Some two-loop threshold corrections and three-loop renormalization group analysis of the MSSM

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Outline

Minimal Supersymmetric Standard Model

Gauge

	Bosons		Fermions		Quantum numbers
$\hat{G}$	gluons	G^a_μ	gluinos	$\tilde{g}^a$	(8,1,0)
$\hat{V}$	$SU(2)$-bosons	A^i_μ	gauginos	$\tilde{A}^i$	(1,3,0)
$\hat{V}'$	$U(1)$-bosons	B_μ	gauginos	$\tilde{B}$	(1,1,0)

Matter

	Fermions		Bosons		
$\hat{Q}$	quarks	$Q = \binom{u}{d}_L$	squarks	$\tilde{Q} = \binom{\tilde{u}}{\tilde{d}}_L$	(3,2,1/3)
$\hat{U}$		$U = u_R$		$\tilde{U} = \tilde{u}_R$	(3,1,4/3)
$\hat{D}$		$D = d_R$		$\tilde{D} = \tilde{d}_R$	(3,1,-2/3)
$\hat{L}$	leptons	$E = \binom{\nu_l}{l}_L$	sleptons	$\tilde{E} = \binom{\tilde{\nu}_l}{\tilde{l}}_L$	(1,2,-1)
$\hat{E}$		$E = l_R$		$\tilde{E} = \tilde{l}_R$	(1,1,-2)
$\hat{H}_1$	higgsino	$(\tilde{H}^0_1, \tilde{H}^-_1)$	higgses	$H_1 = \binom{H^0_1}{H^-_1}$	(1,2,-1)
$\hat{H}_2$		$(\tilde{H}^+_2, \tilde{H}^0_2)$		$H_2 = \binom{H^+_2}{H^0_2}$	(1,2,1)

Minimal Supersymmetric Standard Model

MSSM has the following set of free parameters

1. Three gauge couplings α_i
2. Three Yukawa matrices y_u, y_d, y_l
3. Parameter $\bar{\mu}$ (mixing of higgs superfields)
4. Soft (dim $<$ 4) supersymmetry breaking terms

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    - $m_{1/2}$ – a gaugino mass
    - m_0 – a common mass for squarks and sleptons
    - A_0 – trilinear squark-squark-higgs and slepton-slepton-higgs couplings
    - B – higgs field mixing parameter

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5. $v^2 = v_1^2 + v_2^2$, $\tan\beta = v_2/v_1$, где $v_1 =< H_1 >$, $v_2 =< H_2 >$

$$y_t = \frac{\sqrt{2}m_t}{v_2}, \quad y_b = \frac{\sqrt{2}m_b}{v_1}, \quad y_\tau = \frac{\sqrt{2}m_\tau}{v_1}$$

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$$y_t = \frac{\sqrt{2} m_t}{v \sin\beta}, \quad y_b = \frac{\sqrt{2} m_b}{v \cos\beta}, \quad y_\tau = \frac{\sqrt{2} m_\tau}{v \cos\beta}$$

Constraints on parameters are needed !

Direct non-observation, Dark matter, $(g-2)_\mu$, EWSB, ...

Low-energy input [PDG'09]

- M_Z mass

$$M_Z = 91.1876(21) \text{ GeV}$$

- Strong coupling in $\overline{\text{MS}}$

$$\alpha_s^{(5)}(M_Z) = 0.1176(20)$$

- Fermi constant

$$G_F = 1.16637(1) \times 10^{-5} \text{ GeV}^{-2}$$

- Electric charge in $\overline{\text{MS}}$

$$\alpha^{-1}(M_Z) = 127.925(16)$$

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- Top quark

$$M_t = 170.9(1.9)\ \text{GeV}$$

- Bottom quark

$$m_b(m_b) = 4.196(28)\ \text{GeV}$$

- Tau lepton

$$M_\tau = 1776.84(17)\ \text{MeV}$$

$\overline{\mathrm{DR}}$ renormalization scheme

- Regularization: dimensional reduction (DRED) [Siegel,'79-80],[Avdeev,Chochia&Vladimirov,'81],[Stockinger,'05]
- Renormalization: minimal (modified) subtractions, $\overline{\mathrm{DR}}$.

$$
\begin{aligned}
g^{\mu\nu} &= \hat{g}^{\mu\nu} + \tilde{g}^{\mu\nu} & \qquad g^{\mu\nu} g_{\mu\nu} &= 4 \\
g^{\mu\nu} \hat{g}^{\rho}_{\nu} &= \hat{g}^{\mu\rho} & \qquad \hat{g}^{\mu\nu} \hat{g}_{\mu\nu} &= d \\
g^{\mu\nu} \tilde{g}^{\rho}_{\nu} &= \tilde{g}^{\mu\rho} & \qquad \tilde{g}^{\mu\nu} \tilde{g}_{\mu\nu} &= 2\varepsilon = 4 - d \\
\hat{g}^{\mu\nu} \tilde{g}^{\rho}_{\nu} &= 0 & &
\end{aligned}
$$

All fields in 4D Lagrangian depend only on $\hat{x}_{\mu} = \hat{g}_{\mu\nu} x^{\nu}$.

