# On existence of nonsingular solutions in static braneworlds

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### Abstract

A static $SO(3) \times \mathbb{Z}_2$ symmetric and translation invariant braneworld setup with flat brane is considered. The null energy conditions (NEC) for matter on the brane and in the bulk are discussed. We prove that for the static regular background with broken Lorentz invariance the NEC and positiveness of the total energy density on the brane together with the NEC in the bulk cannot be satisfied simultaneously. Then we give some examples and elaborate some special cases.

## 1    Introduction and Motivation

Models with large extra dimensions were suggested considerably long time ago as possible solutions of fundamental problems of cosmology and particle physics like cosmological constant problem, hierarchy problem or problem of generations (see e.g. Ref. [1]; for a review see Ref. [2] and for earlier works see Refs. [3, 4]). In the recent eight years braneworld models have attracted very much attention because of works [5, 6] by L. Randall and R. Sundrum (we will refer to these models as to RS1 and RS2 models correspondingly). In these models it has been shown that once a four dimensional flat brane is embedded into a five dimensional anti de-Sitter space then static gravitational potential on a brane at small distances is exactly Newtonian up to a power correction with a coefficient depending on a warping factor. Therefore at small energies an observer does not see extra dimension and does not detect deviations from a four dimensional physics. The interest to the braneworld models is also warmed up by upcoming high energy experiments on LHC.

In the RS2 model a bulk is filled by a negative cosmological constant and no other media are considered. In many further contributions different braneworld backgrounds were elaborated and different bulk and brane types of matter were investigated (see for review e.g. [7, 8]). One expects to have a configuration with a "physical" matter, that is the one with an energy density being positively defined and a null energy condition (NEC) being satisfied. A natural question arises whether this kind of solutions exists. The answer to this question for a generic (even $\mathbb{Z}_2$ symmetric) braneworld setup with time dependent metric is, however, a formidable task as it entails rather complicated Einstein equations to be solved. Thus first one can simplify the problem by asking the same question for static configurations where rigorous analysis can be performed. This is the main goal of the present work. We consider static backgrounds with a broken five-dimensional Lorentz invariance. Our Universe is supposed to be located on a 4-d brane representing a slice in a five dimensional spacetime.

The motivation of this paper is twofold. The first one comes from the so-called no-go theorem discussed in [9]. The authors prove that if a static braneworld solution has a singularity at some

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point in a bulk then this singularity is necessarily naked (that is not screened by any horizon from the brane) if NECs both on the brane and in the bulk are satisfied. The existence of the singularity was a starting point of the proof. But still the following question remains unclear whether there exists a self-tuning static setup with nonsingular metric provided that null energy conditions are satisfied on the brane and in the bulk.

The second motivation comes from [10, 11] where the authors in a context of a so-called "trans-Planckian problem" use 'ad hoc' Lorentz violating background and do not provide the reader with energy-momentum tensor which solves field equations. A natural question arises whether the corresponding matter satisfies NEC or not. In the current work we answer both questions from the papers [9, 10, 11] and generically both are negative if the static background is considered. As an answer to the first question we prove a theorem that no non-singular solutions are possible thereby answering the second question posed in [10, 11] where the setup is assumed to be smooth and non-singular[1].

The paper is organized as follows. In the Section 2 we spell out the metric tensor, bulk Einstein equations, impose junction conditions at the brane location, discuss null energy conditions and formulate the main statement. The Section 3 is devoted to proof of the this statement. Then in the Section 4 we discuss various examples illustrating the theorem and specify the conditions under which the theorem can be evaded. In the end we make some concluding remarks.

