# Black holes production in transplanckian collisions in (anti)-de Sitter spacetime

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(based on joint works with
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- The volume of the trapped surface is being interpreted as the cross section of black holes production

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- Beyond the $AdS/CFT$-duality there is also $dS/CFT$-duality (Strominger - 2001). Therefore there is a chance to extend the holographic approaches to QCD on the de Sitter case
- $dS$ background can be considered as the simplest model of spacetime with dark energy. In this context we analyze possible influence of dark energy on the process of black holes production in collisions

# Metric of spacetime

- It is convenient to immerse $D$-dimensional $(A)dS$ spacetime with shock wave as submanifold to $(D+1)$-dimensional Minkowski space. In these terms the metric is following:

$$ds^2 = -dUdV + d\vec{Z}^2 + dZ^{D^2} + F(Z^D)\delta(U)dU^2,$$

$$\vec{Z} = \{Z^2, ..., Z^{D-1}\}, \ -UV + \vec{Z}^2 \pm Z^{D^2} = \pm a^2,$$

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- The shape of shock-wave could be obtained by boosting of the $(A)dS$-Schwarzschild solution (Hotta, Tanaka - 1992):

$$\lim_{v\to 1} \frac{1}{\sqrt{1-v^2}} f\left(\frac{(Z^0 + vZ^1)^2}{1-v^2}; \lambda\right) = \delta(Z^0 + Z^1) \int\limits_{-\infty}^{\infty} f(x^2; \lambda) dx$$

# Shape function properties

- Explicit integral representation:

$$F(Z) = 2Ma^2 \cdot \; p.v. \int_{-\infty}^{\infty} \frac{(a^2(\pm Z^2 + x^2) + Z^2(x^2 \mp Z^2))}{(Z^2 \mp x^2)^2(\pm a^2 + x^2 \mp Z^2)^{\frac{D-1}{2}}} dx$$

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- For example:

$$F_{4,dS}(Z^4) = 4MG_4 \left( -2 + \frac{Z^4}{a} \ln \left( \frac{1 + \frac{Z^4}{a}}{1 - \frac{Z^4}{a}} \right) \right)$$

$$F_{5,AdS}(Z^5) = \frac{3\pi MG_5}{2a} \left( \frac{2\frac{Z^{5\,2}}{a^2} - 1}{\sqrt{\frac{Z^{5\,2}}{a^2} - 1}} - 2\frac{Z^5}{a} \right)$$

# Shape function properties

- In the charged case:

$$F_D(Q,Z) = F_D(Z) - \frac{Q^2}{2M} F_{2D-3}(Z),$$

where rescaled energy and charge are

$$M = \frac{8\pi G_D m}{(D-2)\Omega_{D-2}}, \; Q^2 = \frac{8\pi G_D q^2}{(D-2)(D-3)}$$

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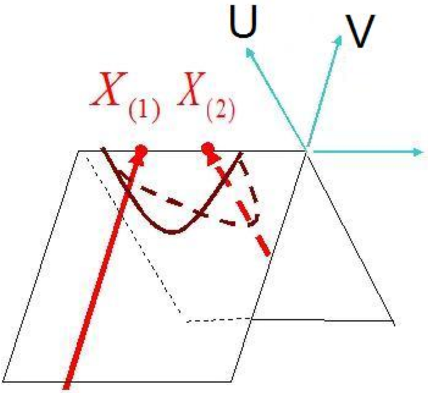
- Th. (Hawking-Penrose) A spacetime (M; g) with a complete future null infinity which contains a closed trapped surface must contain a future event horizon, the interior of which contains the trapped surface
- This has been proved for asymptotically flat spacetimes, but there is a common opinion that it is valid for $dS/AdS$ too

# Geometrical structure




- Parts of trapped surface:

$$S = S_1 \cup S_2;$$

$$S_1 : \ U = 0, \ V = -\Psi_1(\rho),$$

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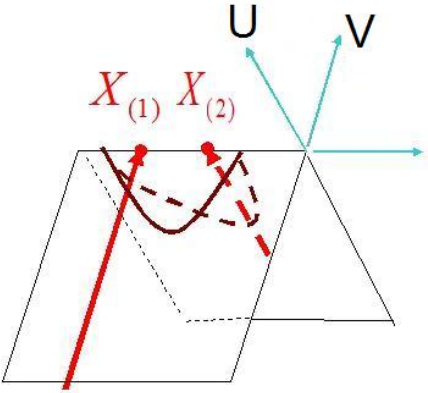


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- On the surface of gluing:

$$\Psi_1 = \Psi_2$$

$$\left(\frac{d\Psi}{d\rho}\right)^2 = 4$$

# Basic mathematical objects

- Trapped surface is a surface of null convergence:

$$\theta = h^{MN}\nabla_N \xi_M; \;\; \theta = 0$$

Here:

$\xi_M$ is a vector tangent to geodesic flow,
$\nabla_M$ is a covariant derivation in the sense of $D$-dimensional metric

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- And (here $M, N = 0..D-1$; $\alpha, \beta = 0..D-3$):

$$h^{MN} = K^M_\alpha g^{\alpha\beta} K^N_\beta,$$

$$K^M_\alpha = (0, -\partial_\alpha \Psi, \delta^M_\alpha),$$

$$g_{\alpha\beta} = \frac{\partial Z^M}{\partial s^\alpha}\frac{\partial Z^N}{\partial s^\beta} g_{MN}$$

# Dirichlet problem

- Equation on the trapped surface:

$$\left(\triangle_{\mathbb{S}^{D-2}/\mathbb{H}^{D-2}} \pm \frac{D-2}{a^2}\right) \frac{\Psi - H}{1 \pm \frac{\rho^2}{2a^2}} = 0, \quad H = \frac{1}{2}(1 \pm \frac{\rho^2}{2a^2})F$$

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- Boundary conditions in arbitrary dimension give us the following equation on the radius of the trapped surface:

$$\frac{1}{4}(1 \pm \frac{\rho_0^2}{2a^2})F'(\rho_0) \pm \frac{\rho_0}{2a^2 \mp \rho_0^2}F(\rho_0) + 2 = 0$$

# Criticality in de Sitter space

- In de Sitter space we deal with equation

$$\frac{1}{4}(1+\frac{\rho_0^2}{2a^2})F'(\rho_0)+\frac{\rho_0}{2a^2-\rho_0^2}F(\rho_0)+2=0$$

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- There is the explicit general formula:

$$\frac{(2+x^2)^{D-2}}{x^{D-3}(2-x^2)}=C_D\frac{a^{D-3}}{p}; \quad x=\rho/a$$

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- These functions have positive minima at the points

$$x_{min,D}=\frac{\sqrt{2(-1+D-2\sqrt{D-2})}}{\sqrt{D-3}}, \quad D>3;$$

$$x_{min,3}=0, \quad D=3$$

# When does the trapped surface exist in dS?

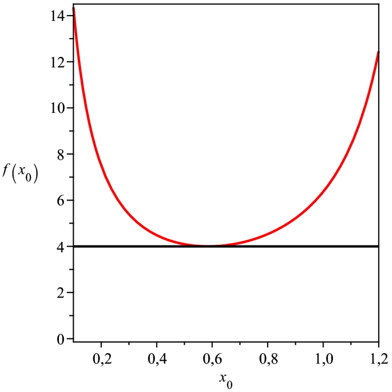


Figure: Critical effect of trapped surface non-formation.

# Charged shock waves in dS

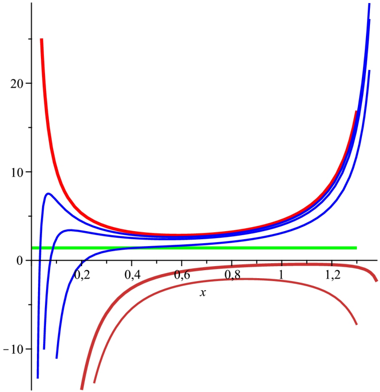


Figure: Influence of charge on the cross section of BH production in dS

# Charged shock waves in AdS

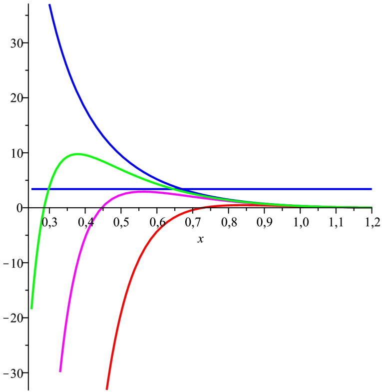


Figure: Influence of charge on the cross section of BH production in AdS

# Volume of the trapped surface

- Volume of the trapped surface can be evaluated in general way and does not depend explicitly on the form of $\Psi$. We consider particular cases of $dS_4$ and $AdS_5$ both in the limit of low energy $M \ll a^{D-3}$:

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- In particular:

$$\mathcal{A}_{dS_4} \approx 4\pi\rho_0^2(1 - \frac{\rho_0^2}{2a^2}) \approx 32\pi M^2(1 - \frac{4M^2}{a^2} - \frac{3\pi}{64}\frac{Q^2}{M^2}))$$

$$\mathcal{A}_{AdS_5} \approx 4\pi a^3(\frac{3\pi}{2}\frac{M}{a^2})^{2/3}\left(1 - \frac{1}{24}(1 + \frac{5Q^2}{Ma^2})(\frac{4\sqrt{2}}{3\pi}\frac{a^2}{M})^{2/3}\right)$$

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- Influence of charge on the cross section of black holes production has been calculated

# Outlook

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- Holographic description. Application of shock wave technics to solution of QCD problems (Gubser, Yarom, Pufu - 2008)