Low-energy input [PDG'09]

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$$M_Z = 91.1876(21)\ \text{GeV} \qquad \Rightarrow \alpha_{1,2}^{\overline{\text{DR}}}(\bar{\mu}),\ v(M_Z)$$

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$$M_t = 170.9(1.9) \text{ GeV} \qquad \Rightarrow m_t^{\overline{\text{DR}}}(\bar{\mu})$$

- Bottom quark

$$m_b^{\overline{\text{MS}}}(m_b^{\overline{\text{MS}}}) = 4.163(16) \text{ GeV (talk by J. Kühn)} \qquad \Rightarrow m_b^{\overline{\text{DR}}}(\bar{\mu})$$

- Tau lepton

$$M_\tau = 1776.84(17) \text{ MeV} \qquad \Rightarrow m_\tau^{\overline{\text{DR}}}(\bar{\mu})$$

Effective theories and Matching

- Advanatatage: Minimal renormalization schemes (such as $\overline{\mathrm{DR}}$) have very simple mass-independent RGEs
  $\Rightarrow$ parameters can be easily extrapolated to high energies (and back!)

- Disadvanatatage: Decoupling theorem [Appelquist&Carazzone,'74] does not hold automatically! In theories with different scales $m \ll M$ large $\log m/M$ usually appear in the expressions.

Solution: At low energies instead of full theory one uses effective one!

Effective theories and Matching

$M_b, M_\tau \sim 1$ GeV $\qquad M_{\rm EW} \sim M_t \sim 100$ GeV $\qquad M_{\rm SUSY} \sim 1$ TeV

Fermi × QCD

$$\alpha_s^{\overline{\rm MS}}(\mu),\;\; m_b^{\overline{\rm MS}}(\mu)$$
$$\alpha^{\overline{\rm MS}}(\mu),\;\; G_F$$

$$A(E) \implies A_{Fermi\times QCD}(E, \alpha_s^{\overline{\rm MS}}(\mu), m_b^{\overline{\rm MS}}(\mu), \cdots, \mu)$$

Effective theories and Matching

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Fermi × QCD

$\alpha_s^{\overline{\text{MS}}}(\mu),\ m_b^{\overline{\text{MS}}}(\mu)$
$\alpha^{\overline{\text{MS}}}(\mu),\ G_F$

SM

$\alpha_s^{\overline{\text{MS}}}(\mu), y_b^{\overline{\text{MS}}}(\mu)$
$+$ more $\mathcal{O}(20)$
parameters

$$A(E) \Longrightarrow A_{Fermi\times QCD}(E, \alpha_s^{\overline{\text{MS}}}(\mu), m_b^{\overline{\text{MS}}}(\mu), \cdots, \mu) + \mathcal{O}\left(\frac{E^2}{M_{EW}^2}\right)$$

$$\Longrightarrow A_{\text{SM}}(E, \alpha_s^{\overline{\text{MS}}}(\mu), y_b^{\overline{\text{MS}}}(\mu), \cdots, \mu)$$

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Fermi $\times$ QCD

$\alpha_s^{\overline{\rm MS}}(\mu),\ m_b^{\overline{\rm MS}}(\mu)$
$\alpha^{\overline{\rm MS}}(\mu),\ G_F$

SM

$\alpha_s^{\overline{\rm MS}}(\mu), y_b^{\overline{\rm MS}}(\mu)$
+ more $\mathcal{O}(20)$ parameters

MSSM

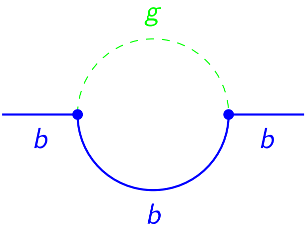
$\alpha_s^{\overline{\rm DR}}(\mu), y_b^{\overline{\rm DR}}(\mu)$
+ more $\mathcal{O}(120)$ parameters

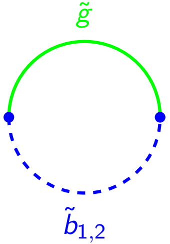
$$
\begin{aligned}
A(E) \quad &\Longrightarrow \quad A_{Fermi\times QCD}(E, \alpha_s^{\overline{\rm MS}}(\mu), m_b^{\overline{\rm MS}}(\mu), \cdots, \mu) + \mathcal{O}\left(\frac{E^2}{M_{EW}^2}\right) \\
&\Longrightarrow \quad A_{\rm SM}(E, \alpha_s^{\overline{\rm MS}}(\mu), y_b^{\overline{\rm MS}}(\mu), \cdots, \mu) + \mathcal{O}\left(\frac{E^2}{M_{SUSY}^2}\right) \\
&\Longrightarrow \quad A_{\rm MSSM}(E, \alpha_s^{\overline{\rm DR}}(\mu), y_b^{\overline{\rm DR}}(\mu), \cdots, \mu)
\end{aligned}
$$

Decoupling of heavy particles

MSSM, $\overline{\mathrm{DR}}$-scheme. up to $\mathcal{O}(m_b^2/M_{\text{hard}}^2) \sim 0.1$ %

$$
\begin{aligned}
M_b &= m_b \left(1+\frac{\alpha_s}{3\pi}\left(4 - 3\log\frac{m_b^2}{\mu^2}\right)+\frac{\alpha}{4\pi}\left(c_1 + c_2 \log\frac{M_{\text{hard}}^2}{\mu^2}\right)\right) \\
&= m_b \left[1 + \frac{\alpha}{4\pi}\left(c_1 + c_2 \log\frac{M_{\text{hard}}^2}{\mu^2}\right)\right]\left[1 + \frac{\alpha_s}{3\pi}\left(4 - 3\log\frac{m_b^2}{\mu^2}\right)\right]
\end{aligned}
$$





