

Non-linear supersymmetry and brane dynamics

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- 1 Introduction and motivations
- 2 $N = 1$: goldstino couplings to matter

I.A.-Tuckmantel '04; Komargodski-Seiberg '09

Non-linear MSSM

I.A.-Dudas-Ghilencea-Tziveloglou to appear

- 3 Extended supersymmetry and brane dynamics

Bagger-Galperin '97; I.A.-Derendinger-Maillard '08

Ambrosetti-I.A.-Derendinger-Tziveloglou '09 + '10

Why study goldstino interactions:

- Effective field theory of SUSY breaking at low energies $m_\chi \ll m_{\text{susy}}$
e.g. gauge mediation dominant vs gravity mediation
 χ : longitudinal gravitino with $m_\chi \ll m_{\text{soft}} \ll m_{\text{susy}}$
- Brane dynamics: half SUSY of the bulk broken but NL realized
 \Rightarrow e.g. strongly constrain coupling of brane to bulk fields
exact NL susy in the large volume limit
broken by the orientifold projection at finite volume
 \Rightarrow important for large volume compactifications, e.g. low scale strings

Non-linear SUSY transformations: [5]

$$\delta\chi_\alpha = \frac{\xi_\alpha}{\kappa} + \kappa \Lambda_\xi^\mu \partial_\mu \chi_\alpha \quad \Lambda_\xi^\mu = -i(\chi\sigma^\mu\bar{\xi} - \xi\sigma^\mu\bar{\chi})$$

κ : goldstino decay constant (SUSY breaking scale) $\kappa = (\sqrt{2}m_{\text{susy}})^{-2}$

Goldstino interactions: 3 formulations

- Standard realization

Volkov-Akulov '73, Clark-Love '96, Clark-Lee-Love-Wu '98

- Superfield formalism

Ivanov-Kapustnikov '78, Samuel-Wess '83

Brignole-Feruglio-Zwirner '97, Luty-Ponton '98, I.A.-Tuckmantel '04

- Constrained superfields

Rocek-Tseytlin '78, Lindstrom-Rocek '79, Komargodski-Seiberg '09

Standard realization

Define the 'metric': $G_{\mu}^{\nu} = \delta_{\mu}^{\nu} + \kappa^2 t_{\mu}^{\nu}$ $t_{\mu}^{\nu} = i\chi \overleftrightarrow{\partial}_{\mu} \sigma^{\nu} \bar{\chi}$

$\delta(\det G) = \kappa \partial_{\mu} \left(\Lambda_{\xi}^{\mu} \det G \right) \Rightarrow$ invariant action:

$$S_{VA} = -\frac{1}{2\kappa^2} \int d^4x \det G = -\frac{1}{2\kappa^2} - \frac{i}{2} \chi \sigma^{\mu} \overleftrightarrow{\partial}_{\mu} \bar{\chi} + \dots$$

Generalization to matter and gauge fields:

$$S_{eff} = \int d^4x \det G \mathcal{L}_{SM}(\phi) \quad \text{invariant if } \delta\phi = \kappa \Lambda_{\xi}^{\mu} \partial_{\mu} \phi \quad \text{and so } \mathcal{L}_{SM}$$

However problem with derivatives \Rightarrow define SUSY covariant ones:

$$\mathcal{D}_{\mu} \phi \equiv (G^{-1})_{\mu}^{\nu} D_{\nu} \phi \quad \mathcal{F}_{\mu\nu} \equiv (G^{-1})_{\mu}^{\lambda} (G^{-1})_{\nu}^{\rho} F_{\lambda\rho}$$

$$\mathcal{L}_{eff} = \det G \mathcal{L}_{SM}(\phi, \mathcal{D}_{\mu} \phi) = \mathcal{L}_{SM}(\phi, D_{\mu} \phi) + \kappa^2 t^{\mu\nu} T_{\mu\nu} + \dots$$

universal coupling to stress-tensor but NOT the most general inv action

Superfield formalism

Recipe: $\phi(x) \rightarrow \Phi(x, \theta, \bar{\theta}) \equiv \phi(\tilde{x}) \quad \tilde{x}^\mu = x^\mu + \Lambda_\theta^\mu(\tilde{x})$ [3] [8]

