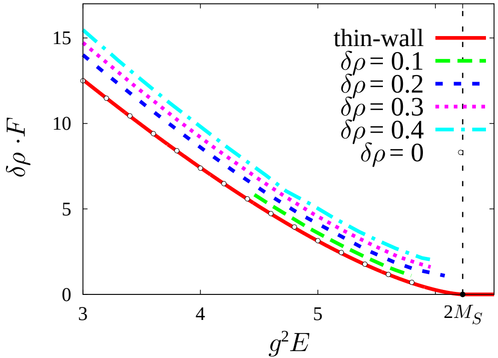
$\delta\rho \to 0$: thin–wall limit!

$$F(\delta\rho) = F_{-1}/\delta\rho + F_0 + O(\delta\rho)$$

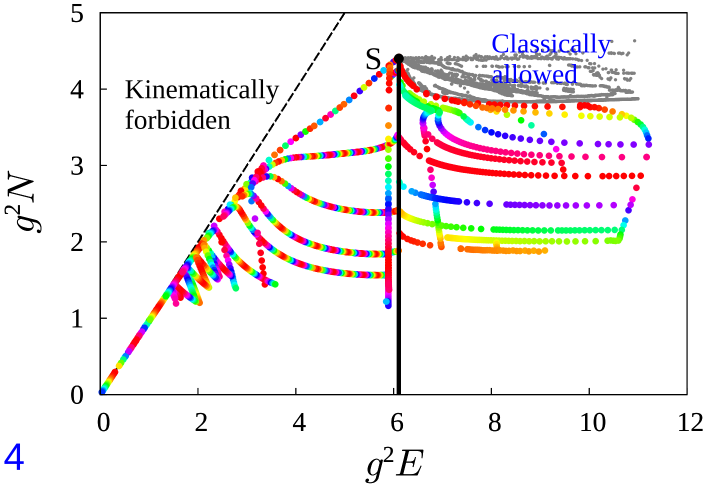
Voloshin, Selivanov, 1986

Rubakov et al, 1991

$$F_{-1}(E, N) = E_S^2 \left(\pi - 2\arcsin\frac{E}{2E_S} - \frac{E}{E_S}\sqrt{1 - \frac{E^2}{4E_S^2}} \right)$$