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$$= m_b\left[1+\frac{\alpha}{4\pi}\left(c_1+c_2\log\frac{M_{\text{hard}}^2}{\mu^2}\right)\right]\left[1+\frac{\alpha_s}{3\pi}\left(4-3\log\frac{m_b^2}{\mu^2}\right)\right]$$

QCD, $\overline{\mathrm{MS}}$-scheme

$$M_b = m_b\left(1+\frac{\alpha_s}{3\pi}\left(4-3\log\frac{m_b^2}{\mu^2}\right)\right)$$

Decoupling of heavy particles

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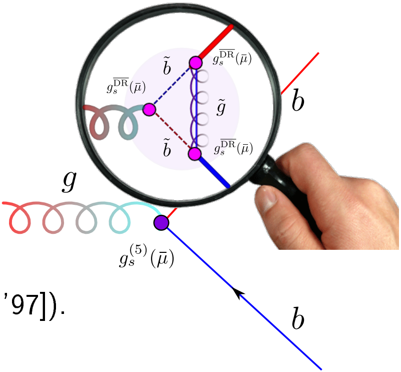
&= m_b \left[1 + \frac{\alpha}{4\pi}\left(c_1 + c_2 \log\frac{M_{\text{hard}}^2}{\mu^2}\right)\right]\left[1 + \frac{\alpha_s}{3\pi}\left(4 - 3\log\frac{m_b^2}{\mu^2}\right)\right]
\end{aligned}
$$

$$
\begin{aligned}
m_b^{\overline{\mathrm{MS}}}(\mu) &= m_b^{\overline{\mathrm{DR}}}(\mu)\left(1 + \frac{\alpha^{\overline{\mathrm{DR}}}(\mu)}{4\pi}\left(c_1 + c_2\log\frac{M_{\text{hard}}^2}{\mu^2}\right)\right) \\
\alpha_s^{\overline{\mathrm{MS}}}(\mu) &= \alpha_s^{\overline{\mathrm{DR}}}(\mu)
\end{aligned}
$$

Effective theory and matching

- Matching at the level of (pseudo-) observables is not efficient.

Due to nice features of Dimensional Regularization (UV and IR poles are treated on equal footing) one usually uses a nice trick (see, e.g.) to perform matching at the level of bare couplings
(See [Chetyrkin, Kniel & Steinhauser,'97]).



Practical prescription for matching

Consider two (renormalizable) theories:

Theory	fields	parameters
Full (FT)	ϕ, Φ	A, B, M
Effective (EFT)	$\underline{\phi}$	$\underline{A}$

(Φ has mass $M = M_{\text{hard}}$, ϕ and A have their counterparts in EFT)

1. One carries out matching at the bare level

$$\underline{A_0} = \zeta_{A,0} \cdot A_0, \qquad \underline{\phi_0} = \zeta_{\phi,0} \cdot \phi_0$$

by demanding that bare Green functions calculated in both theories coincide in the limit when external momenta vanish. In EFT all the diagrams vanish in this limit. So the calculation is reduced to evaluation of bubbles in FT with at least one heavy line .

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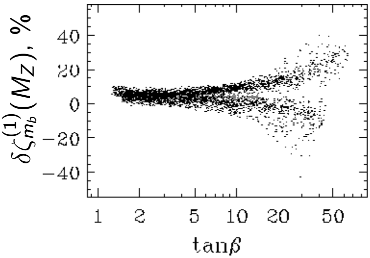
2 Given bare decoupling constants and renormalization constants in both theories ($\underline{Z_A}$, Z_A) one obtains the final result for renormalized decoupling constant $\underline{A}(\bar{\mu}) = \zeta_A(\bar{\mu}) \cdot A(\bar{\mu})$:

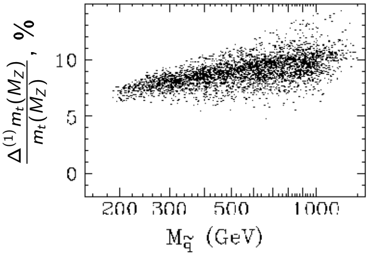
$$\begin{aligned} \underline{A_0} &= \underline{Z_A}\,(\underline{A})\,\underline{A}, \qquad A_0 = Z_A\,(A, B)\,A, \\ \zeta_A &= \Big[Z_A(A,\, B)\Big]\Big[\underline{Z_A}\,(\underline{A})\Big]^{-1} \zeta_{A,0}(Z_A\,A, Z_B\,B,\, Z_M M). \end{aligned}$$

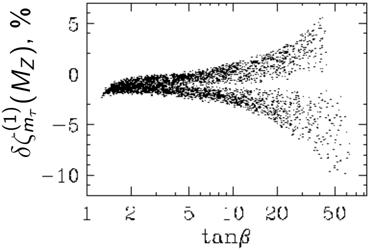
$$\zeta_A(\bar{\mu}) = 1 + \delta\zeta_A^{(1)}(\bar{\mu}) + \delta\zeta^{(2)}(\bar{\mu}) \cdots$$

One-loop results

One-loop corrections from [BPMZ,'96] are used in all the codes [Softsusy,Suspect,Spheno,Isajet].







