Workshop on P- and CP-odd effects in hot and dense matter, BNL, 26-30.April 2010

Abnormal enhancement of dilepton yield in central heavy-ion collisions from local parity breaking

A.A. Andrianov

Санкт-Петербургский
государственный
университет

Sankt-Petersburg State University

ICCUB
Institut de Ciències del Cosmos

in collaboration with V.Andrianov, D. Espriu and X.Planells

Some highlights in:

A. Andrianov, D. Espriu, P. Giacconi and R. Soldati, JHEP 09 (2009) 057

A. Andrianov, D. Espriu, F. Mescia and A. Renau, Phys.Lett. B684 (2010) 101

Photon/vector mesons instability in pseudoscalar background

For slowly growing/decreasing neutral pion (isovector) condensate

$$\eta_\alpha \simeq \partial_\alpha \langle \Pi \rangle \simeq \delta_{\alpha 0} \langle \dot{\Pi}(t) \rangle$$

or large scale isoscalar theta field $\mu_5 = \partial_0 \theta / (2N_f)$ ← **axial chemical potential**

(in central heavy ion collisions)

Induced C-S term

$$\boxed{\Delta \mathcal{L} = \frac{1}{4} \eta \epsilon^{ijk} A_i F_{jk}} \qquad \eta_\alpha = (\eta, 0, 0, 0)$$

Adiabatic approximation: $\dot{\eta}/f_\pi \ll \eta \ll \omega_\gamma$

in units $1/fm \sim f_\pi$ **time derivative of CS vector << photon frequency/ vector meson energy**

P-and CP-odd condensates

(*a long list starting from)* **A.B. Migdal, Zh. Eksp. Teor. Fiz. 61 (1971);**
T. D. Lee and G. C. Wick, Phys. Rev. D 9, 2291 (1974)

Isoscalar condensate → theta vacuum bubbles

D. Kharzeev, R. D. Pisarski and M. H. G. Tytgat, Phys. Rev. Lett. 81, 512 (1998)

K. Buckley, T. Fugleberg, and A. Zhitnitsky, Phys. Rev. Lett. 84 (2000) 4814

D. E. Kharzeev, L. D. McLerran and H. J. Warringa, Nucl. Phys. A 803, 227 (2008)

Isotriplet neutral condensate ="pion" condensate

A.A.Andrianov and D.Espriu, Phys.Lett. B 663 (2008) 450 (*and refs..therein)*

A.A.Andrianov, V.A.Andrianov and D.Espriu, Phys.Lett. B 678 (2009) 416

Photons **of different circular polarizations have different dispersion relation between their frequencies and wave vectors**

$$k^{\mu}_{\pm} = (\omega_{\mathbf{k}\pm}, \mathbf{k}) \qquad \omega_{\mathbf{k}\pm} = \sqrt{\mathbf{k}^2 + m_\gamma^2 \pm \eta|\mathbf{k}|}$$

Effective photon masses $k_{\pm}^2 = m_\gamma^2 \pm \eta\,|\mathbf{k}| \simeq \pm\eta\,|\mathbf{k}|$

Photon "-" is a tachyon

Photon "+"decays $\gamma \to l^+ l^-$

for $m_\gamma \ll m_e$

$$\boxed{|\mathbf{k}| \geq \frac{4m_l^2}{\eta} \equiv k_{\mathrm{th}}}$$

Threshold hierarchy!

If for electrons/positrons the threshold is of order 100 MeV then for muons it is four orders of magnitude higher , i.e. 1 TeV !

No muon pairs excess in the PHENIX data!? Maybe in NA60?

Polarizations $A = T, L, +, -$

$$\varepsilon^{\mu}_{T}(k) \equiv \frac{\mathrm{i}\,k^{\mu}}{\sqrt{k^2}} \qquad (k^2 > 0),$$
$$\varepsilon^{\mu}_{L}(k) \equiv (k^2 \mathrm{D})^{-1/2}\left(k^2 \eta^{\mu} - k^{\mu}\,\eta \cdot k\right) \qquad (k^2 > 0)$$

(Almost) circular polarizations of distorted photons

$$\varepsilon^{\mu}_{\pm}(k) \equiv \left[\frac{\mathbf{k}^2 - (\epsilon \cdot k)^2}{2\mathbf{k}^2}\right]^{-1/2} P^{\mu\nu}_{\pm}\,\epsilon_{\nu}$$

$$P^{\mu\nu}_{\pm} \equiv \frac{S^{\mu\nu}}{S} \pm \frac{\mathrm{i}}{\sqrt{2S}}\,\varepsilon^{\mu\nu\alpha\beta}\,\eta_{\alpha}\,k_{\beta}$$

$$S^{\nu}_{\lambda} \equiv \varepsilon^{\mu\nu\alpha\beta}\,\eta_{\alpha}\,k_{\beta}\,\varepsilon_{\mu\lambda\rho\sigma}\,\eta^{\rho}\,k^{\sigma}$$
$$S = S^{\nu}{}_{\nu} = 2[(\eta \cdot k)^2 - \eta^2 k^2]$$

Polarized decay !! $\gamma \to l^{+} l^{-}$

Bounds on lepton momenta

$$\text{for } k \gg k_{\text{th}} \qquad \min p_- \simeq \frac{m_e^2}{\eta} \leq p \leq \max p_+ \simeq k - \frac{m_e^2}{\eta}$$

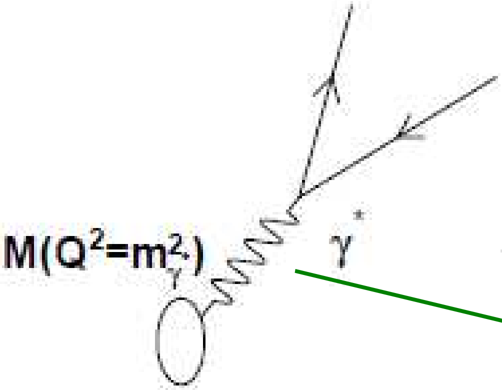
Asymmetric!

Decay width $$\Gamma_+ = \tau_+^{-1} \simeq \frac{\alpha\eta}{3}$$

Constant at high energies !

But the distorted photon is not a proper *Breit-Wigner* resonance as its position (effective mass) moves with momentum!

Dilepton pair creation

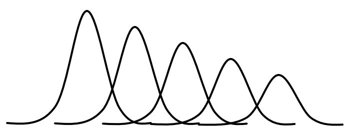


$$\frac{d^2 N_{ee}}{dM^2} = \frac{2\alpha}{3} \frac{M^2}{(M^2 - m_\gamma^2)^2} \Phi(M) dN_{\gamma_*}$$

$$M \equiv m_{ee} \equiv m_{\gamma^*} \qquad \Phi(M) = \sqrt{1 - \frac{4m_e^2}{M^2}} \left(1 + \frac{2m_e^2}{M^2}\right)$$

"Giant" resonance
with variable position

$$\frac{1}{3}\Theta\left(|\vec{k}| - \frac{4m_e^2 - m_\gamma^2}{\eta}\right) \frac{\Gamma_+ \omega_+(|\vec{k}|)}{(M^2 - m_\gamma^2 - \eta|\vec{k}|)^2 + \Gamma_+^2 \omega_+^2(|\vec{k}|)}$$



$$\omega_+(|\vec{k}|) = \sqrt{\vec{k}^2 + m_\gamma^2 + \eta|\vec{k}|}; \quad \Gamma_+ \simeq \frac{\alpha\eta}{3} \quad \text{for} \quad |\vec{k}| \gg \frac{4m_e^2 - m_\gamma^2}{\eta}$$

$|\vec{k}|$

"On-shell" enhancement in a range of | k | !

$$\sim \frac{1}{\Gamma_+ \omega_+(|\vec{k}|)} \sim \frac{3}{\alpha M^2} \qquad |\vec{k}| \gg \frac{4m_e^2 - m_\gamma^2}{\eta}, \eta, m_\gamma$$

