The worldsheet low-energy limit of the $\mathrm{AdS}_4 \times \mathbb{CP}^3$ superstring

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The AdS/CFT correspondence relates (super)conformal field theories in various dimensions to string theories in 10 dimensions

We will focus on gauge group SU(N), so a generic QFT has two parameters: N and the coupling constant g

The 't Hooft limit: $\lambda = g^\alpha N$ is fixed, whereas $\mathbf{N} \to \infty$

String sigma models with target space $\mathbf{AdS_{D+1}} \times \mathcal{M}_\mathbf{c}$ descibe planar diagrams in QFT

The most prominent example is the $\mathbf{AdS_5 \times S^5}$ vs. maximally supersymmetric Yang-Mills theory in D=4

The example we will focus on is the $\mathbf{AdS_4 \times \mathbb{C}P^3}$ versus a Chern-Simons theory with N=6 (out of a maximal number of 8) complex susy's in D=3

$$S^{\mathcal{N}=2} = \int \frac{k}{4\pi}\mathrm{Tr}(A \wedge dA + \frac{2}{3}A^3) + D_\mu \bar{\phi}_i D^\mu \phi_i + i\bar{\psi}_i \gamma^\mu D_\mu \psi_i$$
$$- \frac{16\pi^2}{k^2}(\bar{\phi}_i T^a_{R_i} \phi_i)(\bar{\phi}_j T^b_{R_j} \phi_j)(\bar{\phi}_k T^a_{R_k} T^b_{R_k} \phi_k) - \frac{4\pi}{k}(\bar{\phi}_i T^a_{R_i} \phi_i)(\bar{\psi}_j T^a_{R_j} \psi_j)$$
$$- \frac{8\pi}{k}(\bar{\psi}_i T^a_{R_i} \phi_i)(\bar{\phi}_j T^a_{R_j} \psi_j).$$

k is the coupling constant

In the planar limit the ratio $\mathbf{\frac{N}{k}}$ is kept fixed

In a generic gauge theory there's a class of the so-called large-spin operators

Gross and Wilczek, 1974

$$ {}^{n}O^{V}_{\mu_1 \cdots \mu_n} = i^{n-2} S \,\mathrm{Tr}\, F_{\mu_1 \alpha} \nabla_{\mu_2} \cdots \nabla_{\mu_{n-1}} F^{\alpha}{}_{\mu_n} $$

$$ - \text{trace terms}, $$

$$ {}^{n}O^{F\pm,0}_{\mu_1 \cdots \mu_n} = \tfrac{1}{2} i^{n-1} S \bar{\psi} \gamma_{\mu_1} \nabla_{\mu_2} \cdots \nabla_{\mu_n} (1 \pm \gamma_5) \psi $$

$$ - \text{trace terms}, $$

$$ {}^{n}O^{F\pm,a}_{\mu_1 \cdots \mu_n} = \tfrac{1}{2} i^{n-1} S \bar{\psi} \gamma_{\mu_1} \nabla_{\mu_2} \cdots \nabla_{\mu_n} (1 \pm \gamma_5) \tfrac{1}{2} \lambda^a \psi $$

$$ - \text{trace terms}, $$

Their anomalous dimension
has the following characteristic behaviour

$$ \Delta = f(\lambda) \; \log(S) $$

Anomalous dimensions are observable

Consider $(p, q) \to (p, q)$ forward scattering

Introduce the spectral function

$$Q^2 = -q^2, \quad p^2 = 1, \quad \nu = p \cdot q, \quad \omega = \frac{2\nu}{Q^2}$$

$$W(\nu > 0, Q^2 > 0) = \mathrm{Im}(T(\nu, Q^2))$$

$$\mathrm{T}(\nu, Q^2) \sim \int dx\, e^{iqx} \langle p | T(j(x) j(0)) | p \rangle$$

The moments are defined as follows

$$\mu_n(Q^2) = \int_1^{\infty} d\omega\, \frac{W(\nu, Q^2)}{\omega^{n+1}}$$

The moments behave for large Q^2 as

$$\mu_n(Q^2) \sim (Q^2)^{-\frac{1}{2}D_n}$$

where

$$D_n = (d_n - n - 2d_j + 4 + \Delta_n)$$

There is a unitarity bound on the behavior of the anomalous dimension Δ (which follows from the positivity of W)

Nachtmann, 1973

$$\Delta(S) \lesssim S$$

One can reproduce the characteristic behaviour of anomalous dimensions using the spinning string solution of the sigma-model equations of motion