$$\delta\zeta^{(1)}_{\alpha_s}(M_Z) \simeq 10 - 20~\%$$

Sometimes corrections are *big* and two-loop calculations are required at least to estimate the error of the result

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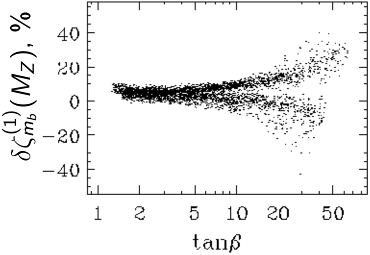

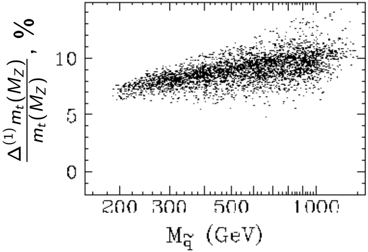

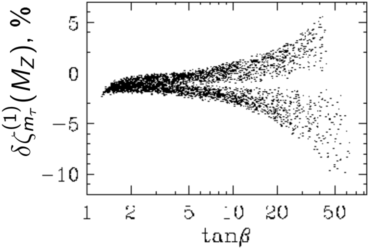

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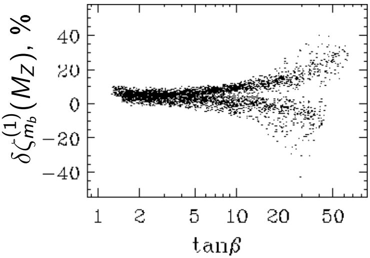

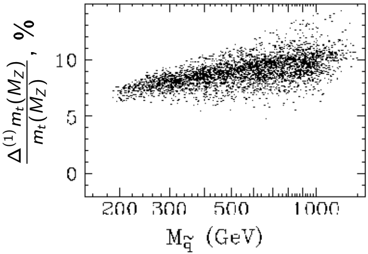

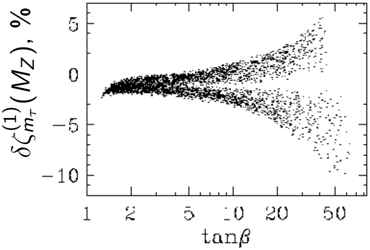

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"Effective" and "Fundamental" theories

Matching the SM and the MSSM is not an easy task at two-loop level. To simplify the problem one can neglect electroweak gauge couplings and only consider contributions proportional to α_s and Yukawa coupling constants of heavy SM fermions ($\alpha_b = y_b^2/(4\pi), \alpha_t = y_t^2/(4\pi), \alpha_\tau = y_\tau^2/(4\pi)$).
In this case

- A theory of free tau-lepton and five-flavour QCD with massive bottom-quark can be considered as "effective" one with parameters $\alpha_s^{(5)}(M_Z),\ m_b^{\overline{\rm MS}}(m_b),\ M_\tau$.
- The the so-called gaugeless limit of the MSSM [Haestier et al,'05] can be treated as "fundamental" theory.

NB: Running mass of the t-quark can be extracted *directly* from the pole mass [Bednyakov et al,'02].

$$M_t = m_t^{\overline{\rm DR}}(\bar\mu)\left(1 + \frac{\Delta^{(1)} m_t}{m_t} + \frac{\Delta^{(2)} m_t}{m_t} + \cdots\right)$$

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$$G_\mu^{(4)} \to \left\{ G_\mu \equiv \hat{g}_{\mu\nu} G^\nu_{(4)},\, W_\sigma \equiv \tilde{g}_{\sigma\rho} G^\rho_{(4)} \right\}$$

G_μ vector fields (gauge bosons),

W_σ unphysical ε-scalars.

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$$
\begin{aligned}
\mathcal{L}_B^{\varepsilon} &= \frac{1}{2}(D_\mu W_\sigma)_a^+ (D^\mu W_\sigma)^a + \frac{1}{2} m_\varepsilon^2 W_\sigma^a W_a^\sigma \\
&- \frac{g_s^2}{4} f^{abc} f^{ade} W_\sigma^b W_\rho^c W_\sigma^d W_\rho^e - g_s \overline{q} \gamma^\sigma T^a q W_\sigma^a \\
&+ i \frac{g_s}{2} f^{abc}\, \overline{\tilde{g}^a} \gamma_\sigma \tilde{g}^b W_\sigma^c + g_s^2 \tilde{q}^* T^a T^b \tilde{q}\, W_\sigma^a W_\sigma^b
\end{aligned}
$$

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- Renormalization: minimal (modified) subtractions, $\overline{\mathrm{DR}}$.

The problem with ε-scalar mass m_ε^2, can be solved in two equivalent ways ($\overline{\mathrm{DR}}'$).