$$\boxed{R_{enh}\Big|_{M \gg m_\gamma} = 3/\alpha^2 \sim 10^5}$$

Taking into account thermal distribution

Boltzmann $$f^B(k^0,T)=\frac{1}{e^{k^0/T}-1}$$

For transversal polarizations

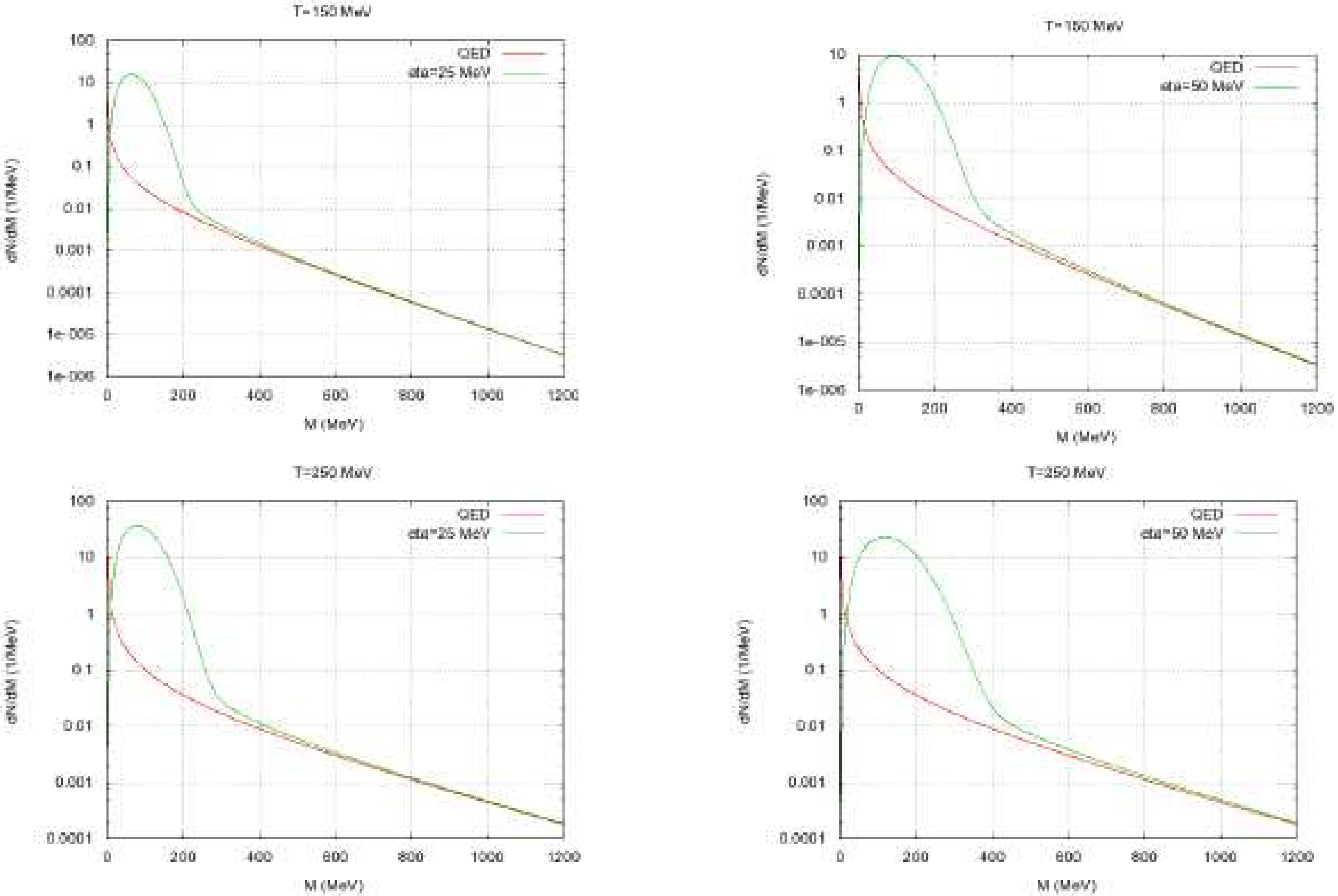
$$\frac{dN_{ee}}{dM}=M^3\frac{\alpha^2}{9\pi^2}\int\frac{1}{[M^2-m_{\rm eff}^2(|\vec{k}|)]^2+\Gamma_\pm^2 w_\pm^2(|\vec{k}|)}\frac{\sqrt{(k^0)^2-M^2}}{e^{k^0/T}-1}dk^0 \qquad |\vec{k}|=\sqrt{(k^0)^2-M^2}$$

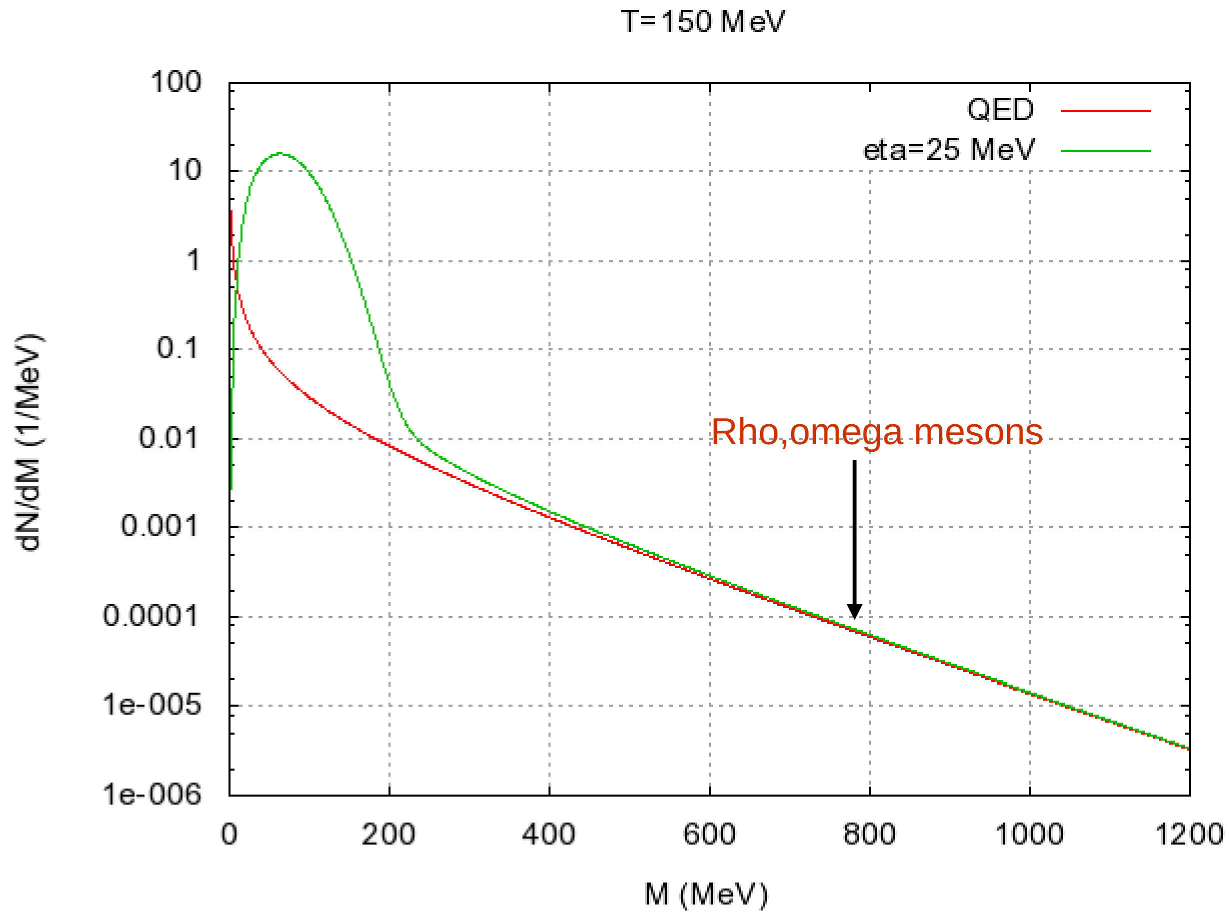
Production rate is sensitive to temperatures via photon effective mass and width

Maximums at $M^2\sim 2T\eta$

RHIC temperatures T = 150 – 250 MeV

Photon thermal distribution makes resonances broader





Does it provide the abnormal ee excess in the range 100 – 700 MeV?

Only partially! **Eta scale is plausibly of order of few MeV's,** $\eta \sim \alpha f_\pi \sim 1\ \mathrm{MeV}$
ρ, ω **mesons must enter the game.**

Finite-volume suppression (qualitatively)

A typical size of nuclear fireball $L \sim 5 \div 10$ fm

Time spent by photons in nuclear medium $\tau_\eta \simeq L$

Resonance wave function and amplitude

$$\psi[\tau] = \exp\left((-i\omega - \frac{1}{2}\Gamma)\tau\right) \Longrightarrow D[E] = i\int_0^{\tau_\eta} d\tau \exp\left((i\Delta E - \frac{1}{2}\Gamma)\tau\right) = \frac{\exp\left((i\Delta E - \frac{1}{2}\Gamma)\tau_\eta\right) - 1}{\Delta E + \frac{1}{2}i\Gamma},$$

$$\Delta E = E - \omega$$

Breit-Wigner

In the peak for $\Delta E = 0$ $\Gamma\tau_\eta \ll 1$ $D_\eta(0)/D(0) \simeq \Gamma\tau_\eta/2 \ll 1$

Absolute enhancement

$$\sim \frac{\Gamma_+\tau_\eta}{2}\frac{2}{3\Gamma_+\omega_+(|\vec{k}|)} \sim \frac{\tau_\eta}{3|\vec{k}|} \sim \frac{\tau_\eta\eta}{3M^2}; \qquad |\vec{k}| \gg \eta, m_\gamma$$

Relative enhancement

$$R_{enh,fin}\Big|_{M\gg m_\gamma} \sim \frac{\tau_\eta\eta}{2\alpha}$$

For $\eta \sim 10$ MeV $\tau_\eta \sim 5$ fm $\sim 1/40MeV^{-1}$ $R_{enh,fin} \sim 17$ **rather large**