$$= \phi(x) + \kappa \Lambda_\theta^\mu \partial_\mu \phi + \dots \Rightarrow$$

Goldstino (spinor) superfield: $\mathcal{G}_\alpha = \frac{\theta_\alpha}{\kappa} + \chi_\alpha(\tilde{x})$

space-time derivatives: use the 'metric' $G(\tilde{x})$

$$\text{e.g. } \mathcal{F}_{\mu\nu}(x, \theta, \bar{\theta}) \equiv \left[(G^{-1})_\mu^\lambda (G^{-1})_\nu^\rho F_{\lambda\rho} \right] (\tilde{x})$$

$$O = \int d^2\theta d^2\bar{\theta} \mathcal{O} = \sum_{n \geq 0} \kappa^n O^{(n)} \quad \text{even/odd } n \leftrightarrow \text{even/odd number of } \chi\text{'s}$$

$$\text{dims: } [O] = d \geq 0 \Rightarrow [O] = d + 2, [O^{(n)}] = d + 2 + 2n$$

$$\text{Effective operators of dimension } \leq 8 \Rightarrow d \leq 2, n \leq 2$$

List of lowest dim operators

2 operators of dim 6 linear in χ [12]

$$S_1 = C_1 \int d^4x \kappa F_{\mu\nu} \psi \sigma^\mu \partial^\nu \bar{\chi} + h.c. \quad S_2 = C_2 \int d^4x \kappa (\psi \partial_\alpha \chi) D^\alpha \phi + h.c.$$

Quadratic in χ : 1 operator of dim 7

$$S_7 = C_7 \int d^4x \kappa^{3/2} \phi_1 \phi_2 \partial_\mu \chi J_{(\frac{1}{2},0)}^{\mu\nu} \partial_\nu \chi + h.c. \quad J_{(\frac{1}{2},0)}^{\mu\nu} = \frac{i}{4} \sigma^{[\mu} \bar{\sigma}^{\nu]}$$

+ 5 operators of dim 8

$$S_3 = C_3 \int d^4x \kappa^2 (\psi_1 \partial^\mu \chi) (\bar{\psi}_2 \partial_\mu \bar{\chi}) + h.c. \quad S_4 = C_4 \int d^4x \kappa^2 (\psi_1 \psi_2) (\partial_\mu \chi \partial^\mu \chi) + h.c.$$

$$S_5 = C_5 \int d^4x \kappa^2 \phi_1 \overleftrightarrow{D}_\mu \phi_2 i \partial_\alpha \chi \sigma^\mu \partial^\alpha \bar{\chi} + h.c.$$

$$S_6 = C_6 \int d^4x \kappa^2 \partial^\alpha \chi \sigma^\mu \partial^\nu \bar{\chi} \partial_\alpha F_{\mu\nu} + h.c.$$

$$S_8 = C_8 \int d^4x \kappa^2 \phi_1 \phi_2 \phi_3 (\partial_\mu \chi J_{(\frac{1}{2},0)}^{\mu\nu} \partial_\nu \chi) + h.c.$$

Constrained superfields

spontaneous global SUSY: no supercharge but still conserved supercurrent

⇒ superpartners exist in operator space (not as 1-particle states)

⇒ constrained superfields: 'eliminate' superpartners

Goldstino: chiral superfield X_{NL} satisfying $X_{NL}^2 = 0$ ⇒ [22]

$$\begin{aligned} X_{NL}(y) &= \frac{\chi^2}{2F} + \sqrt{2}\theta\chi + \theta^2 F & y^\mu &= x^\mu + i\theta\sigma^\mu\bar{\theta} \\ &= F\Theta^2 & \Theta &= \theta + \frac{\chi}{\sqrt{2}F} \end{aligned}$$

$$\mathcal{L}_{NL} = \int d^4\theta X_{NL}\bar{X}_{NL} - \frac{1}{\sqrt{2}\kappa} \left\{ \int d^2\theta X_{NL} + h.c. \right\} = \mathcal{L}_{VA}$$

$$F = \frac{1}{\sqrt{2}\kappa} + \dots$$

Goldstino couplings to matter

Coupling to superfields: $m_{\text{soft}} \ll E \ll m_{\text{susy}} \sim 1/\sqrt{\kappa}$

replace auxiliary superfield spurion $S = m_{\text{soft}}\theta^2$ by $\sqrt{2\kappa}m_{\text{soft}}X_{NL}$ [15]

