A simple way to take into account back reaction on pair creation[1]

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Motivation

Why we should study problem of back reaction on classical background due to the quantim pair creation?

- Imprortance of the problem
  For high intensity field dynamics, for BH physics, for cosmology...
- Manifistation of new physics
  Especially in the gravity context of the problem
- Absence of simple systematic way of consideration
  Even in the simplest case of *scalar QED*!

Formulation of the problem and Strategy

Let's assume that at $t = 0$

- somehow a constant (everywhere in space) electric field was created (we neglect the boundary effects);
- there are no electrically charged particles present on top of the field ($\phi(x)|_{t=0} = 0$)
- one turns on interactions and *the pair creation begins*

Strategy:
One has to calculate the electric current which is created by pairs and to take into account the field due to this current.

$$\partial^\mu F_{\mu\nu}(x,t) = \langle in | j_\nu(x,t) | in \rangle$$

What is the most simple, but nevertheless illustrative example?

Scalar QED with Dirichlet boundary conditions at $t = 0$

If one recall the definition of current for charged scalars

$$\begin{aligned} j_\nu(x) &= \phi^*(x)\Big(\mathrm{i}e\overset{\leftrightarrow}{\partial}_\nu + 2e^2 A_\nu\Big)\phi(x) = \\ &= \left\{\left[\mathrm{i}e\left(\frac{\partial}{\partial y^\nu} - \frac{\partial}{\partial z^\nu}\right) + 2e^2 A_\nu\right]\phi^*(y_\mu)\phi(z_\mu)\right\}_{y=z=x} \end{aligned}$$

then one could rewrite all through the *in-in propagator* of the theory

$$\langle in|j_\nu(x,t)|in\rangle = \left\{\left[\mathrm{i}e\left(\partial_{y^\nu} - \partial_{z^\nu}\right) + 2e^2 A_\nu\right] G^{hs}_{in-in}(y,z)\right\}_{y=z=x}$$

where

$$G^{hs}_{in-in}(y,z) = \langle in|\phi^*(y)\phi(z)|in\rangle$$

is the in-in propagator in the background electric field on the half of $R^{3,1}$ ($t \in [0,+\infty)$)

Relationships of propagators

At the present moment our central equation reduced to the following form

$$\partial^\mu F_{\mu\nu}(x) = \left\{ \left[\mathrm{ie}\left(\partial_{y^\nu} - \partial_{z^\nu}\right) + 2e^2 A_\nu \right] G^{hs}_{in-in}(y,z) \right\}_{y=z=x}$$

For further simplifications let's express this *in–in propagator for the half of* $R^{3,1}$ through the *in–out propagator for the full* $R^{3,1}$. Two basic steps:

- From the consideration of Bogolubov transformation one can obtain, that for the full $R^{3,1}$ space

$$G_{in-in}(z,y) = 2\mathrm{Re}\left[G_{out-in}(z,y)\right]$$

- Using *mirror sources* one can connect propagators for full space and for half space

$$G^{hs}_{out-in}(z,y) = G_{out-in}(z,y) - G_{out-in}(z,\overline{y})$$

where $\overline{y} = (-y_0, \vec{y})$ is the position of the mirror source

Summarize all above transformations and using Euclidean reformulation one arrived to the following version of our central equation

$$\partial_\mu F_{\mu\nu} = -2\left[\mathrm{ie}\left(\frac{\partial}{\partial y^\nu}-\frac{\partial}{\partial z^\nu}\right)+2e^2A_\nu\right]\times$$
$$\times \quad \mathrm{Re}\left[\left\langle z\left|\frac{1}{-D_\mu^2+m^2}\right|y\right\rangle-\left\langle z\left|\frac{1}{-D_\mu^2+m^2}\right|\bar{y}\right\rangle\right]\Bigg|_{y=z=x}$$

here

$$G_{out-in}(z,y)=\left\langle z\left|\frac{1}{-D_\mu^2+m^2}\right|y\right\rangle=$$
$$\left\langle z\left|\int_0^\infty dT e^{-\left(-D_\mu^2+m^2\right)T}\right|y\right\rangle=\int_0^\infty dT e^{-m^2T}\left\langle z\left|e^{D_\mu^2\,T}\right|y\right\rangle=$$
$$=\int_0^\infty dT e^{-m^2T}\int_{x(0)=y;x(T)=z}\mathcal{D}x(\tau)\; e^{-\int_0^T\left(\frac{1}{4}\dot{x}^2+\mathrm{ie}A_\mu\dot{x}_\mu\right)d\tau}.$$