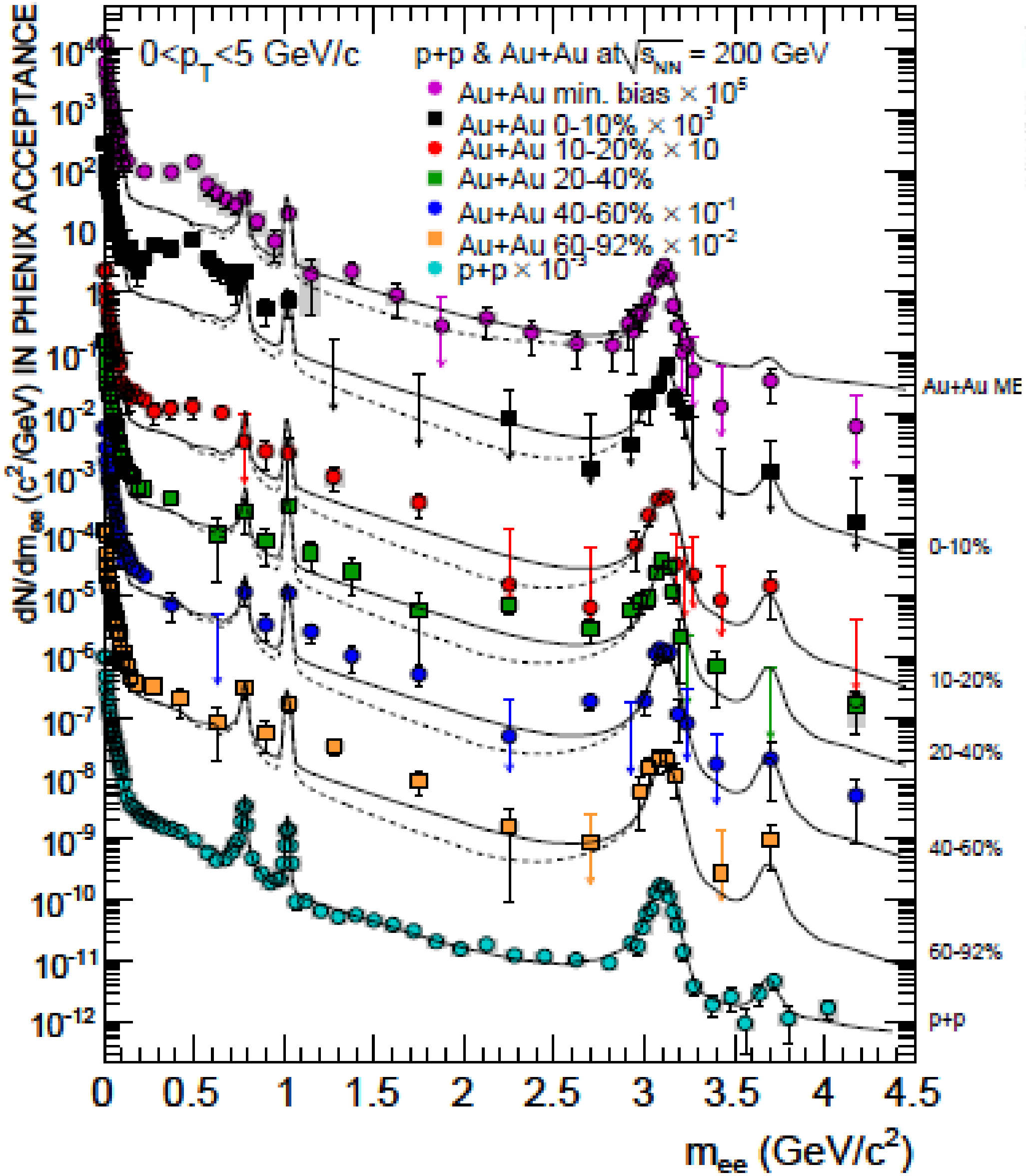
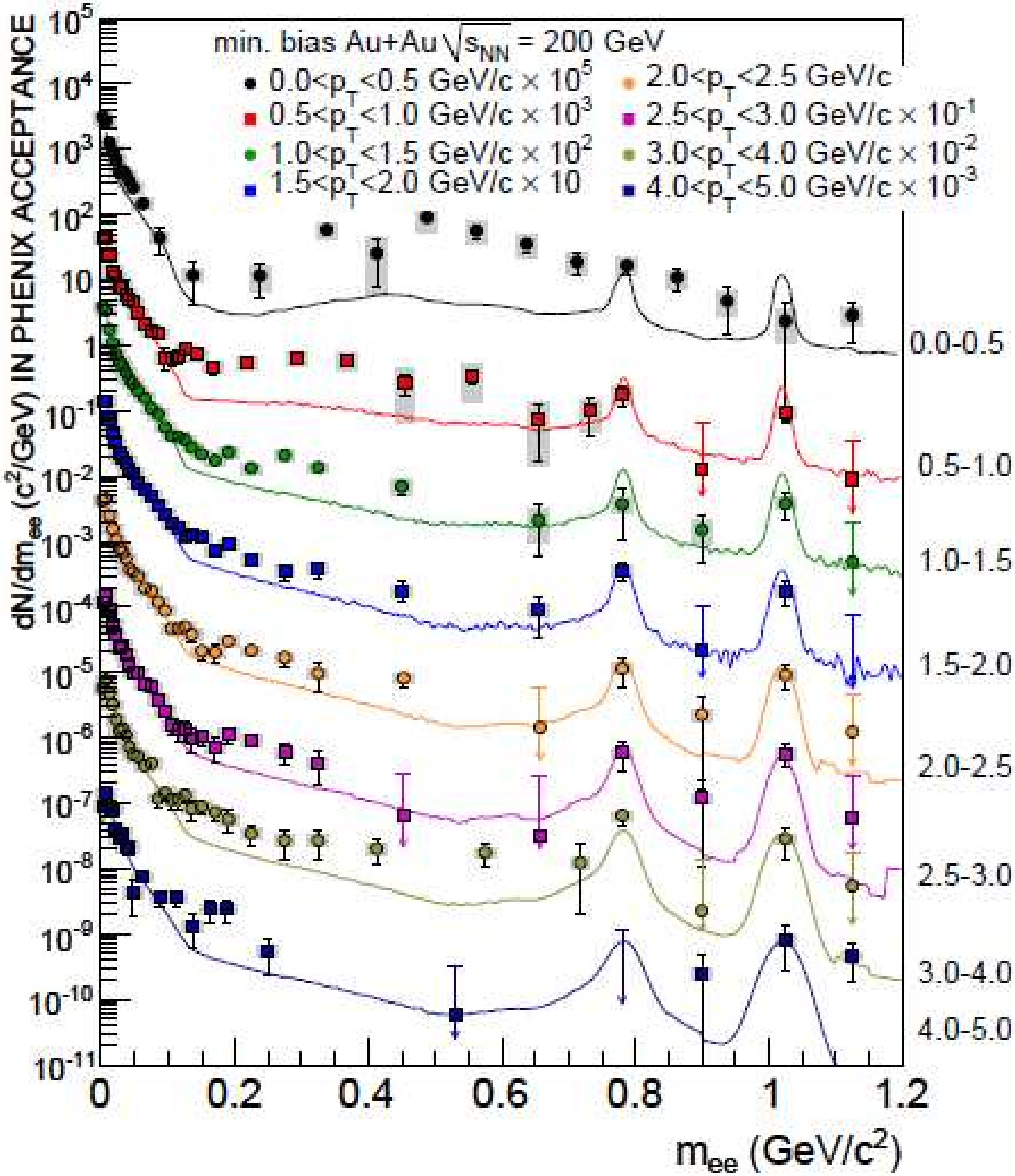


FIG. 27: (Color online) Invariant mass spectrum of e^+e^- pairs inclusive in p_T compared to expectations from the model of hadron decays for $p+p$ and for different Au + Au centrality classes.

From: 0912.0244v1 [nucl-ex]
(PHENIX Collaboration)

TABLE IX: The enhancement factor, defined as the ratio between the measured yield and the expected yield for $0.15 < m_{ee} < 0.75$ GeV$/c^2$, for different centrality bins. The meaning of the errors is defined in the text.

