Neutrino energy quantization in rotating medium

I. Balantsev

Department of Theoretical Physics
Moscow State University

Quarks - 2010

Introduction

- The problem considered in this paper has originated from the studies of neutrino electromagnetic properties and related items [1].
- The studies of flavor conversion in neutrino fluxes give an evidence of non-zero neutrino mass.
- Massive neutrino has nontrivial electromagnetic properties (magnetic and electric dipole moments, electric charge).

[1] C. Giunti, A. Studenikin, *Phys. Atom. Nucl.* **72**, 2151 (2009), arXiv: 0812.3646.

Neutrino electromagnetic processes

1. Neutrino radiative decay $\nu_1 \to \nu_2 + \gamma$.
2. Photon decay $\gamma \to \nu\bar{\nu}$.
3. Neutrino scattering off electrons.
4. Neutrino spin precession in magnetic field.

Quantum theory of such processes [2],[4],[5] has been developed within the method of exact solutions of the modified Dirac equations for the neutrino wave function in matter.

[2] A. Studenikin, A. Ternov *Phys. Lett. B*,**608**, 107, 2005;
A. Grigoriev, A. Studenikin, A. Ternov *Phys. Lett. B*, **622**, 199, 2005, hep-ph/0502231; *Grav. & Cosm.***11**, 132, 2005.
[4] A. Studenikin *J. Phys. A: Math. Theor.*, **41**, 164047, 2008.
[5] A. Studenikin *J. Phys. A: Math. Gen.*, **39**, 6769, 2006;
A. Grigoriev, A. Studenikin, A. Ternov *Phys. Atom. Nucl.*, **72**, 718, 2009.

The modified Dirac equations for the neutrino wave function in matter

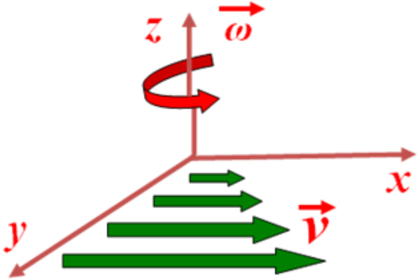
$$\Big\{ i\gamma_\mu \partial^\mu - \frac{1}{2}\gamma_\mu(1+\gamma_5)f^\mu - m \Big\}\Psi(x) = 0$$

In the case of matter composed of neutrons only

$$f^\mu = -G(n, n\mathbf{v}), \quad G = \frac{G_F}{\sqrt{2}}$$

The problem with linear velocity field distribution

$$\vec{v} = (\omega y, 0, 0)$$



The problem with linear velocity field distribution

$$\Big[i\left(\partial_0-\partial_3\right)+Gn\Big]\psi_1+\Big[-\left(i\partial_1+\partial_2\right)+Gn\omega y\Big]\psi_2=m\psi_3,$$
$$\Big[\left(-i\partial_1+\partial_2\right)+Gn\omega y\Big]\psi_1+\Big[i\left(\partial_0+\partial_3\right)+Gn\Big]\psi_2=m\psi_4,$$
$$i\left(\partial_0+\partial_3\right)\psi_3+\left(i\partial_1+\partial_2\right)\psi_4=m\psi_1,$$
$$\left(i\partial_1-\partial_2\right)\psi_3+i\left(\partial_0-\partial_3\right)\psi_4=m\psi_2.$$

Limit $m \to 0$
Right-handed neutrino

$$\psi_R \sim L^{-\frac{3}{2}} \exp\{i(-p_0 t + p_1 x + p_2 y + p_3 z)\}\psi$$

$$\begin{aligned} (p_0 - p_3)\,\psi_3 - (p_1 - ip_2)\,\psi_4 &= 0, \\ -\,(p_1 + ip_2)\,\psi_3 + (p_0 + p_3)\,\psi_4 &= 0. \end{aligned}$$

$$\psi_R = \frac{\mathrm{e}^{-ipx}}{L^{3/2}\sqrt{2p_0(p_0 - p_3)}} \begin{pmatrix} 0 \\ 0 \\ -p_1 + ip_2 \\ p_3 - p_0 \end{pmatrix}$$

Limit $m \to 0$
Left-handed neutrino

$$\left(p_0 + p_3 + Gn\right)\psi_1 - \sqrt{\rho}\left(\frac{\partial}{\partial\eta} - \eta\right)\psi_2 = 0,$$
$$\sqrt{\rho}\left(\frac{\partial}{\partial\eta} + \eta\right)\psi_1 + \left(p_0 - p_3 + Gn\right)\psi_2 = 0,$$

$$\eta = \sqrt{\rho}\left(x_2 + \frac{p_1}{\rho}\right), \quad \rho = Gn\omega, \;\; G = \frac{G_F}{\sqrt{2}}$$

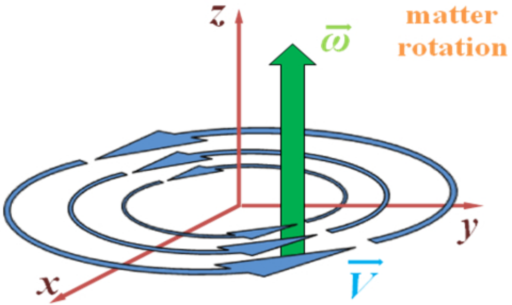
$$n - matter\, density, \; \omega - angular\, frequency.$$

$$\psi_L = \frac{\rho^{\frac{1}{4}} \mathrm{e}^{-ip_0t + ip_1x + ip_3z}}{L\sqrt{(p_0 - p_3 + Gn)^2 + 2\rho N}} \begin{pmatrix} (p_0 - p_3 + Gn)\, u_N(\eta) \\ -\sqrt{2\rho N} u_{N-1}(\eta) \\ 0 \\ 0 \end{pmatrix}$$