## 2   The Setup and the No–Go Theorem

First we specify the metric we are going to employ in the current paper. The following coordinates are used: $t$, $\mathbf{x} = (x^1, x^2, x^3)$ are four-coordinates parallel to the brane and $z$ is the bulk coordinate with $z$-axis being orthogonal to the brane. We impose $SO(3) \times T^3 \times \mathbb{Z}_2$ symmetry which acts on spatial coordinates $(x^1, x^2, x^3)$ by rotations and translations, and $\mathbb{Z}_2$ reflection of $z$ axis as $z \mapsto -z$ (the latter is imposed for simplicity). Moreover we shall consider only flat brane. One can show that under these symmetries the general static metric can be written as follows[2]

$$ds^2 = e^{-2a(z)}dt^2 - e^{-2b(z)}d\mathbf{x}^2 - dz^2 \tag{1}$$

The brane is located at $z = 0$ and the functions $a(z)$ and $b(z)$ are assumed to be $z$ even. For each fixed value of $z$ the Universe represents a slice warped by factors $e^{-2a(z)}$ and $e^{-2b(z)}$. If the functions $a$ and $b$ are different then Lorentz invariance in the bulk is violated. We shall also impose the condition that at the location of the brane at $z = 0$ Lorentz invariance is reproduced and by choosing an appropriate normalization one can put $a(0) = b(0) = 0$ thereby setting the metric to the Minkowskian one at this point.

**Einstein Equations and Junction Conditions.**   The bulk Einstein equations relating Einstein tensor $G_{AB}$ and energy momentum tensor read[3]

$$G^A_B = T^A_B + \delta^A_B \Lambda, \tag{2}$$

where $T^A_B$ is the bulk energy-momentum tensor and $\Lambda$ is the cosmological constant which is assumed to be negative ($\Lambda < 0$). The non zero Einstein tensor components for the metric (1)

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[1]It is worth to stress here, however, that in [10, 11] non-static metric has been considered, so our present result can be viewed only as an indication that the setup in [10, 11] is pathological.

[2]Indeed, $SO(3)$ rotations fix $\mathbf{x}$ dependence of the functions $a$ and $b$ as follows: $a(z, x) = a(z, |\mathbf{x}|)$ and $b(z, x) = b(z, |\mathbf{x}|)$. The requirement that the spacetime in question is translation invariant along $(x^1, x^2, x^3)$ rules out $x$ dependence completely.

[3]For simplicity we put the coefficient in front of the energy-momentum tensor to be equal to 1 since it does not play any role in our further discussions.

are the following

$$G_0^0 = 3b'' - 6b'^2$$
$$G_1^1 = 2b'' + a'' - 3b'^2 - a'^2 - 2a'b'$$
$$G_5^5 = -3b'^2 - 3b'a'$$

(3)

We introduce Minkowski brane at $z = 0$ with the following five-dimensional energy-momentum tensor

$$T_{B,\,b}^A = \operatorname{diag}(\rho_b + \sigma, \ -p_b + \sigma, \ -p_b + \sigma, \ -p_b + \sigma, \ 0)\,\delta(z)$$

(4)

where $p_b = \omega_b \rho_b$ is the brane equation of state and $\sigma$ is the brane tension. Israel junction condition [12] applied to our case reads

$$\left[K_B^A\right]\Big|_{z=0} = \int\limits_{-\infty}^{+\infty} \left(T_{B,\,b}^A - \frac{1}{3}T_{C,\,b}^C h_B^A\right)dz,$$

where we put the extrinsic curvature junction at the brane location in the l.h.s and $h_{AB}$ is the induced metric on the brane. Applying the above condition to our metric (1) we obtain

$$-2a'(0) = p_b + \frac{2}{3}\rho_b - \frac{\sigma}{3},$$
$$-2b'(0) = -\frac{\rho_b}{3} - \frac{\sigma}{3}.$$

(5)

From these equations brane matter quantities can be extracted.

**The Null Energy Condition and the No-Go Theorem.**   A NEC can be expressed in the covariant way via energy momentum tensor as follows

$$T_{AB}\xi^A\xi^B \geq 0, \quad g_{AB}\xi^A\xi^B = 0$$

(6)

for every null vector $\xi^A$. Physically a NEC implies that in adiabatic media a speed of sound does not exceed a speed of light.