Centrality	Enhancement (±stat ±syst ±model)
00-10 %	$7.6 \pm 0.5 \pm 1.5 \pm 1.5$
10-20 %	$3.2 \pm 0.4 \pm 0.1 \pm 0.6$
20-40 %	$1.4 \pm 1.3 \pm 0.02 \pm 0.3$
40-60 %	$0.8 \pm 0.3 \pm 0.03 \pm 0.2$
60-92 %	$1.5 \pm 0.3 \pm 0.001 \pm 0.3$
Min. Bias	$4.7 \pm 0.4 \pm 1.5 \pm 0.9$



min. bias Au+Au $\sqrt{s_{NN}}$ = 200 GeV
0.0<p_T<0.5 GeV/c × 10^5
0.5<p_T<1.0 GeV/c × 10^3
1.0<p_T<1.5 GeV/c × 10^2
1.5<p_T<2.0 GeV/c × 10
2.0<p_T<2.5 GeV/c
2.5<p_T<3.0 GeV/c × 10^{-1}
3.0<p_T<4.0 GeV/c × 10^{-2}
4.0<p_T<5.0 GeV/c × 10^{-3}
dN/dm_{ee} (c^2/GeV) IN PHENIX ACCEPTANCE
m_{ee} (GeV/c^2)
0.0-0.5
0.5-1.0
1.0-1.5
1.5-2.0
2.0-2.5
2.5-3.0
3.0-4.0
4.0-5.0

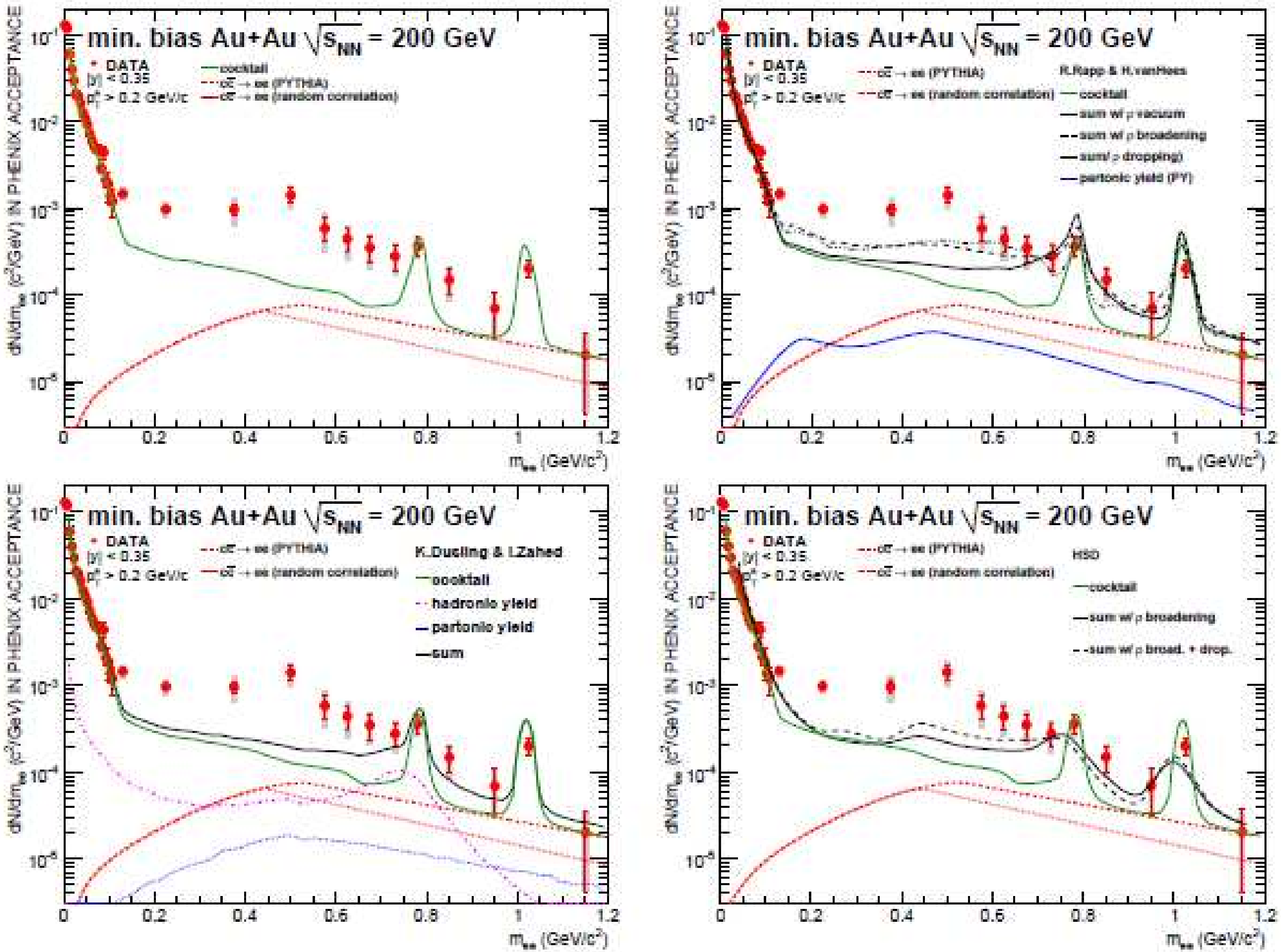


FIG. 42: (Color online) Invariant mass spectra of e^+e^- pairs in Au + Au collisions in the LMR. The data are compared to the sum of cocktail+charm (top left). The data are also compared to the sum of cocktail+charm and hadronic+partonic contributions from different models. The calculations are from The calculations are from (top right) Rapp and van Hees [15, 18, 83], (bottom right) Dusling and Zahed [19, 84, 85], and Cassing and Bratkowskaya [20, 27, 86, 87].

This is an old puzzle!

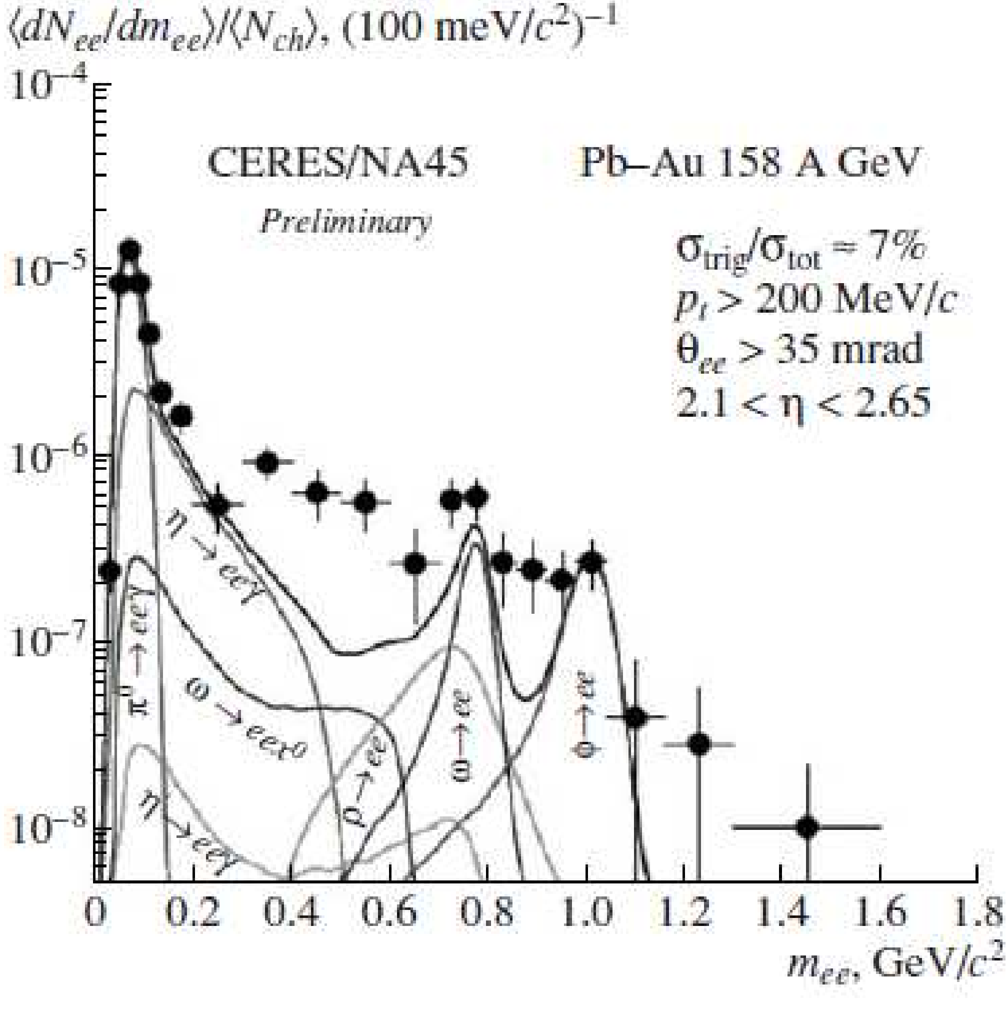
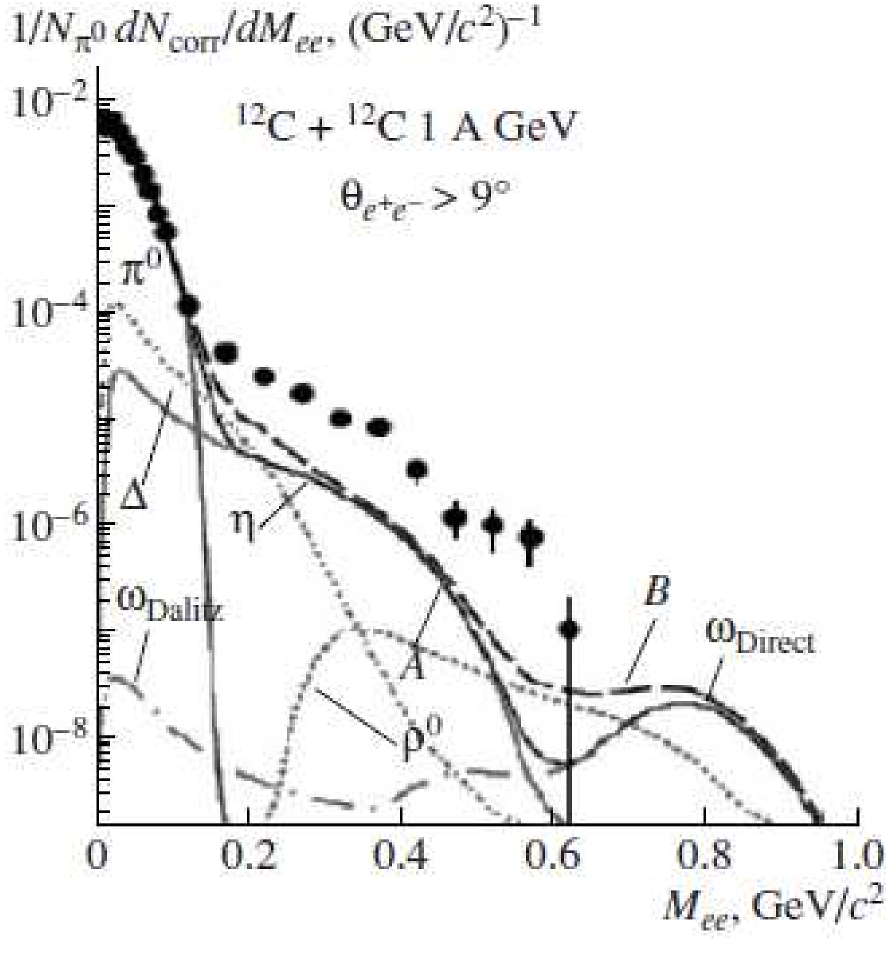