- Minimal subtractions, but $m_\varepsilon^2 \neq 0$, with corresponding redefinition of masses of scalar superpartners [Jack et al'94].
- Non-minimal subtractions, $m_\varepsilon^2 = 0$ [Avdeev&Kalmykov,'97]

$\overline{\mathrm{DR}} \to \overline{\mathrm{MS}}$ transition

- Since ε-scalar mass m_ε is an unphysical parameter one can take it to be infinitely large, $m_\varepsilon \gg m_b, M_{\text{hard}}$. One can formally decouple ε-scalars together with superpartners. This trick allows one to make a simultaneous transition from MSSM to QCD and from $\overline{\mathrm{DR}}$-scheme to $\overline{\mathrm{MS}}$ [Bednyakov,'07].
- There is another ("standard") way [Harlander et al,'06]. The transition from $\overline{\mathrm{DR}}$ to $\overline{\mathrm{MS}}$ is realized by two (independent?) steps:

$$
\begin{array}{ccccc}
(\text{MSSM},\ \overline{\mathrm{DR}}) & \xrightarrow{(1)} & (\text{QCD},\ \overline{\mathrm{DR}}) & \xrightarrow{(2)} & (\text{QCD},\ \overline{\mathrm{MS}}). \\
(\alpha_s, m_b, M_{\text{SUSY}}, \ldots) & \longrightarrow & (\alpha_s, m_b, \alpha_y, \lambda) & \longrightarrow & (\alpha_s, m_b)
\end{array}
$$

Higgs tadpoles and EW gauge-parameter dependence

It is known that in the MSSM radiative corrections can lead to the tadpole diagrams. They should be canceled. One can use two different schemes to do this (see. Passarino for review)
We choose the scheme (β_H of [Bardin & Passarino, '99], also used in [BPMZ,'96]) in which loop-generated tadpoles t_1 and t_2 are canceled by "tree-level" terms $T_1 = -t_1$ and $T_2 = -t_2$

$$
\begin{aligned}
T_1 &= \frac{g^2 + g'^2}{8}(v_1^2 - v_2^2)v_1 + m_1^2 v_1 - m_3^2 v_2, \\
T_2 &= \frac{g^2 + g'^2}{8}(v_2^2 - v_1^2)v_2 + m_2^2 v_2 - m_3^2 v_1,
\end{aligned}
$$

In such a scheme tadpoles give contribution only to Higgs masses.
NB: Gauge invariance wrt $SU(2) \times U(1)$ transformation is not obvious.

Higgs tadpoles and EW gauge-parameter dependence

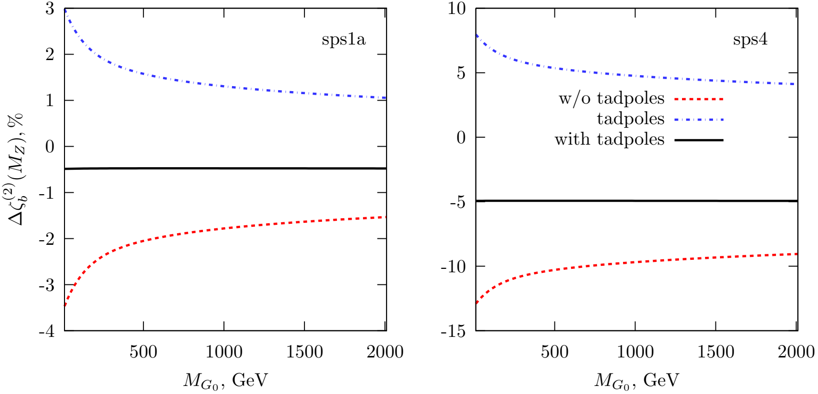
$$
\begin{aligned}
V_2 \quad = \quad & T_H\,H + T_h\,h + \frac{1}{2}\left(M_H^2\,H^2 + M_h^2\,h^2 + M_A^2\,A^2\right) + M_{H^\pm}^2\,H^+H^- \\
+ \quad & \begin{pmatrix} H & h \end{pmatrix} \begin{pmatrix} b_{HH} & b_{hH} \\ b_{hH} & b_{hh} \end{pmatrix} \begin{pmatrix} H \\ h \end{pmatrix} + \begin{pmatrix} G^0 & A \end{pmatrix} \begin{pmatrix} b_{G^0G^0} & b_{G^0A} \\ b_{G^0A} & b_{AA} \end{pmatrix} \begin{pmatrix} G^0 \\ A \end{pmatrix} \\
+ \quad & \begin{pmatrix} G^+ & H^+ \end{pmatrix} \begin{pmatrix} b_{G^+G^-} & b_{G^+H^-} \\ b_{G^+H^-} & b_{H^+H^-} \end{pmatrix} \begin{pmatrix} G^- \\ H^- \end{pmatrix}. \qquad (1)
\end{aligned}
$$

- T_h and T_H (linear combination of $T_{1,2}$) serve as counter-terms for one-loop tadpoles.
- $b_{\Phi_i\Phi_j}$ are expressed in terms of T_H, T_h and mixing angles for α and β.

(See [Pierce & Papadopoulos, '92] and [BPMZ,'96])

Higgs tadpoles and EW gauge-parameter dependence

In our setup the importance of tadpoles can be indirectly inferred from the fact that without them two-loop decoupling corrections to b-quark mass significantly depend on unphysical parameter — Goldstone mass (formally related to gauge-parameter dependence in R_ξ - gauge, e.g. $M^2_{G_0} = \xi M^2_Z$).