Let us first sketch a couple of examples. Given a theory of an usual scalar field the NEC is satisfied automatically. For the perfect fluid with pressure $p$, energy density $\rho$ and state equation $p = w\rho$ it reads $w > -1$ and $w = -1$ for a vacuum.

The statement we prove in the current paper approves the result obtained in [9]. The authors state that it is impossible to shield the singularity from the brane by a horizon, unless the NEC is violated in the bulk or on the brane. Here we are going to prove the following statement:
*Let the following conditions hold*

1. *bulk NEC*    $T_{AB}\xi^A\xi^B \geq 0, \quad g_{AB}\xi^A\xi^B = 0$

2. *brane NEC*   $T_{b,\,\mu\nu}\xi^\mu\xi^\nu \geq 0, \quad g_{b,\,\mu\nu}\xi^\mu\xi^\nu = 0$

3. *positiveness of brane energy density*   $\rho_b + \sigma \geq 0$

4. *3d brane flatness*  $k = 0$

5. *bulk Lorentz invariance violation*  $a(z) \neq b(z)$.

*Then and only then a generic background of type* (1) *without bulk singularities*[4] *does not exist.*

Note that the conditions 1,2 and 3 in the above theorem are physical but the conditions 4 and 5 might be relaxed in principle. In the later case the theorem does not hold and we shall provide the reader with corresponding examples further.

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[4]By singularity a physical one is assumed, i.e. the case when the energy–momentum tensor has poles.

## 3 The Proof

Here we adduce the rigorous proof of the statement from previous Section which can be treated as a mathematical theorem. We note that it holds only for generic $a$ and $b$. The special case $a = b$ corresponding to the Lorentz invariant setup enables us to evade it under certain choice of the function and will be considered later.

**The NEC inequalities.** The bulk NEC reads $\mathcal{T} = T_{AB}\xi^A\xi^B \geq 0$ or equivalently

$$T_0^0 g_{00}(\xi^0)^2 + \sum_{i=1}^{3} T_i^i g_{ii}(\xi^i)^2 + T_5^5 g_{55}(\xi^5)^2 \geq 0$$

where the vector $\xi^A$ lives on the 5D light cone $g_{00}(\xi^0)^2 + \sum g_{ii}(\xi^i)^2 + g_{55}(\xi^5)^2 = 0$. Thus in order to provide the above inequality one can find an absolute minimum of the bilinear form $\mathcal{T}(\xi)$

$$\mathcal{T} = g_{00}T_0^0(\xi^0)^2 - |g_{11}|T_1^1\sum_{i=1}^{3}(\xi^i)^2 - T_5^5(\xi^5)^2 \tag{7}$$

on the light cone

$$g_{00}(\xi^0)^2 - |g_{11}|\sum(\xi^i)^2 = (\xi^5)^2 \tag{8}$$

and require that $\mathcal{T}_{\min} \geq 0$.

Expressing $\xi^0$ from (8) and substituting it into (7) one has

$$\mathcal{T} = |g_{11}|\left(T_0^0 - T_1^1\right)\sum_{i=1}^{3}(\xi^i)^2 + \left(T_0^0 - T_5^5\right)(\xi^5)^2$$

The form $\mathcal{T}$ is nonnegatively defined iff the inequalities $T_0^0 - T_1^1 \geq 0$ and $T_0^0 - T_5^5 \geq 0$ are satisfied. Using the Einstein equations (2) and Einstein tensor (3) one can express these inequalities in terms of the metric coefficients

$$b'' - a'' - 3b'^2 + a'^2 + 2a'b' \geq 0 \tag{9}$$
$$b'' - b'^2 + b'a' \geq 0 \tag{10}$$

These constraints should be completed by the brane NEC which can be extracted from Israel junction condition (5). Subtraction of the second equation from the first one in (5) yields

$$2(b'(0) - a'(0)) = p_b + \rho_b$$

We also consider $\rho_b + \sigma \geq 0$. Thus the brane NEC and positiveness of brane energy density imply that at $z = 0$ we have

$$b'(0) - a'(0) \geq 0, \quad b'(0) \geq 0 \tag{11}$$

which means that these inequalities are also satisfied at least at some vicinity of $z = 0$.