Fig. 2. CERES experiment data collected in 2000 [17].



HADES-experiment data on the dilepton yield

From K. O. Lapidus and V. M. Emel'yanov, Phys. Part. Nucl. 40 (2009) 29

In order to derive the value of η one has to resolve

Mixing with vector mesons $\quad (V_{\mu,a}) \equiv (A_\mu,\, \omega_\mu,\, \rho_\mu)$

In the lagrangian $\quad \mathcal{L}_{mixing}(k) \;=\; \eta\, \epsilon_{jkl}\, V_{j,a}\, N_{ab}\, \partial_k V_{j,b}$

coupling constants N_{ab} are known from the anomaly or, independently, can be derived from radiative decays $\quad \pi_0 \to \gamma\gamma \qquad \omega \to \pi_0\gamma \qquad \rho_0 \to \pi_0\gamma$

One can, in principle, disentangle the isospin of pseudoscalar condensate: (after normalization on photon channel)

for isoscalar $\quad \eta \sim \alpha\langle\dot\theta\rangle/\pi f_\pi$

$$(N^\theta_{ab}) \simeq \begin{pmatrix} 1 & -\frac{3g}{10e} & -\frac{9g}{10e} \\ -\frac{3g}{10e} & \frac{9g^2}{10e^2} & 0 \\ -\frac{9g}{10e} & 0 & \frac{9g^2}{10e^2} \end{pmatrix}; \quad \det\left(N^\theta\right) = 0.$$

for pion condensate $\quad \eta \sim \alpha\langle\dot\pi_0\rangle/\pi f_\pi$

$$(N^\pi_{ab}) \simeq \begin{pmatrix} 1 & -\frac{3g}{2e} & -\frac{g}{2e} \\ -\frac{3g}{2e} & 0 & \frac{3g^2}{2e^2} \\ -\frac{g}{2e} & \frac{3g^2}{2e^2} & 0 \end{pmatrix}; \quad \det(N^\pi) = 0.$$

where coupling constants

$$\begin{array}{ccc} \gamma & \omega & \rho \\ e & g_\omega \simeq & g_\rho \equiv g \end{array}$$

These coupling constants are in a very good correspondence with data on

$$\pi_0 \to \gamma\gamma, \quad \omega \to \pi_0\gamma, \quad \rho_0 \to \pi_0\gamma, \quad \omega \to \pi\pi\pi, \ldots$$

Mass splitting for transversal polarizations

Mass shell

$$\left[P_\perp^{\mu\nu}(k)\left(k^2\delta_{ab} - (\hat{m}^2)_{ab}\right) + i\,\epsilon^{\mu\nu\sigma\rho}\eta_\sigma k_\rho\, N_{ab}\right] V_{\nu,b}(k) \;=\; 0;$$

$$\hat{m}^2 \simeq \begin{pmatrix} m_\gamma^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + m_V^2 \begin{pmatrix} \frac{10e^2}{9g^2} & -\frac{e}{3g} & -\frac{e}{g} \\ -\frac{e}{3g} & 1 & 0 \\ -\frac{e}{g} & 0 & 1 \end{pmatrix},$$

for transversal polarizations $\left[k^2\delta_{ab} - (\hat{m}^2)_{ab}\right) \,\pm\, \eta|\vec{k}|\, N_{ab}\Big] V_b^{\pm}(k) \;=\; 0;$

For massless photons and isoscalar condensate

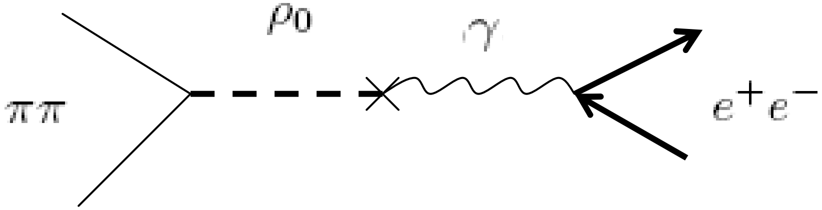
$$N^\theta \;=\; \mathrm{diag}\left[0,\, \frac{9g^2}{10e^2},\, \frac{9g^2}{10e^2} + 1\right];\quad \hat{m}^2 \;=\; m_V^2\,\mathrm{diag}\left[0,\, 1,\, 1 + \frac{10e^2}{9g^2}\right] \;\simeq\; \mathrm{diag}[0,\, 1,\, 1].$$

Transversal photon are not distorted and not decaying!

But for a mixture of isoscalar and isovector condensates they do decay.

Natural scale $\eta_{vec} = \eta\left(9g^2/10e^2\right) \simeq 360\eta \sim 360$ Mev for $\eta \sim 1$ MeV .

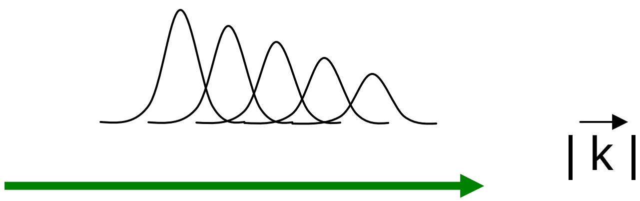
$\pi\pi \to e^+e^-$ channel and VMD



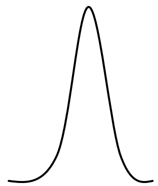
$$\simeq \frac{e^2\left(m^2_{\rho,eff} - i m_{\rho,eff}\Gamma_\rho\right)}{(k^2 - m^2_{\rho,eff} + i m_{\rho,eff}\Gamma_\rho)k^2}$$