3-loop RGEs

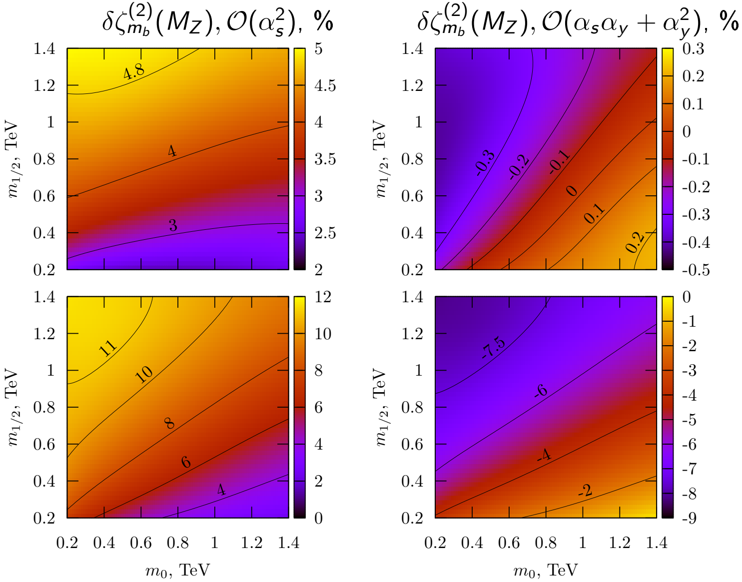
- For self-consistent study of N-loop threshold corrections one needs (N+1)-loop RGEs
- Three-loop beta-functions for rigid (supersymmetric) part of the Lagrangian can be calculated with the help of superfield formalism (see [Ferreira, Jack & Jones,'96] for results).
- With the help of spurion formalism (see [Yamada,'94] and [Avdeev, Kazakov & Kondrashuk, '97]) beta-functions for soft-breaking terms are found [Jack, Jones & Pickering, '98].
- The first three-loop analysis of the MSSM without two-loop threshold has been performed in [Jack, Jones & Kord,'04]

  `http://www.liv.ac.uk/~dij/betas`

  $\Rightarrow$ The effect of three-loop running corrections on SUSY mass spectrum is small for weakly interacting particles but larger for squark masses (ranges from 1-5 %)