Coupling to (non-SUSY) matter: $E \ll m_{\text{soft}}, m_{\text{susy}}$

→ constrained matter superfields

- Fermions: Q_{NL} satisfying $Q_{NL}X_{NL} = 0$ (eliminate sfermions) \Rightarrow

$$Q_{NL} = \sqrt{2} \left(\psi - \frac{F_Q \chi}{F} \right) \Theta + F_Q \Theta^2$$

- Complex scalars: H_{NL} with $X_{NL}\bar{H}_{NL} = \text{chiral}$ (eliminate 'higgsinos') [5]

$$\Rightarrow H_{NL} = H(\hat{y}) \quad \hat{y} = y^\mu + i\sqrt{2}\theta\sigma^\mu\bar{\chi}(\hat{y})/\bar{F}(\hat{y})$$

- Real scalars: A_{NL} with $X_{NL}\bar{A}_{NL} = X_{NL}A_{NL}$

$$\Rightarrow \text{Im}A = \frac{1}{2FF} (\chi\sigma^\mu\bar{\chi}) \partial_\mu \text{Re}A + \dots$$

- Gauge fields: V_{NL}

gauge transformations: $\delta V = \Omega + \bar{\Omega}$ with $X_{NL}(\Omega_{NL} + \bar{\Omega}_{NL}) = 0$

e.g. charged matter $H_{NL} = e^{iR_{NL}}$; $X_{NL}(R_{NL} - \bar{R}_{NL}) = 0$: $\delta R_{NL} = i\Omega_{NL}$

\Rightarrow convenient gauge choice $X_{NL}V_{NL} = 0$

eliminate gaugino: $X_{NL}W_{NL} = 0$ field strength $W = -\frac{1}{4}\bar{D}^2 DV$

$\Rightarrow V_{NL} = -\Theta \left(\sigma^m V_m + \frac{D}{|F|^2} \chi \bar{\chi} \right) \bar{\Theta} + \frac{1}{2} \Theta^2 \bar{\Theta}^2 D + \text{derivatives}$

D-brane dynamics

Type II (closed) strings on $4d$ Minkowski $M_4 \times X_6$ internal $6d$ manifold

X_6 flat $\Rightarrow N = 8$ SUSY ; X_6 Calabi-Yau $\Rightarrow N = 2$ SUSY

Single stack of N Dp -branes \Rightarrow half SUSY is spontaneously broken $p \geq 3$

$(p - 3)$ dims wrapped around cycles in $X_6 \Rightarrow 4d$ effective field theory

- Gauge group: $G = U(N)$ (generically)
- SUSY: half remains unbroken Q_e ; other half NL realized Q_o
broken SUSY commutes with $G \Rightarrow$ goldstino = $U(1)$ gaugino of Q_e

Intersecting branes: useful framework for model building

Standard Model embedding

Two D-brane stacks: $N_1 Dp_1$ and $N_2 Dp_2$

⇒ bifundamental matter on their intersections: chiral fermions

L-SUSY: generally broken but preserved for special intersection angles

e.g. for $X_6 = T^2 \times T^2 \times T^2$ when $\theta_1 + \theta_2 + \theta_3 = 0$

NL-SUSY: generally all (both $U(1)_1 \times U(1)_2$ gauginos = goldstinos)

special angles ⇒ only a linear combination

Remark: string consistency (e.g. tadpole cancellation) ⇒ need orientifolds

non-dynamical planes ⇒ break half-SUSY explicitly

⇒ goldstino gets a volume suppressed mass

NL-SUSY only locally → restored in the large volume limit [19]

1) Goldstino decay constant: sum of brane tensions

$$\frac{1}{2\kappa^2} = T_1 + T_2 \quad T_i = \frac{M_s^4}{4\pi^2 g_i^2} N_i$$

2) Goldstino couplings: only 3 non-vanishing up to order κ^2 [6]

$$C_1 = \sqrt{2} \quad ; \quad C_2 = 2 \quad ; \quad C_3 = 2$$

- universal coefficients independent of brane-angles
- C_3 : fixes the field theory ambiguity of 4-fermion operator