$$m^2_{\rho,eff} = m^2_\rho \pm \frac{9g^2}{10e^2}\eta|\vec{k}|$$

"Giant" resonances for transversal polarizations with variable position



Normal rho meson resonance for longitudinal polarization

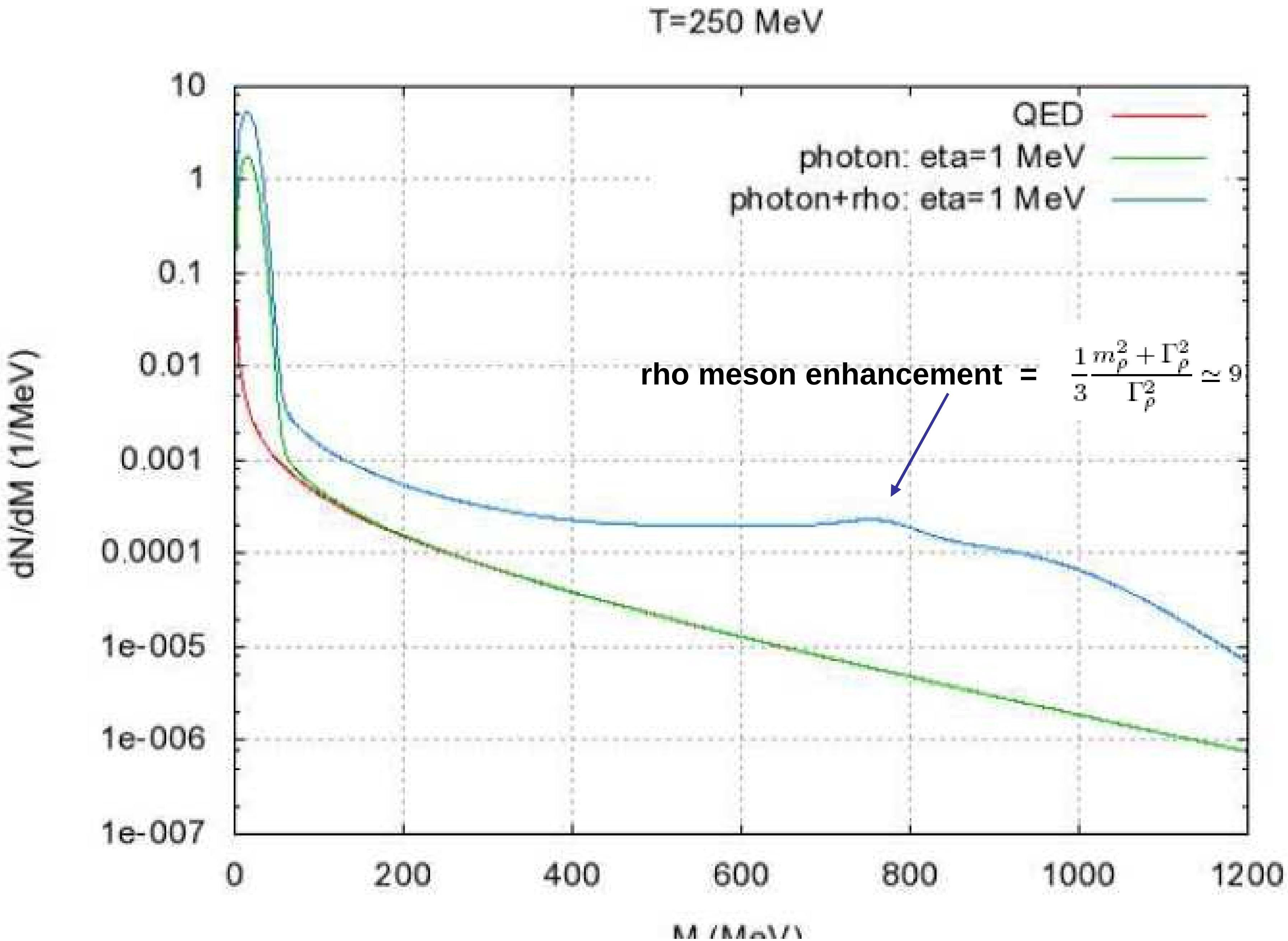


$$m_\rho \simeq 770 Mev$$

e^+e^- production rate

$$\frac{1}{[M^2 - \varepsilon\eta|\vec{k}|]^2 + \frac{\alpha^2}{9}M^4} \Longrightarrow \frac{\left(m^4_{\rho,eff} + m^2_{\rho,eff}\Gamma^2_\rho\right)}{\left(\left(M^2 - m^2_{\rho,eff}\right)^2 + m^2_{\rho,eff}\Gamma^2_\rho\right)\left([M^2 - \varepsilon\eta|\vec{k}|]^2 + \frac{\alpha^2}{9}M^4\right)}$$

and to convolute with thermal distribution


T=250 MeV
QED
photon: eta=1 MeV
photon+rho: eta=1 MeV
rho meson enhancement = $\frac{1}{3}\frac{m_\rho^2+\Gamma_\rho^2}{\Gamma_\rho^2} \simeq 9$
dN/dM (1/MeV)
10
1
0.1
0.01
0.001
0.0001
1e-005
1e-006
1e-007
0
200
400
600
800
1000
1200
M (MeV)

Signatures and searches of parity breaking

1) Photon of "+" polarization decays in dilepton pair $\gamma \rightarrow l^+ l^- -$
when

$$|\,\mathbf{k}\,| \geq \frac{4m_l^2}{\eta} \equiv k_{\rm th}$$

2) For photons different thresholds for different dilepton species!
For vector mesons much weaker! Search for dimuon excess

3) Decay width is energy independent $\Gamma_+ = \tau_+^{-1} \simeq \frac{\alpha\eta}{3}$ for $k \gg k_{\rm th}$

4) Distorted photon/vector meson resonances enhance considerably the yield of dileptons

5) Positions of resonances with transversal polarizations move with photon/vector meson wave vector **k** and therefore convolution with photon thermal distribution makes it broader

6) Mixing with vector mesons to disentangle the condensate isospin

Program for RHIC → CBM FAIR + NICA

Five-dimensional interpretation

5-dim abelian gauge field $$(A_B) = (A_0, ..., A_3, A_5) \equiv (A_\mu, A_5)$$

5-dim Chern-Simons interaction $$\mathcal{L}^5_{CS} = c_5 \int d^5x \epsilon^{BCDEF} A_B \partial_C A_D \partial_E A_F$$

KK or brane reduction

$$A_B(X_C) \Longrightarrow A_B(x_\mu)$$

$$\Longrightarrow \tilde{c}_5 \int d^4x \epsilon^{\mu\nu\sigma\rho 5} A_\mu \partial_\nu A_\sigma \partial_\rho A_5$$

Time dependent fifth component

$$A_5(x_\mu) \Longrightarrow A_5(t) \equiv \theta(t)$$

$$\Longrightarrow \int d^4x \, \frac{1}{2} \eta \, \epsilon^{jkl} A_j \partial_k A_l$$

induces electric field $$\partial_0 A_5 = E_5 = \dot{\theta}$$

Pair creation by a 5-dim Schwinger-like mechanism!

Estimation of C.-S. coupling in dense (and hot) nuclear/quark matter

$$\eta \sim \frac{\alpha}{2\pi} \frac{\dot{\varrho}_B}{f_\pi} \frac{\partial \langle \Pi \rangle}{\partial \varrho_B}$$

(+ temperature when it is relevant)

from nuclear evolution

from a model of P-breaking

f_π in-medium pion decay constant

ϱ_B time-dependent baryon density

$\langle \Pi \rangle (\varrho_B)$ density dependent P-breaking condensate

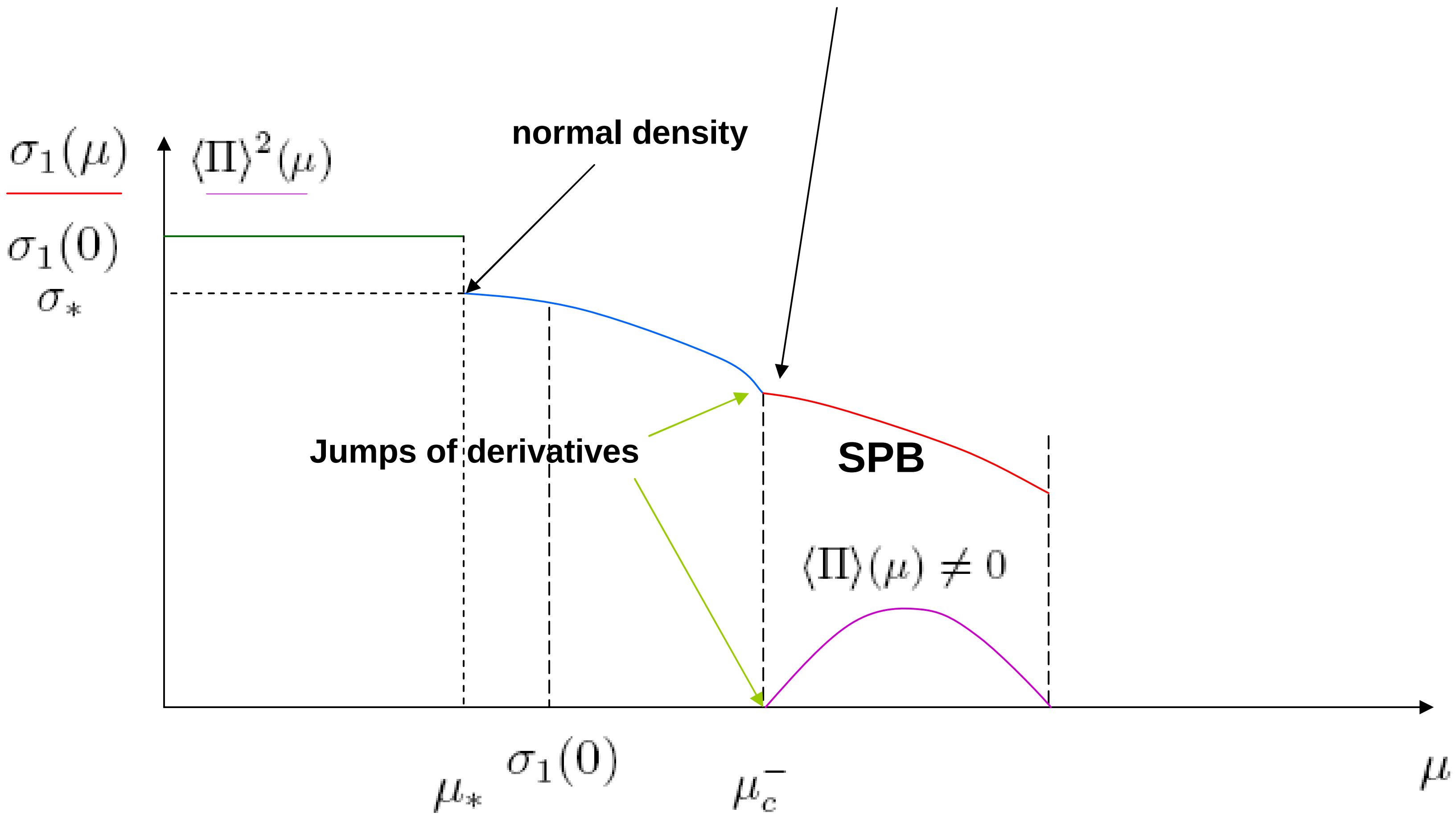
(isosinglet $\langle \theta \rangle$ or isotriplet $\langle \pi_0 \rangle$)