$$p_0 = \sqrt{p_3^2 + 2\rho N} - Gn, \quad N = 0, 1, 2, \ldots$$

Neutrino in rotating medium

$$\mathbf{v} = (-\omega y, \omega x, 0)$$



Neutrino in rotating medium

$$-(p_0+p_3+Gn)\psi_1 + i\mathrm{e}^{-i\phi}\left\{\frac{\partial}{\partial r} - \frac{i}{r}\frac{\partial}{\partial\phi} + \rho r\right\}\psi_2 = -m\psi_3,$$
$$i\mathrm{e}^{i\phi}\left\{\frac{\partial}{\partial r} + \frac{i}{r}\frac{\partial}{\partial\phi} - \rho r\right\}\psi_1 + (p_3-p_0-Gn)\psi_2 = -m\psi_4,$$
$$(p_0-p_3)\psi_3 + i\mathrm{e}^{-i\phi}\left(\frac{\partial}{\partial r} - \frac{i}{r}\frac{\partial}{\partial\phi}\right)\psi_4 = m\psi_1,$$
$$+i\mathrm{e}^{i\phi}\left(\frac{\partial}{\partial r} + \frac{i}{r}\frac{\partial}{\partial\phi}\right)\psi_3 + (p_0+p_3)\psi_4 = m\psi_2.$$

$$[H, J_z] = 0 \Rightarrow \psi(t,x,y,z) = \mathrm{e}^{-ip_0t+ip_3z}\begin{pmatrix} i\chi_1(r)e^{i(l-1)\phi} \\ \chi_2(r)e^{il\phi} \\ i\chi_3(r)e^{i(l-1)\phi} \\ \chi_4(r)e^{il\phi}\end{pmatrix}$$

Rising and decreasing operators

$$
\begin{aligned}
-(p_0 + p_3 + Gn)\chi_1 + \left\{\tfrac{d}{dr} + \tfrac{l}{r} + \rho r\right\}\chi_2 &= -m\chi_3, \\
\left\{\tfrac{d}{dr} - \tfrac{l-1}{r} - \rho r\right\}\chi_1 + (p_0 - p_3 + Gn)\chi_2 &= m\chi_4, \\
(p_0 - p_3)\chi_3 + \left(\tfrac{d}{dr} + \tfrac{l}{r}\right)\chi_4 &= m\chi_1, \\
\left(\tfrac{d}{dr} - \tfrac{l-1}{r}\right)\chi_3 - (p_0 + p_3)\chi_4 &= -m\chi_2.
\end{aligned}
$$

$$
R^+ = \frac{d}{dr} - \frac{l-1}{r} - \rho r, \qquad R^- = \frac{d}{dr} + \frac{l}{r} + \rho r.
$$

$$
\begin{aligned}
R^-R^+\chi_1 + \left((p_0 + Gn)^2 - p_3^2 - m^2\right)\chi_1 &= m(Gn\chi_3 + \rho r\chi_4), \\
R^+R^-\chi_2 + \left((p_0 + Gn)^2 - p_3^2 - m^2\right)\chi_2 &= m(Gn\chi_4 + \rho r\chi_3).
\end{aligned}
$$

Approximate solution for the neutrino in rotating medium

$$R^- R^+ \chi_1 + \left((p_0 + Gn)^2 - p_3^2 - m^2\right) \chi_1 = 0,$$
$$R^+ R^- \chi_2 + \left((p_0 + Gn)^2 - p_3^2 - m^2\right) \chi_2 = 0.$$

Wave functions

$$\begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} = \begin{pmatrix} C_1 \mathcal{L}_s^{l-1}(\rho r^2) \\ C_2 \mathcal{L}_s^{l}(\rho r^2) \end{pmatrix},$$

$$R^+ \, \mathcal{L}_s^{l-1}(\rho r^2) = -2\sqrt{\rho(s+l)}\, \mathcal{L}_s^{l}(\rho r^2),$$
$$R^- \, \mathcal{L}_s^{l}(\rho r^2) = 2\sqrt{\rho(s+l)}\, \mathcal{L}_s^{l-1}(\rho r^2).$$

The spectrum

$$p_0 = \sqrt{m^2 + p_3^2 + 4N\rho} - Gn, \quad N = 0, 1, 2, \ldots \,.$$

Neutrino trapping on circular orbits inside of the neutron star

To make estimation let us take into consideration:
- star radius $R_{NS} = 10\ km$,
- density $n = 10^{37} cm^{-3}$
- angle velosity $\omega = 2\pi \times 10^3\ s^{-1}$

For this set of parameters the radius of an orbit is less than the typical star radius if the quantum number

$$N \leq N_{max} = 10^{10}.$$

The energy is of the order $\tilde{p}_0 \sim 1\ eV$.
To consider the motion of neutrino on circular orbits in such a case we can use quasiclassical aproach.

References

- [1] Giunti C., Studenikin A., *Phys. Atom. Nucl.* **72**, 2151 (2009), arXiv: 0812.3646.
- [2] Studenikin A., Ternov A. *Phys. Lett. B*,**608**, 107, 2005; Grigoriev A., Studenikin A., Ternov A. *Phys. Lett. B*, **622**, 199, 2005, hep-ph/0502231; *Grav. & Cosm.***11**, 132, 2005; *Phys. Atom. Nucl.***69**, 1940, 2006.
- [3] Lobanov A. *Dokl. Phys.***50**, 286, 2005; *Phys. Lett.B*, **619**, 136, 2005.
- [4] Studenikin A. *J. Phys. A: Math. Theor.*, **41**, 164047, 2008.
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- [6] Balantsev I., Popov Yu., Studenikin A. *Nuovo Cimento C*, **32**, 53, 2009.