Some kinds of matrix models at large N

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Abstract

One-matrix, Goldstone, two-matrix, multi-matrix, multitrace, Goldstone muititrace matrix models are briefly discussed. Multitrace multi-matrix models are proposed because of they can be investigated.

Matrix models appear in the different branches of science, for example, nuclear physics and economics. It is known from the work of 't Hooft [1] that the only planar diagrams survive in the large N limit of $SU(N)$ gauge field theory. The zero-dimensional matrix models are important since they count number of planar diagrams for many-dimensional field theories. Note that more famous application of zero-dimensional matrix models consists in the use to two-dimensional quantum gravity [2]. In this text for simplicity and without loss of generality we will study the models with ϕ^4 interaction. Let us consider the properties of usual hermitian matrix model ϕ^4 with action

$$S(\Phi) = \frac{1}{2} tr\Phi^2 + \frac{g}{N} tr\Phi^4. \tag{1}$$

Statistical sum Z and a planar vacuum energy E_0 of matrix model are

$$Z = \int d\Phi e^{-S(\Phi)}, \tag{2}$$

$$E^0(g) = -\lim_{N\to\infty} \frac{1}{N^2} \ln Z. \tag{3}$$

When $N \to \infty$, we can obtain following the equation for eigenvalue distribution function at segment $[-2a; 2a]$

$$\pm\frac{1}{2}\lambda + 2g\lambda^3 = \fint_{-2a}^{2a} d\mu \frac{u(\mu)}{\lambda - \mu}, \qquad |\lambda| \leq 2a. \tag{4}$$

Via an eigenvalue distribution function it is possible to calculate correlation functions. For example, the propagator at large N is

$$D =< \frac{1}{N^2} tr\Phi^2 >= \int_L u(\lambda)\lambda^2 d\lambda. \tag{5}$$

The planar approximation was tested other approaches of planar investigation for one-matrix model. The results of [3] were verified by correct method of orthogonal polynomials. There are many approximate methods. Some of them are the variational method [4], the effective action procedure [5], the quantum collective field method [6], renormalization group approach to matrix models [7]. A planar parquet approximation was introduced by Arefeva and Zubarev [8], is one most exact amount approximate methods. Here the closed system of algebraic equations replaces integral equation (4).

It was observed that distribution function can exist at many segments. Simplest such solution appear in [9]-[12] Goldstone matrix model

$$S(\Phi) = -\frac{1}{2} tr\Phi^2 + \frac{g}{4N} tr\Phi^4. \tag{6}$$

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Here solution of semiclassical equation exits at two segments [a,b] and [c,d]. This two-cut solution determines a special parameter

$$\xi = \int_a^b u(\lambda)\, d\lambda. \tag{7}$$

When $\xi = \frac{1}{2}$ we have symmetric solution [9]. Exact planar vacuum energy for symmetric phase looks as

$$E^0(g) - E^0(0) = -\frac{1}{16g} + \frac{1}{4}\log(4g) - \frac{3}{8}. \tag{8}$$

It is possible also to examine two-dimensional quantum gravity with the major terms of curvature. More simple matrix model with the multitrace terms for this case was solved in ref. [13]. If we leave in action only free term with twotrace summand at large N, this model behave as a vector model at $N \to \infty$. The action for this model is

$$S(\Phi) = \frac{1}{2}tr\Phi^2 + \frac{g}{4N}tr\Phi^4 + \frac{h}{N^2}(tr\Phi^2)^2. \tag{9}$$

Goldstone multitrace matrix model was considered in [14]. It was observed that here exist phases

$$S(\Phi) = -\frac{1}{2}tr\Phi^2 + \frac{g}{N}tr\Phi^4 + \frac{h}{N^2}(tr\Phi^2)^2. \tag{10}$$

The vacuum energy for symmetric solution can be written as

$$E^0(g,h) - E^0(0) = -\frac{g}{16(g+h)^2} + \frac{1}{4}\log(4g) - \frac{3}{8}. \tag{11}$$

There are many variants of matrix models with interacting two matrices. The most plausible case was supposed by Itzykson and Zuber [15].

Here an action for two-matrix model is

$$S(\Phi_1, \Phi_2) = S(\Phi_1) + S(\Phi_2) - ctr\Phi_1\Phi_2, \tag{12}$$

where

$$S(\Phi_1) = \frac{1}{2}tr\Phi_1^2 + \frac{g}{4N}tr\Phi_1^4. \tag{13}$$

This model can be analyzed by the method of orthogonal polynomials.

Let us use formula of Itzykson-Zuber

$$Z \sim \int \prod_i dx_i dy_i e^{-(S(x)+S(y)-c\sum_i x_i y_i)}. \tag{14}$$

Biorthogonal polynomials $P_i(x)$ satisfy the relation

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} exp(-S(x) - S(y) + cxy)P_i(x)P_j(y)dxdy = h_i\delta_{ij}. \tag{15}$$

Recursion equation takes the form

$$xP_i(x) = P_{i+1} + R_iP_{i-1}(x) + T_iP_{i-3}(x). \tag{16}$$

In large N limit ($f_i = h_i/h_{i-1}, x = i/N$) we can obtain the closed system of algebraic equations for $R(x)$, $T(x)$ and $f(x)$. Using those functions we can calculate correlation functions and vacuum energy.

Multi-matrix models are natural generalization of two-matrix model. Open matrix chain has action

$$S_{op}(\Phi_1, ..., \Phi_n) = \frac{1}{2}\sum_{i=1}^n tr\Phi_i^2 + \sum_{i=1}^n \frac{g_i}{4N}tr\Phi_i^4 +$$

$$-c\sum_{i}^{n-1} tr\Phi_i\Phi_{i+1}. \quad (17)$$

Here it is possible to use orthogonal polynomials. Closed matrix chain defines as

$$S_{cl}(\Phi_1,...,\Phi_n) = S_{op}(\Phi_1,...,\Phi_n) - ctr\Phi_1\Phi_n. \quad (18)$$

The exact method of solving this model is unknown. This chains in the limit of big number matrices lead to matrix quantum mechanics in a straight line and a circle.

Another two-matrix model is multitrace two-matrix one. The simplest model with twotrace term expresses as

$$S(\Phi_1,\Phi_2) = \frac{1}{2}tr\Phi_1^2 + \frac{g_1}{4N}tr\Phi_1^4 + \frac{1}{2}tr\Phi_2^2 + \\ + \frac{g_2}{4N}tr\Phi_2^4 - ctr(\Phi_1)^2 tr(\Phi_2)^2. \quad (19)$$

When $g_1 = g_2$ this case reduces to one-matrix multitrace model.

A multitrace matrix open chain has action

$$S(\Phi_1,...,\Phi_n) = \frac{1}{2}\sum_{i=1}^{n} tr\Phi_i^2 + \sum_{i=1}^{n}\frac{g_i}{4N}tr\Phi_i^4 + \\ -c\sum_{i} tr(\Phi_i)^2 tr(\Phi_{i+1})^2. \quad (20)$$

Similarly the one-matrix the case multimatrix multitrace models can be resolved using master equation and equations for correlator functions.

This models can be applied in two-dimensional gravity [17]- [18], supersymmetric gauge theories [19] and AdS/CFT-correspondes [20]- [21].

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