And vice versa:
when measuring η **one can reconstruct the density dependence with a simulated time evolution of density**

Crude estimation (order of magnitude)

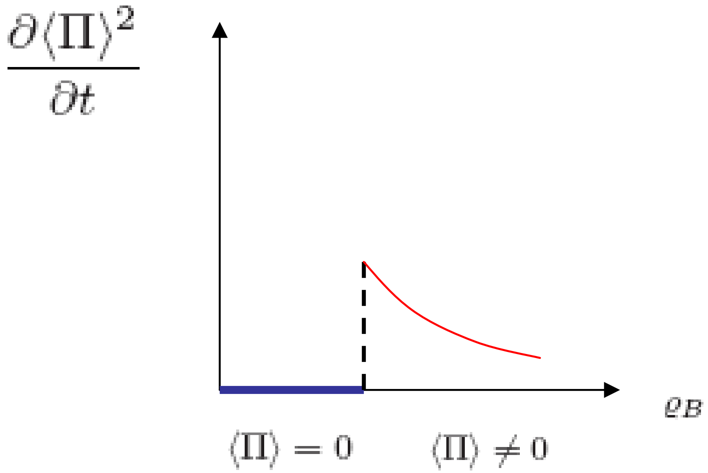
$$\eta \sim \alpha \dot{\theta} / f_\pi \sim \alpha f_\pi \sim 1 MeV$$

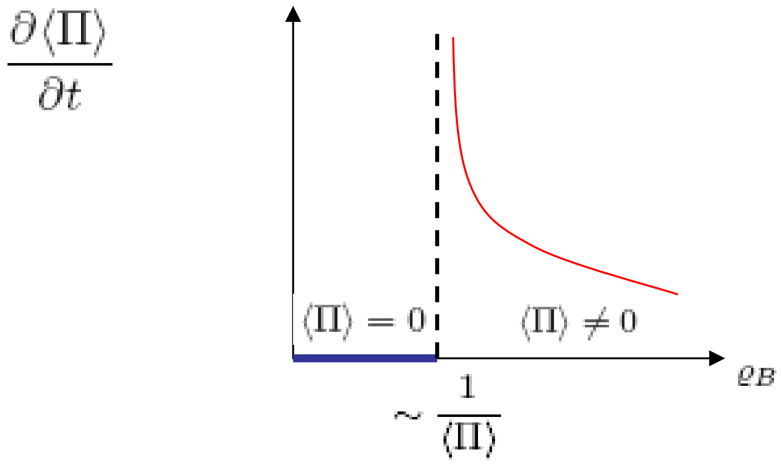
Spontaneous P-parity breaking (IId order phase transition)



A.A.Andrianov and D.Espriu, Phys.Lett. B 663 (2008) 450
A.A.Andrianov, V.A.Andrianov and D.Espriu, Phys.Lett. B 678 (2009) 416

Infinite jump in order parameter





Second order phase transition
in the absence of manifest parity breaking

relevant for anomaly vertex

Back-up slides

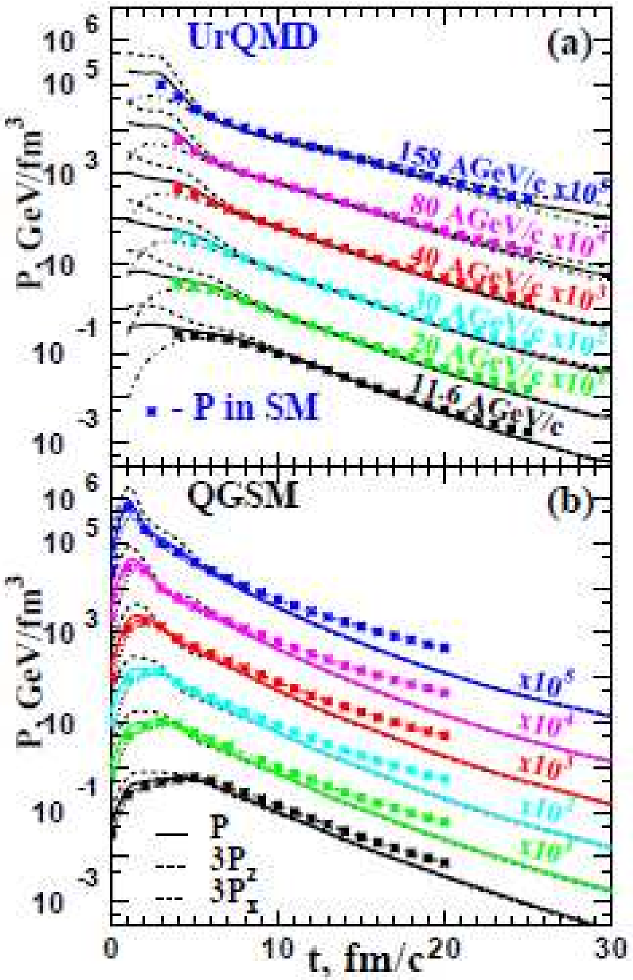


FIG. 2: (Color online) The longitudinal ($3P_z$, dashed curves) and the transverse ($3P_x$, dash-dotted curves) diagonal components of the microscopic pressure tensor in the central 125 fm^3 cell in (a) UrQMD and (b) QGSM calculations of central Au+Au collisions at energies from 11.6A GeV to 158A GeV. Asterisks indicate the pressure given by the statistical model and solid lines show the total microscopic pressure.

From L.Bravina et al arXiv:0804.1484 [hep-ph]

TABLE I: The time evolution of the thermodynamic characteristics of hadronic matter in the central cell of volume $V = 125$ fm^3 in central Au+Au collisions at bombarding energy 20A GeV. The temperature, T, baryochemical potential, $\mu_{\rm B}$, strange chemical potential, $\mu_{\rm S}$, pressure, P, entropy density, s, and entropy density per baryon, $s/\rho_{\rm B}$, are extracted from the statistical model of ideal hadron gas, using the microscopically evaluated energy density, $\varepsilon^{\rm cell}$, baryonic density, $\rho_{\rm B}^{\rm cell}$, and strangeness density, $\rho_{\rm S}^{\rm cell}$, as input. Of each pair of numbers, the upper one corresponds to the UrQMD calculations, and the lower one to the QGSM calculations.

Time fm/c	$\varepsilon^{\rm cell}$ MeV/fm^3	$\rho_{\rm B}^{\rm cell}$ fm^{-3}	$\rho_{\rm S}^{\rm cell}$ fm^{-3}	T MeV	$\mu_{\rm B}$ MeV	$\mu_{\rm S}$ MeV	P MeV/fm^3	s fm^{-3}	$s/\rho_{\rm B}^{\rm cell}$
11	464.2	0.210	-0.0143	144.5	450.5	92.7	59.6	2.97	14.16
	522.6	0.257	-0.0059	150.2	487.8	116.1	73.8	3.13	12.19
12	343.2	0.160	-0.0115	137.9	459.2	86.4	44.0	2.27	14.18
	385.7	0.197	-0.0051	141.9	498.1	109.4	53.1	2.40	12.16
13	255.2	0.124	-0.0093	131.5	469.5	80.4	32.6	1.75	14.15
	286.9	0.153	-0.0046	134.0	609.5	103.1	38.5	1.85	12.09
14	189.9	0.096	-0.0072	124.9	481.7	75.8	24.1	1.34	14.06
	214.2	0.117	-0.0035	127.2	515.9	97.1	28.2	1.43	12.22
15	143.9	0.075	-0.0064	119.2	492.8	68.6	18.1	1.05	13.97
	162.3	0.091	-0.0028	121.0	522.3	91.5	20.1	1.12	12.35
16	108.8	0.059	-0.0052	113.7	502.5	62.7	13.6	0.82	13.97
	125.4	0.072	-0.0025	115.4	529.2	85.4	15.9	0.89	12.43
17	83.6	0.046	-0.0043	108.7	511.0	57.0	10.4	0.65	14.02
	98.3	0.058	-0.0022	110.4	535.9	80.1	12.3	0.72	12.52
18	65.0	0.037	-0.0035	103.5	523.7	52.4	8.0	0.52	13.88
	78.1	0.047	-0.0019	105.9	541.3	75.4	9.6	0.59	12.66
19	50.9	0.030	-0.0029	98.8	534.5	47.6	6.2	0.41	13.82
	62.9	0.039	-0.0016	101.1	552.7	72.2	7.6	0.49	12.52
20	40.6	0.025	-0.0027	94.6	544.2	38.9	4.8	0.34	13.76
	51.0	0.033	-0.0014	97.0	560.1	67.4	6.0	0.40	12.54

Accompanying processes

Lepton decays $l^{\pm} \to l^{\pm}\gamma$

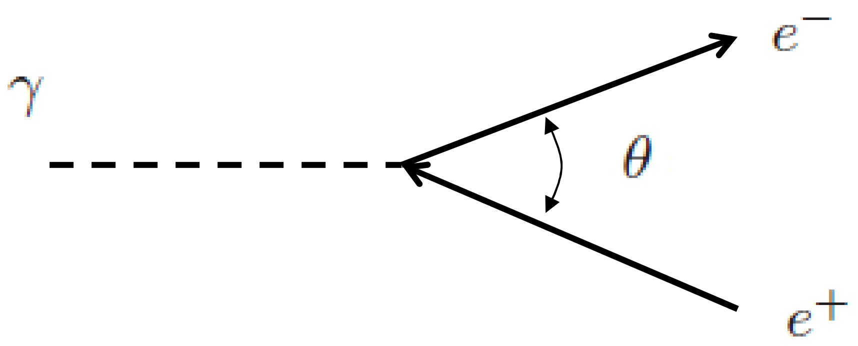
A. Andrianov, D. Espriu, F. Mescia and A. Renau, Phys.Lett. B684 (2010) 101

Photon splitting $\gamma \to \gamma_{-}\gamma_{-}\gamma_{-}$ **more rare** $\Gamma_{\gamma\gamma\gamma} \sim \alpha^{2}$

Red shift of lepton and photon spectra!