Computation for constant electric field

Let's put $F_{\mu\nu} = -\mathrm{i}E(x_0)\delta_{3[\mu}\delta_{\nu]0}$ in the LHS of the equation ($\partial_\mu F_{\mu\nu} \to dE(x_0)/dx_0$), and $A_\mu = Ex_0\delta_{\mu 3}$ into the propagators:

$$\left\langle z \middle| \frac{1}{-D_\mu^2 + m^2} \middle| y \right\rangle = e^{-S_{cl}[z,y]} \int_0^\infty dT e^{-m^2 T} \frac{eET}{(4\pi T)^2 \sinh(eET)},$$

where

$$\begin{aligned} S_{cl}[z,y] &= \frac{(y_1 - z_1)^2 + (y_2 - z_2)^2}{4T} + \frac{e\,E\,\coth(e\,E\,T)}{4} \times \\ &\times \left[(y_0 - z_0)^2 + (y_3 - z_3)^2\right] + \mathrm{i}\frac{eE}{2}(z_0 + y_0)(z_3 - y_3) \end{aligned}$$

Substituting all expressions into the equation one obtain

$$\frac{dE}{dt} = -\mathrm{Re}\left(\mathrm{i}\frac{e^3E^2t}{8\pi^2}\int_0^\infty dT e^{-m^2T}\frac{1}{T\sin(eET)}\right).$$

Using that

$$\mathrm{Im}\left(\int_0^\infty dT e^{-m^2T}\frac{1}{T\sin(eET)}\right) = -\ln\left(1+e^{-\frac{m^2\pi}{eE}}\right),$$

one finally arrived to the one loop answer for the decay rate of the background electric field in our approximation

$$\boxed{\frac{dE}{dt} = -\frac{e^3E^2t}{4\pi^2}\ln\left(1+e^{-\frac{m^2\pi}{eE}}\right)}$$

Semiclassical consideration

One could restore the leading approximation of our one loop result on the *general physical grounds*:
Energy of the electric field per unit volume $E^2/8\pi$ is spent on the work on the creation of pairs, which is proportional to $e\,E\,z = e\,E\,t$, here z is the separation distance between the members of the pair reached during the observation time t and also to the $w(E) \propto e^2\,E^2\,e^{-\frac{m^2\pi}{eE}}$ approximate Schwinger's pair creation probability rate per unit time and unit volume. Hence, one could write

$$\frac{d}{dt}E^2 \propto -2e\,E\,t\,w(E),$$

and obtain from here the leading approximation of our result

$$\frac{dE}{dt} \propto -2e^3\,E^2\,t\,e^{-\frac{m^2\pi}{eE}}$$

Discussion

$$\frac{dE}{dt} = -\frac{e^3 E^2 t}{4\pi^2} \ln\left(1 + e^{-\frac{m^2 \pi}{eE}}\right)$$

- *weak* electric field ($eE << m^2$) changes slowly in time, because in this limit $\ln\left(1 + e^{-\frac{m^2 \pi}{eE}}\right) \to 0$ and therefore $dE(t)/dt \to 0$
- In the *strong* field limit ($eE >> m^2$) there will be a fast decay $E(t) \propto 1/t^2$ – it's just a hint, because in the case of the overcritical field we can not apply our approximation

Conclusions

- Simple and systematic way for acounting back reaction on pairs creation was analized
- Simplest model, which admit analytical investigation, was proposed
- One loop answer for the decay rate of the weak background field was obtained

Thank you for your attention!