Going to $E > 2M_S$

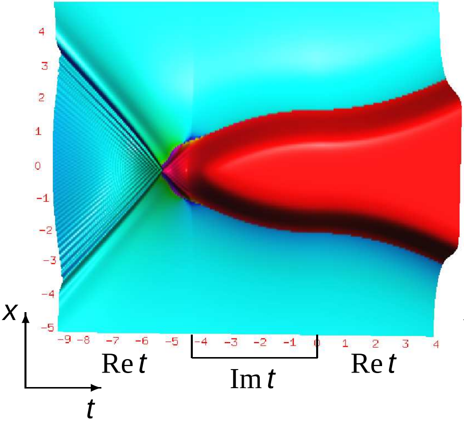


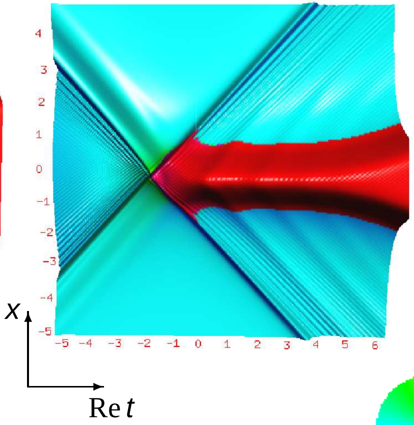
$\delta\rho = 0.4$

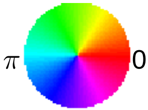
Going to $E > 2M_S$

$E \approx 5.48$ $(2M_S \approx 6.23)$ $E \approx 9.06$

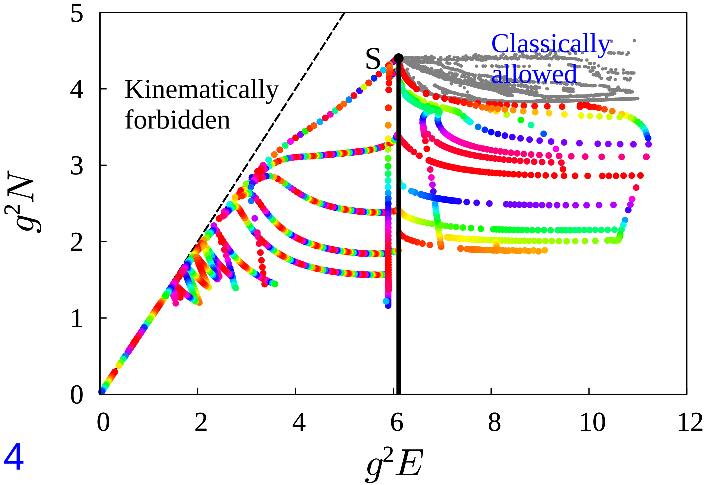
$N \approx 2.39,\ \delta\rho = 0.4$ $N \approx 2.47,\ \delta\rho = 0.4$







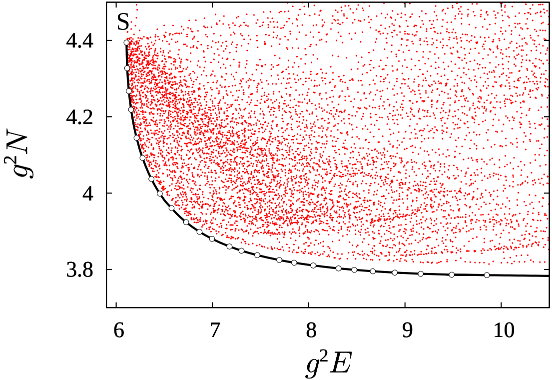
Extrapolating to $N \to 0$



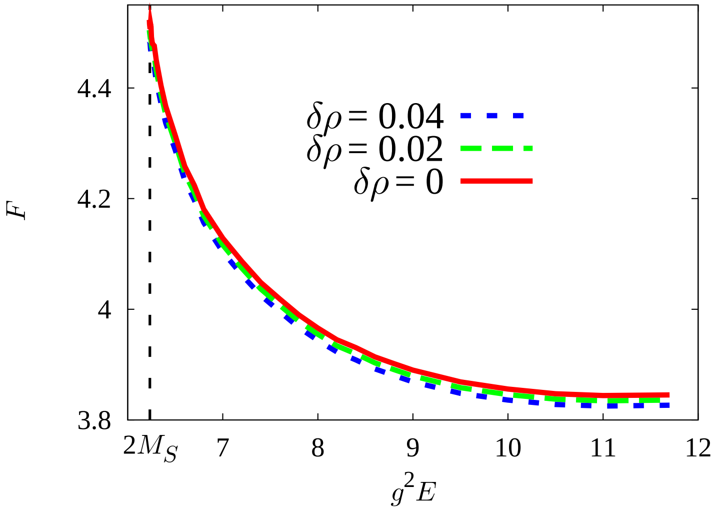
$\delta\rho = 0.4$

Classically allowed region

1. $\phi_0(x) = \phi_{cr.bubble}(x) + \delta\phi(x)$
2. Evolution according to classical field equations
3. Find N in asymptotic future



Result



$\delta\rho = 0.04$
$\delta\rho = 0.02$
$\delta\rho = 0$
F
4.4
4.2
4
3.8
$2M_S$
7
8
9
10
11
12
g^2E

Conclusions

- Method is applicable in $2D$ scalar field models.
- The probability of SA creation in high–energy collisions is

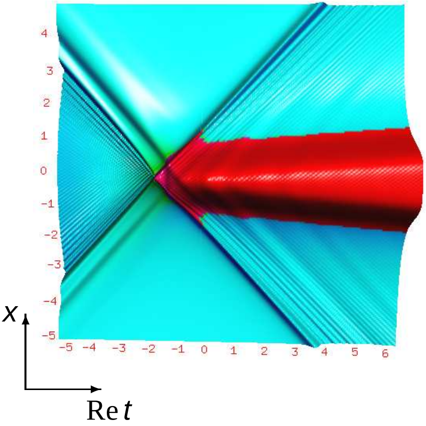
$$\mathcal{P}(E) \approx \mathrm{e}^{-F(E)/g^2}$$

Generalizations to other models?

Limit $\delta\rho \to 0$

$E \approx 8.95$

$N \approx 2.42$, $\delta\rho = 0.02$



$E \approx 9.06$

$N \approx 2.47$, $\delta\rho = 0.4$