Numerical results for two-loop decoupling corrections

b-quark

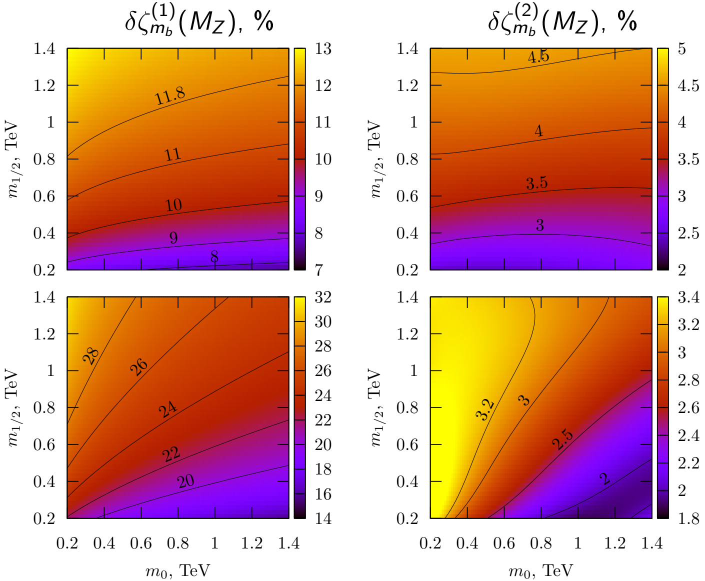


$\tan\beta = 10$

$\tan\beta = 50$

Numerical results for two-loop decoupling corrections

b-quark



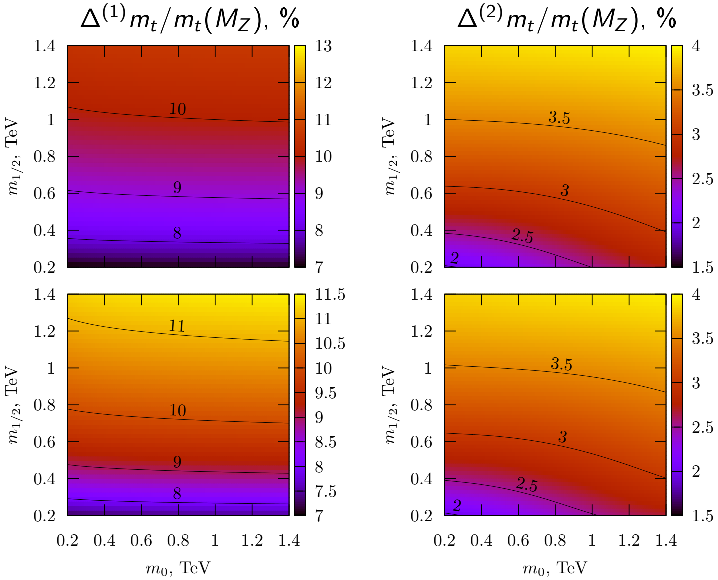
tan $\beta = 10$

tan $\beta = 50$

Numerical results for two-loop decoupling corrections

t-quark

$\tan\beta = 10$

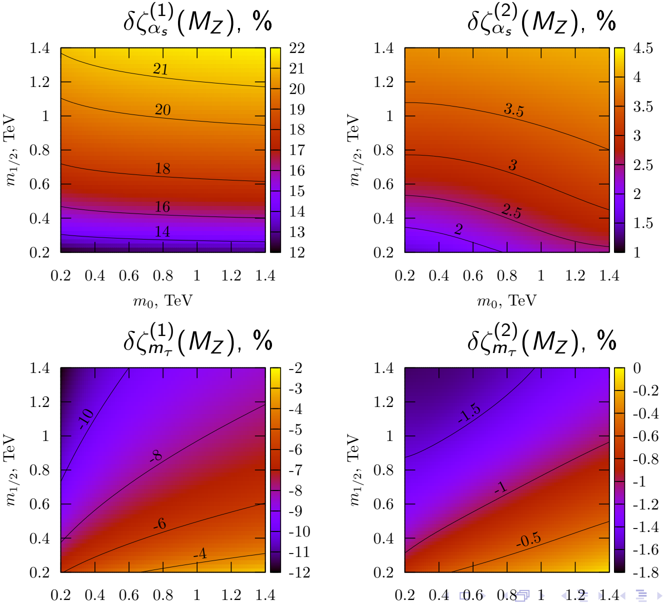


$\tan\beta = 50$

Numerical results for two-loop decoupling corrections

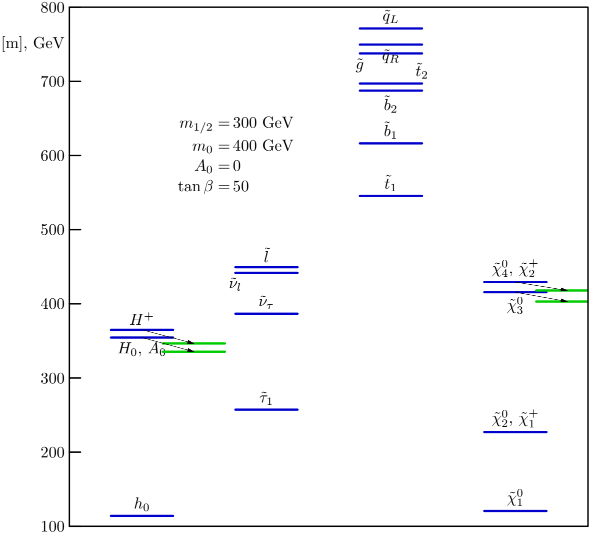
α_s, $\tan\beta = 50$

τ-lepton, $\tan\beta = 50$



Numerical results (SPS4)

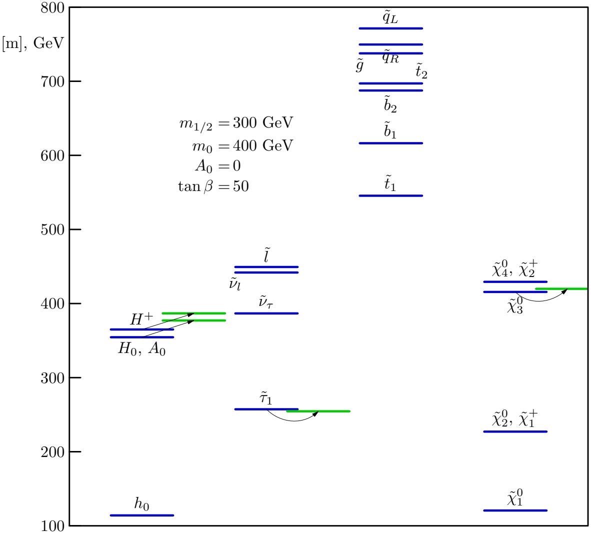
t-quark decoupling



The task is to incorporate the corrections and RGEs in computer codes, e.g. in [SoftSusy].

Numerical results (SPS4)

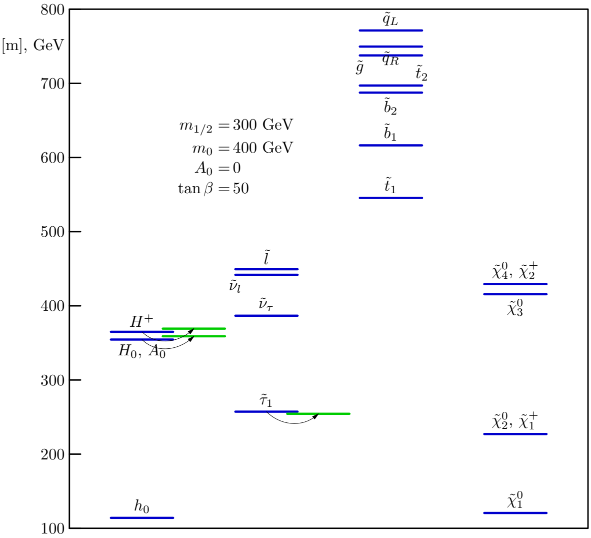
b-quark decoupling



The task is to incorporate the corrections and RGEs in computer codes, e.g. in [SoftSusy].

Numerical results (SPS4)

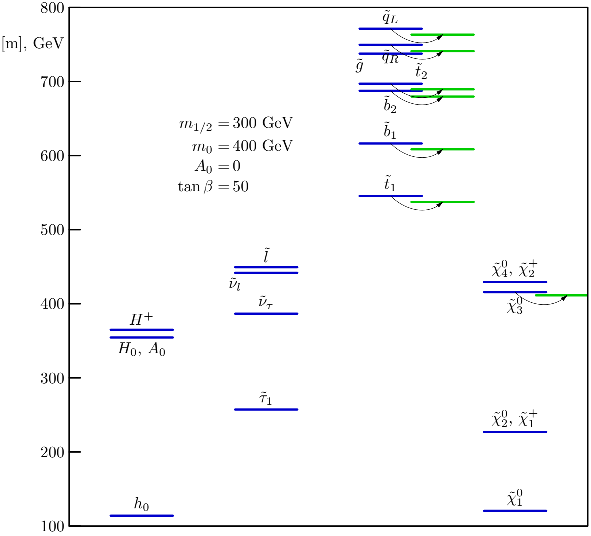
α_s decoupling



The task is to incorporate the corrections and RGEs in computer codes, e.g. in [SoftSusy].