Gubser, Klebanov and Polyakov, 2002

AdS metric in global coordinates

$$(ds^2)_{AdS_3} = R^2 \left(-\cosh^2(\rho)dT^2 + d\rho^2 + \sinh^2(\rho)d\phi^2\right)$$

The solution

$$T = \kappa\tau,\ \phi = \kappa\tau,\ \rho = \pm\kappa\sigma + \rho_0$$

Relation between charges

$$E - S = \frac{\sqrt{\lambda}}{\pi}\log(S)$$

To consider the low-energy limit we first introduce additional rotation along a circle in the compact space

$$\varphi = \omega_2 \tau \qquad \omega_2 = u\kappa$$

Parameter describing the rotation is $u = \frac{J}{\sqrt{\lambda}\,\log(S)}$

Energy (anomalous dimension) of large-spin operators with an additional U(1) charge

$$\Delta = (f(\lambda) + \epsilon(\lambda, u))\,\log(S)$$

$$\epsilon(\lambda, u) = u^2 \sum \frac{a_n}{(\sqrt{\lambda})^n} (\log(u))^{n-1}, \text{ as } u \to 0$$

In the AdS_5 x S^5 case the leading logs are described by the $\mathrm{SO}(6)$ sigma model

Alday and Maldacena, 2007

On the sigma-model side one needs to build a type IIA Green-Schwarz action for the AdS_4 x CP^3 string

$AdS_5 \times S^5$ case Metsaev and Tseytlin, 1998

$\frac{OSP(6|4)}{U(3)\times SO(1,3)}$ coset Arutyunov and Frolov, 2008

The coset action has 24 fermions and represents a partially kappa-gauge-fixed Green-Schwarz action

The spectrum of fluctuations

Frolov and Tseytlin, 2002
Frolov, Tirziu and Tseytlin, 2005

$$f(\lambda) = \frac{\sqrt{\lambda}}{\pi} + a_1 + \frac{a_2}{\sqrt{\lambda}} + O(\frac{1}{\lambda}) \qquad (u \neq 0)$$

$$\langle H \rangle = \sum_{n \in \mathrm{Z}^+} \left(\sum_{i \in \mathrm{Bosons}} \sqrt{n^2 + m_i^2} - \sum_{i \in \mathrm{Fermions}} \sqrt{n^2 + m_i^2} \right)$$

$AdS_5 \times S^5$ case: $a_1 \approx -\frac{3 \ln 2}{\pi}$

The spectrum of fluctuations. $AdS_4 \times \mathbb{C}P^3$ case

6 massless modes from $\mathbb{C}P^3$

Alday, Arutyunov and DB, 2007
McLoughlin, Roiban and Tseytlin, 2007

$$\mathscr{D} = 2^8 \omega_2^{16} \left[(2k_0 - \omega_2)^2 - 4(k_1^2 + \varkappa^2)\right]^2 \left[(2k_0 + \omega_2)^2 - 4(k_1^2 + \varkappa^2)\right]^2 \times$$
$$\times \left[k_0^4 - k_0^2(2k_1^2 + \varkappa^2) + k_1^2(k_1^2 - \omega_2^2 + \varkappa^2)\right]^2$$

- 2 fermions with frequency $\frac{\omega_2}{2} + \sqrt{n^2 + \varkappa^2}$
- 2 fermions with frequency $-\frac{\omega_2}{2} + \sqrt{n^2 + \varkappa^2}$
- 2 fermions with frequency $\sqrt{n^2 + \frac{1}{2}\varkappa^2 + \frac{1}{2}\sqrt{\varkappa^4 + 4\omega_2^2 n^2}}$
- 2 fermions with frequency $\sqrt{n^2 + \frac{1}{2}\varkappa^2 - \frac{1}{2}\sqrt{\varkappa^4 + 4\omega_2^2 n^2}}$

The rank of kappa-symmetry increases when $\omega_2 \to 0$. Use of the coset is problematic!

How can one build the complete action?