Brignole-Feruglio-Zwirner '97, I.A.-Benakli-Laugier '01

- $C_{1,2}$: dim 6 operators linear in χ can be written as

$$\mathcal{L}_{\text{linear}} = \frac{\kappa}{\sqrt{2}} J^\mu \partial_\mu \chi + h.c. \quad J^\mu : N = 1 \text{ supercurrent of linear SUSY}$$

present in the intersection if massless scalars

Phenomenological analysis in the Standard Model

$$\mathcal{L}_\chi = -\frac{i}{2}\chi\sigma^\mu\overleftrightarrow{\partial}_\mu\bar{\chi} + i\kappa^2(\chi\overleftrightarrow{\partial}^\mu\sigma^\nu\bar{\chi})T_{\mu\nu} + \delta\mathcal{L}_\chi$$

$$\delta\mathcal{L}_\chi = i\sqrt{2}\kappa F_{\mu\nu}\psi\sigma^\mu\partial^\nu\bar{\chi} + 2\kappa D_\mu\phi(\psi\partial^\mu\chi) + h.c.$$

$$+ 2\kappa^2(\partial_\mu\chi\psi_1)(\partial^\mu\bar{\chi}\bar{\psi}_2) + \mathcal{O}(\kappa^3)$$

- 1st term: hypercharge + fermion singlet
- 2nd term: higgs + lepton doublets I.A.-Tuckmantel-Zwirner '04

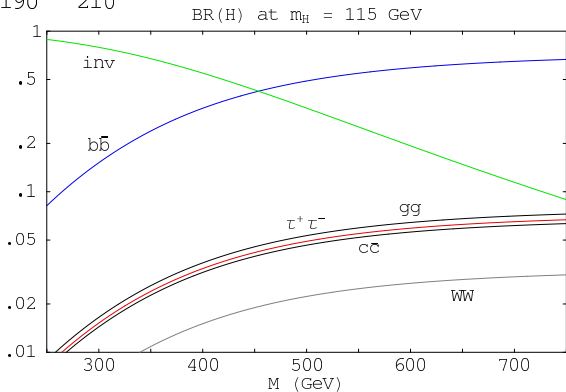
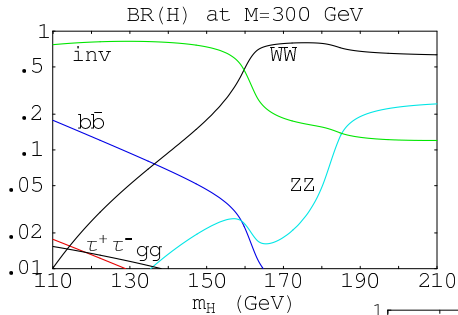
preserves lepton number if $L(\chi) = -1$

$$Z, H \rightarrow \nu\chi \quad W^\pm \rightarrow l^\pm\chi \Rightarrow M = m_{\text{susy}} |\sqrt{2}/C_2|^{1/2} \simeq M_s/2$$

- bounds: $M \gtrsim 270 \text{ GeV} \Rightarrow M_s \gtrsim 500 \text{ GeV}$ (e.g. invisible Z width)

- signal: invisible Higgs decay

dominant or non-negligible in a large range of (M, m_H)



Phenomenological analysis in the MSSM [8]

$$E \sim m_{\text{soft}} \gg m_\chi$$

I.A.-Dudas-Ghilencea-Tziveloglou to appear

Higgs potential is modified:

$$V = V_{\text{MSSM}} + 2\kappa^2 |m_1^2| |h_1|^2 + m_2^2 |h_2|^2 + B\mu h_1 h_2 + \mathcal{O}(\kappa^4) \Rightarrow$$