Numerical results (SPS4)

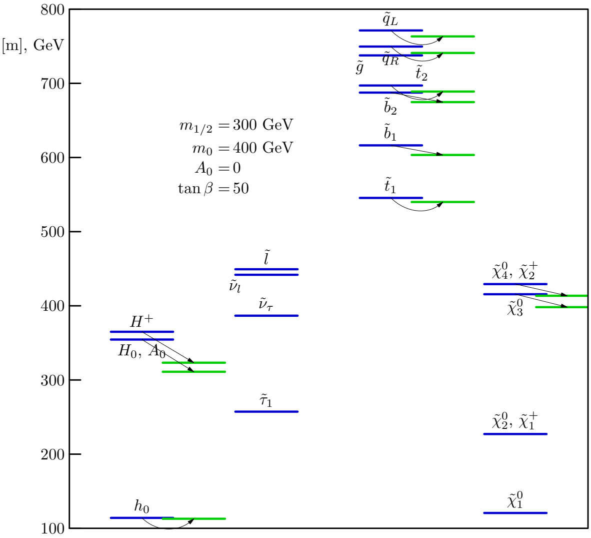
3-loop RGE only



The task is to incorporate the corrections and RGEs in computer codes, e.g. in [SoftSusy].

Numerical results (SPS4)

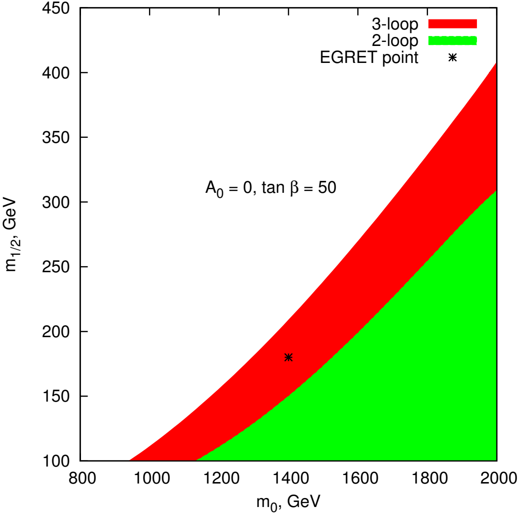
3-loop RGE with thresholds



The task is to incorporate the corrections and RGEs in computer codes, e.g. in [SoftSusy].

Influence on “no EWSB” boundary

Dominant shift in the boundary comes from the inclusion of two-loop t-quark decoupling constant.



Conclusions

Main results include

1. Calculation of two-loop treshold corrections to α_s, m_b, m_t and m_τ. (Issue with "gauge-dependence" of $\delta\zeta_{m_b}^{(2)}$ is resolved).
2. Three-loop RGEs for rigid theory and soft supersymmetry-breaking terms are implemented in the SOFTSUSY code together with obtained decoupling corrections.
3. Influence on the spectrum of SUSY particles and on allowed regions of CMSSM is (partially) studied.
4. Obtained results can be used to estimate theoretical uncertainty of the parameters fitted with the help of 2-loop RGEs (More reliable than comparison between the results from different computer codes [Softsusy,Suspect,Spheno,Isajet])

Conclusions

Issues and outlook:

1. The expressions are very lengthy $\Rightarrow$ optimization of numerical code is required.
2. Electroweak gauge couplings are ingnored so far. $\Rightarrow$ Calculate both two-loop threshold corrections to electroweak gauge couplings and missing pieces of already obtained expressions for other decoupling constants.
3. Tricky corners of the CMSSM parameter space still produce large uncertanties. $\Rightarrow$ A more careful study is required.

Thank you for attention!

Influence of Yukawa couplings on SUSY spectrum (backup)

- Strong dependence of μ-parameter on the t-quark mass for large values of m_0:

$$\frac{dm^2_{H_2}}{dt} \sim y_t^2\left(M^2_{Q_L} + M^2_{t_R} + M^2_{H_2} + A_t^2\right)$$

$$\mu^2 = \frac{m^2_{H_1} - m^2_{H_2}\tan^2\beta}{\tan^2\beta - 1} - \frac{1}{2}M_Z^2$$

- For large $\tan\beta$ the b-quark and τ-lepton Yukawa couplings becomes important and significantly affects the value of M_A^2:

$$\frac{dm^2_{H_1}}{dt} \sim y_b^2\left(M^2_{Q_L} + M^2_{d_R} + M^2_{H_1} + A_b^2\right) + y_\tau^2\left(M^2_{E_L} + M^2_{\tau R} + M^2_{H_1} + A_\tau^2\right)$$

$$M_A^2 = m^2_{H_1} + m^2_{H_2} + |\mu|^2 = \frac{m^2_{H_2} - m^2_{H_1}}{\cos 2\beta} - M_Z^2$$

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