Kinematics

$$\omega_{\mathbf{k}\pm} = \sqrt{\mathbf{k}^2 + m_\gamma^2 \pm \eta\,|\mathbf{k}|} = \sqrt{\mathbf{p}^2 + m_e^2} + \sqrt{(\mathbf{p}-\mathbf{k})^2 + m_e^2}$$



$$\sin^2\theta \le \frac{\eta^2}{4m_e^2}\left(1 - \frac{4m_e^2}{\eta k}\right)$$

Narrow cone $\quad \theta_{\max} < \eta/2m_l \qquad \eta \ll 2m_l$

For muons it is plausible but for e^+e^- **??**

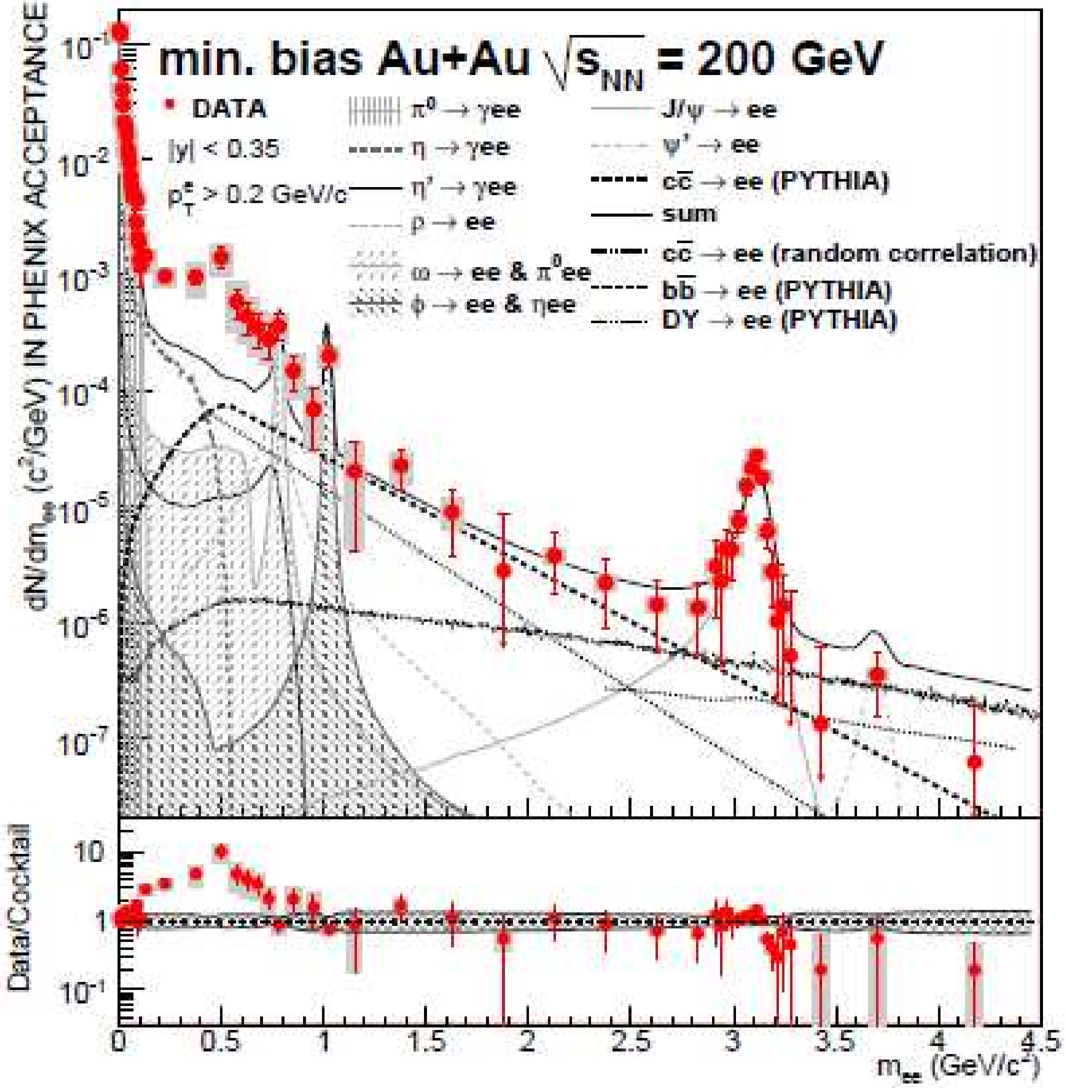


FIG. 26: (Color online) Inclusive mass spectrum of e^+e^- pairs in the PHENIX acceptance in minimum-bias Au + Au compared to expectations from the decays of light hadrons and correlated decays of charm, bottom and Drell-Yan. The

From: 0912.0244v1 [nucl-ex]
(PHENIX Collaboration)

Two-multiplet sigma model

$$V_{\text{eff}} = -\sum_{j,k=1}^{2} \sigma_j \Delta_{jk} \sigma_k - \Delta_{22} (\pi_2^a)^2 + \lambda_2 \Big((\pi_2^a)^2 \Big)^2 + (\pi_2^a)^2 \Big((\lambda_3 - \lambda_4) \sigma_1^2 + \lambda_6 \sigma_1 \sigma_2 + 2\lambda_2 \sigma_2^2 \Big)$$
$$+ \lambda_1 \sigma_1^4 + \lambda_2 \sigma_2^4 + (\lambda_3 + \lambda_4) \sigma_1^2 \sigma_2^2 + \lambda_5 \sigma_1^3 \sigma_2 + \lambda_6 \sigma_1 \sigma_2^3.$$

$$\Delta V_{\text{eff}}(m_q) = 2m_q \left[-(d_1\sigma_1 + d_2\sigma_2) \cos \frac{|\pi_1^a|}{F_0} + d_2 \frac{\pi_1^a \pi_2^a}{|\pi_1^a|} \sin \frac{|\pi_1^a|}{F_0} \right]$$

Mass-gap equations
for spontaneous P-breaking $\langle \pi_1^a \rangle = \langle \pi^0 \rangle \delta^{0a}, \qquad \langle \pi_2^a \rangle = \rho \delta^{0a}.$

$$(d_1\sigma_1 + d_2\sigma_2) \sin \frac{\langle \pi^0 \rangle}{F_0} = -d_2 \rho \cos \frac{\langle \pi^0 \rangle}{F_0},$$
$$-m_q d_2 \sin \frac{\langle \pi^0 \rangle}{F_0} = \rho \frac{m_q d_2^2}{(d_1\sigma_1 + d_2\sigma_2)} \cos \frac{\langle \pi^0 \rangle}{F_0}$$
$$= \rho \Big(-\Delta_{22} + (\lambda_3 - \lambda_4)\sigma_1^2 + \lambda_6 \sigma_1 \sigma_2 + 2\lambda_2 \sigma_2^2 + 2\lambda_2 \rho^2 \Big)$$

At a critical point typically both v.e.v. emerge simultaneously

for $\mu > \mu_{crit}$
$$\cos \frac{\langle \pi^0 \rangle}{F_0} = \frac{d_1\sigma_1 + d_2\sigma_2}{\sqrt{d_2^2 \rho^2 + (d_1\sigma_1 + d_2\sigma_2)^2}}$$

Beyond the chiral limit: $\mathbf{m_q \neq 0}$

Two new lowest-dimensional operators

$$\frac{1}{2} m_q d_1 \mathrm{tr}(H_1 + H_1^\dagger) \qquad \frac{1}{2} m_q d_2 \mathrm{tr}(H_2 + H_2^\dagger)$$

The spectrum in dense matter

$\mu = 0$	$m_\pi^2 \sim m_q$	$m_\Pi^2 \gg m_\pi^2$	heavy
$\mu = \mu_c$	$m_\pi^2 = 0$	$m_\Pi^2 \sim m_q$	very light
$\mu > \mu_c$	$m_{\pi_\pm}^2 = 0$	$m_{\Pi_\pm}^2 \sim m_{\pi_0}^2 \sim m_q$	light

Two Goldstone bosons of isospin breaking

Three light massive "pions"