$m_{1,2}, B\mu$: soft mass parameters, μ : higgsino mass

classical value of light higgs mass can be increased above the LEP bound

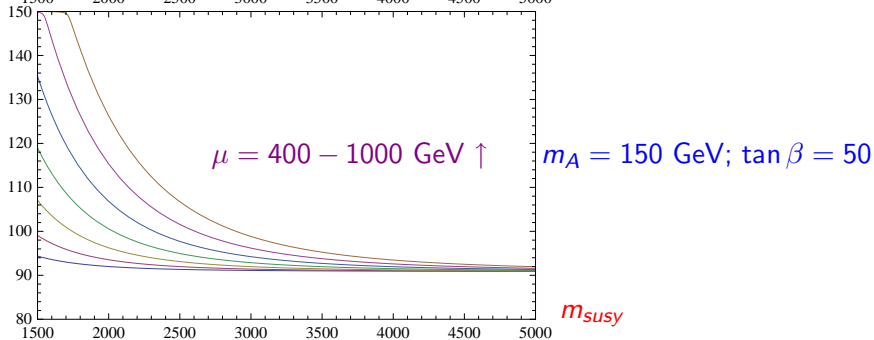
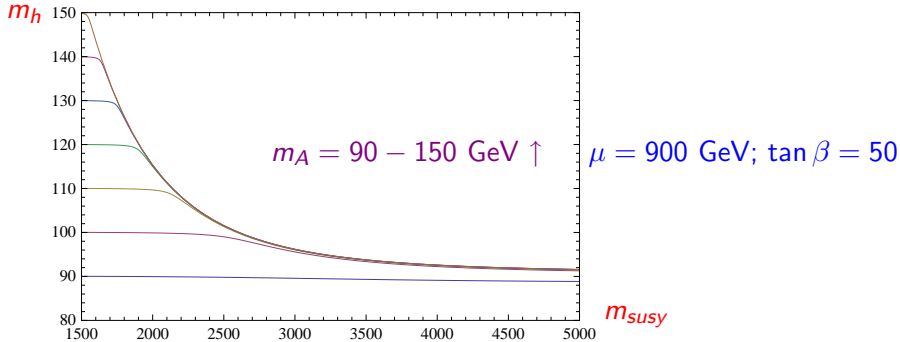
$$\text{large } \tan \beta \text{ limit: } m_h^2 = m_Z^2 + \frac{v^2}{2m_{\text{SUSY}}^2} (2\mu^2 + m_Z^2)^2 + \dots$$

$$\rightarrow \text{e.g. } \mu = 900 \text{ GeV, } m_{\text{SUSY}} = 2 \text{ TeV} \Rightarrow m_h = 114.4 \text{ GeV}$$

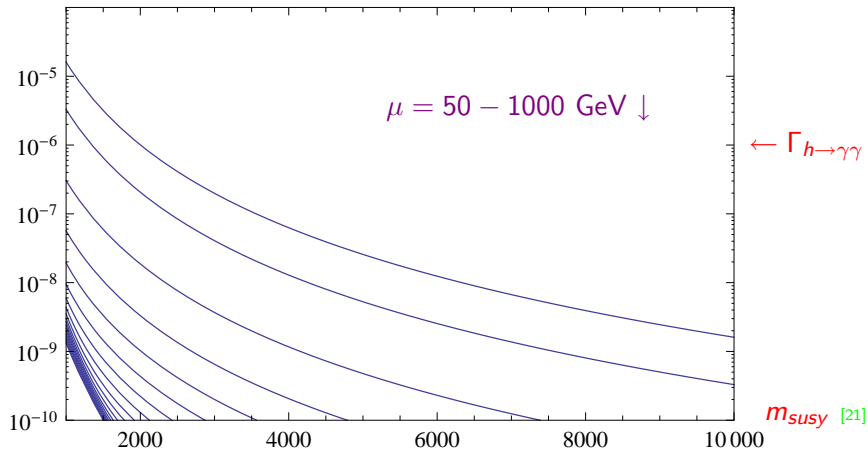
Quartic higgs coupling increases for large soft masses \Rightarrow

MSSM 'little' fine tuning is alleviated

New couplings \Rightarrow invisible higgs decay $h \rightarrow \chi + \text{NLSP}$



$\Gamma_{h \rightarrow \chi + \text{NLSP}}$ $m_A = 150 \text{ GeV}; \tan \beta = 50; (m_{\lambda_1}, m_{\lambda_2}) = (70, 150) \text{ GeV}$



Conclusions

Non-linear supersymmetry: powerful tool for studying:

- low energy SUSY breaking $E \ll m_{SUSY} \sim 1/\sqrt{\kappa}$

Volkov-Akulov action and goldstino χ couplings to matter

standard coupling to stress-tensor *not* the most general

→ detailed analysis \Rightarrow dim 6 operators linear in χ

$E \gg m_{soft} \Rightarrow$ goldstino \equiv spurion coupled to supermultiplets

→ Non-linear MSSM

$E \ll m_{soft} \Rightarrow$ goldstino coupling to Standard Model fields

- brane effective actions \Rightarrow brane dynamics

$N = 1$ string computation: 3 independent couplings up to dim 8

Brane supersymmetry breaking

I.A.-Dudas-Sagnotti, Aldazabal-Uranga '99

Stable configurations of branes with orientifolds

- absence of tachyons
- bulk susy breaking suppressed by (transverse) volume

	D	\bar{D}	O	\bar{O}
RR charge	+	-	-	+
tension	+	+	-	-
linear SUSY	Q_e	Q_o	Q_e	Q_o
NL SUSY	Q_o	Q_e		

Model I: DO or $\bar{D}\bar{O}$ \Rightarrow local charge conservation, brane SUSY (locally)

Model II: $\bar{D}O$ or $D\bar{O}$ \Rightarrow brane SUSY breaking (linear), NL SUSY

- Model I: N D p -branes on O p -plane \Rightarrow $SO(2N)$ SUSY
- Model II: N D p branes on $\bar{O}p$ -plane \Rightarrow $SO(2N)$ with fermions in the symmetric representation = traceless + trace \leftarrow goldstino
 Dudas-Mourad, Pradisi-Riccioni '01
 orientifold: antisymmetrizes bosons but symmetrizes fermions
 \Rightarrow breaks SUSY
- Model III: D-branes away from O-plane \Rightarrow $U(N)$ SUSY \leftarrow L + NL
 partial breaking $N = 2 \rightarrow N = 1$ $U(1)$: goldstino multiplet [11]

Goldstino in multiplet of $N = 1$ SUSY: **vector or chiral?**

brane dynamics \Rightarrow Maxwell goldstino multiplet

gauge chiral multiplet $|_{N=2} \mathcal{W} = (\text{vector } W + \text{chiral } X)_{N=1}$

$$\mathcal{W}(y, \theta, \tilde{\theta}) = X(y, \theta) + i\sqrt{2}\tilde{\theta}W(y, \theta) - \tilde{\theta}^2 \left[\frac{1}{4}\overline{DDX}(y, \theta) + \frac{1}{2\kappa} \right]$$

allow partial SUSY breaking $N = 2 \rightarrow N = 1$ 

$$\delta^* X = i\sqrt{2}\eta^\alpha W_\alpha \quad \delta^* W_\alpha = \frac{i}{\sqrt{2}\kappa}\eta_\alpha + \dots \leftarrow \text{linear SUSY}$$

$$\mathcal{L}_{Maxwell}^{N=2} = -\frac{1}{8} \int d^2\theta d^2\tilde{\theta} \mathcal{W}^2 + h.c. = \int d^2\theta \left[\frac{1}{2} W^2 - \frac{1}{4} X \overline{DDX} - \frac{1}{2\kappa} X \right] + h.c.$$

Partial SUSY breaking: non trivial prepotential $f(\mathcal{W})$

I.A.-Partouche-Taylor '96

Non-linear $N = 2$ constraint: $\mathcal{W}_{NL}^2 = 0$

$$\Rightarrow X^2 = 0 \quad , \quad XW_\alpha = 0 \quad , \quad WW - \frac{1}{2}X\overline{DDX} = \frac{1}{\kappa}X \quad [7]$$

$$X = \kappa W^2 - \kappa^3 \bar{D}^2 \frac{W^2 \bar{W}^2}{1+A_+ + \sqrt{1+2A_+ + A_-^2}} \quad A_\pm = \frac{\kappa^2}{2} \left(D^2 W^2 \pm \bar{D}^2 \bar{W}^2 \right) = \pm A_\pm^*$$

$$\Rightarrow \mathcal{L}_{NL}^{N=2} = \frac{1}{4\kappa} \int d^2\theta X + h.c.$$

$$= \frac{1}{8\kappa^2} \left(1 - \sqrt{-\det(\eta_{\mu\nu} + 2\sqrt{2}\kappa F_{\mu\nu})} \right) + \dots = \mathcal{L}_{\text{DBI}} \leftarrow \text{D-brane}$$