There exists an M2 brane worldsheet action with kappa-symmetry

Bergshoeff, Sezgin and Townsend, 1987

Under a Hopf reduction the $\mathbf{AdS_4} \times \mathbf{S^7}$ background of 11D SUGRA described by the

$$\frac{OSP(8|4)}{SO(7)\times SO(1,3)}$$ coset

transforms into the AdS_4 x CP^3 IIA background

Gomis, Sorokin and Wulff, 2009

To obtain the low-energy limit we need to drop the irrelevant terms, that is the ones of dimension > 2

DB, 2010

$$\mathcal{L} = \eta^{\alpha\beta}\,\overline{\mathcal{D}_\alpha z^j}\,\mathcal{D}_\beta z^j + i\overline{\Psi}\gamma^\alpha\widehat{\mathcal{D}}_\alpha\Psi + \frac{1}{4}(\overline{\Psi}\gamma^\alpha\Psi)^2$$

where $\mathcal{D}_\alpha = \partial_\alpha - i\,\mathcal{A}_\alpha$ and $\widehat{\mathcal{D}}_\alpha = \partial_\alpha + 2\,i\,\mathcal{A}_\alpha$

Besides, the z's are restricted to lie on the sphere

$$\sum_{j=1}^{4} |z^j|^2 = 1$$

No supersymmetry

Comparison with SUSY CP^1 model

Di Vecchia, Ferrara, 1977
Zumino, 1979
Witten, 1981

CP^n models in D=2 with and without fermions have interesting physical properties:

- They are asymptotically free
- Have instanton solutions
- The bosonic model exhibits confinement
- The SUSY model is integrable
- Other non-SUSY integrable models of CP^n with fermions

D'Adda, Di Vecchia, Luescher, 1978
Abdalla et al, 1981-1985
Koeberle, Kurak, 1982
Shankar, Witten, 1977

It is an interesting question whether the model we have obtained is integrable at the quantum level. If yes, what is its S-matrix?

Extra 1

N=4 SYM vs. AdS_5 x S^5 superstring

Maldacena, 1997

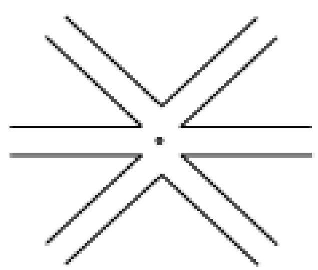
$\mathrm{PSU}(2,2 \mid 4)$

Conformal group $\mathrm{SO}(2,4)$

R-symmetry group $\mathrm{SU}(4)$

The dilatation operator hierarchy

Minahan and Zarembo, 2002
Beisert and Staudacher, 2005



1-loop chiral SU(2) sector ----> XXX spin chain

The Lax pair for the superstring

Bena, Polchinski and Roiban, 2003

Extra 2

The dimensional reduction: from 11D SUGRA to IIA

$$g_{\mu\nu},\ \psi^a,\ H_{\alpha\beta\gamma}\, dx^\alpha \wedge dx^\beta \wedge dx^\gamma$$

Duff, Howe, Inami and Stelle, 1987

Howe, and Sezgin, 2005

«The reduction of supergravity formulated in D=11 superspace to ten dimensions was outlined in [DHIS], but the full details were not given there. Here we provide them.»

$$\mathcal{F} = \mathcal{K}^{(3)} \wedge d\varphi + \mathcal{G}^{(4)} \qquad d\mathcal{K}^{(3)} = d\mathcal{G}^{(4)} = 0$$

$$\mathcal{K}^{(3)} = d\mathcal{B}^{(2)} \qquad \mathcal{D}^{(4)} = \mathcal{G}^{(4)} - \mathcal{A}^{(1)} \wedge \mathcal{K}^{(3)}$$

The Hopf fibration of 11D supergravity Nilsson and Pope, 1984

Extra 3

The Hopf fibre bundle

$$\pi \; : \; S^{2n+1} \rightarrow \mathbb{C}P^n$$

n=1: $|z_1|^2 + |z_2|^2 = 1$ $\pi(z_1, z_2) = \frac{z_2}{z_1}$

$$z_1 = \frac{y_1}{\sqrt{|y_1|^2+|y_2|^2}}, \; z_2 = \frac{y_2}{\sqrt{|y_1|^2+|y_2|^2}}$$

The metric:

$$(ds^2)_{S^3} = \frac{dy_i d\bar{y}_i}{\rho^2} - \frac{|dy_i \bar{y}_i|^2}{\rho^4} + \left(i \frac{dy_i \bar{y}_i - y_i d\bar{y}_i}{2\rho^2} \right)^2, \text{ where } \rho^2 = |y_i|^2$$

$$(ds^2)_{S^3} = \frac{dZ d\bar{Z}}{(1 + Z\bar{Z})^2} + (d\varphi - A)^2$$