The FI-term is also invariant under NL SUSY

$$\mathcal{L}_{FI} = \xi \int d^4\theta V; \quad W = -\frac{1}{4}\bar{D}^2 DV; \quad \delta^* V = \frac{i}{2\kappa} (\eta D + \bar{\eta} \bar{D}) \theta^2 \bar{\theta}^2 + \dots$$

$$\Rightarrow \mathcal{L}_{Max}^{NL} = \mathcal{L}_{NL}^{N=2} + \mathcal{L}_{FI} \quad [24]$$

Coupling to bulk hypermultiplets e.g. the universal dilaton

at least one isometry \rightarrow single-tensor multiplet (RR 2-forms)

$$N = 2 \text{ tensor } \mathcal{Y} = (\text{tensor } L + \text{chiral } \Phi)_{N=1} \quad D^2 L = \bar{D}^2 L = 0$$

general action: $\mathcal{L}_{ST} = \int d^4 \theta \mathcal{H}(L, \Phi, \bar{\Phi})$ with $(\partial_L^2 + 2\partial_\Phi \partial_{\bar{\Phi}}) \mathcal{H} = 0$

$$L = D^\alpha \ell_\alpha + h.c. \quad \ell_\alpha: \text{chiral spinor superfield}$$

off-shell $N = 2 \Rightarrow$ add auxiliary chiral superfield $Y \sim \theta^2 \epsilon \cdot C_4 \leftarrow$ 4-form

$$\mathcal{Y}(y, \theta, \tilde{\theta}) = Y(y, \theta) + i\sqrt{2} \tilde{\theta} \ell(y, \theta) - \frac{i}{2} \tilde{\theta}^2 \Phi(y, \theta)$$

coupling to $N = 2$ vector \mathcal{W} : Chern-Simons interaction:

$$\mathcal{L}_{CS} \sim g \int d^2 \theta d^2 \tilde{\theta} \mathcal{Y} \mathcal{W} = g \int d^2 \theta \left(\ell^\alpha W_\alpha + \frac{1}{2} \Phi X - \frac{i}{2\kappa} Y \right) + h.c.$$

\Rightarrow global SUSY limit of D-brane coupling to bulk hypermultiplets

$N = 2$ NL QED and novel super-higgs mechanism

General action: $\mathcal{L}_{tot}^{NL} = \mathcal{L}_{CS} + \mathcal{L}_{Max}^{NL}(W) + \mathcal{L}_{ST}(L, \Phi) \Rightarrow$ [22]

superhiggs mechanism without gravity:

Maxwell goldstino $\mathcal{W}_{NL}(W)$ is 'absorbed' by $N = 2$ tensor $\mathcal{Y}(L, \Phi)$

→ $N = 1$ massive vector (W, L) + massless chiral Φ :

- tensor of L + vector of W → massive vector
- scalar of $L \in$ same massive vector multiplet
- goldstino + fermion of L → Dirac spinor

System identical to Higgs phase of $N = 2$ NL QED (up to \mathcal{L}_{ST})

$\Phi \sim Q_1 Q_2$ $L \sim |Q_1|^2 - |Q_2|^2$ (Q_1, Q_2) : charged hypermultiplet

adding mass-term $m \int d^2\theta \Phi \Rightarrow$ also Coulomb phase for $\xi = 0$

Vacuum structure of $N = 2$ NL QED

3 parameters: κ, ξ, m

- $m = 0$: Higgs phase and super-higgs without gravity

$$\langle Q_1 \rangle = v \text{ (real) arbitrary} \quad \langle Q_2 \rangle = \sqrt{\xi + v^2}$$

- $m \neq 0, \xi = 0$: Coulomb phase with $N = 2$ NL SUSY unbroken
- $m \neq 0, \xi \neq 0$: $N = 1$ (linear) SUSY is also broken [18]

Ambrosetti-I.A.-Derendinger-Tziveloglou '09