Reshuffling (9) and (10) one has

$$b'' - a'' - 3(a' - b')^2 - 4a'(b' - a') \geq 0 \tag{12}$$
$$b'' - (b' - a')^2 - a'(b' - a') \geq 0 \tag{13}$$

Thus the bulk and the brane NECs can be reformulated as (11)–(13).

**The proof.** Now we shall consider all possible cases assuming smoothness of the functions $a(z)$ and $b(z)$ and figure out whether it is possible or not to satisfy (11), (12) and (13) simultaneously. Let us first assume that in (11) $b'(0) > a'(0)$. Then there exists some positive $z_c$ and $0 < z < z_c$ such that the inequality (11) holds in this vicinity. In this region the inequality (12) can be replaced by the following equation

$$b'' - a'' - 3(a' - b')^2 - 4a'(b' - a') = \phi(b' - a') \tag{14}$$

where $\phi = \frac{T_0^0 - T_1^1}{b' - a'}$ is a nonnegative function for $0 < z < z_c$ and has to be finite (as we will see $b'(z) \neq a'(z)$) for regular solutions. (The fact that $\phi \geq 0$ is crucial in the proof whereas its finiteness will not be used further). The equation (14) can be solved in terms of $b'$ generically as follows[5]

$$b'(z) = a'(z) - \frac{\exp\left(4a(z) + 4\int\limits_0^z \phi(y)\, dy\right)}{3\int\limits_0^z dy\, \exp\left(4a(y) + 4\int\limits_0^y \phi(t)\, dt\right) - C} \tag{15}$$

and boundary conditions fix the constant of integration $1/C = b'(0) - a'(0) \geq 0$. Special case $b'(0) = a'(0)$ we shall consider later. Since $b' - a' \geq 0$ for $0 < z < z_c$ then the function $b - a$ increases at this interval. Let us first assume that $z_c$ is finite and $b' - a' \leq 0$ for $z > z_c$. The function $b - a$ stops increasing and starts decreasing at $z_c$ which means that the fraction in the r.h.s of (15) becomes zero at $z_c$. This is potentially possible in the two following cases:

1) the numerator vanishes;

2) the denominator turns to infinity.

The case 1) cannot be realized since in order to make the exponent in the numerator to be equal to zero (as the integral of $\phi$ is always nonnegative) $a$ has to have a pole $a \rightarrow -\infty$ at $z_c$. It means that the metric becomes singular and no smooth solution is possible. The case 2) can be excluded as well because of the following considerations. The integral $\int_0^{z_c} dy\, \exp\left(4a(y) + 4\int\limits_0^y \phi\, dt\right)$ has to diverge at $z_c$. It means there exists some point $z_*$ from $0 < z_* < z_c$ such that $\int_0^{z_*} dy\, \exp\left(4a(y) + 4\int\limits_0^y \phi\, dt\right) = C$. This entails a pole for $b' - a'$ according to (15). Hence this case is also not allowed.

Thus we showed that $b - a$ has to increase at the whole half-line. Let us consider the inequality (10). Analogously with (9) it can be reformulated as an equation with some nonnegative function $\chi$ involved

$$b'(z) = \frac{\exp\left(-a(z) + \int\limits_0^z \chi(y)\, dy\right)}{-\int\limits_0^z dy\, \exp\left(-a(y) + \int\limits_0^y \chi(t)\, dt\right) + \tilde{C}} \tag{16}$$

Here $\tilde{C} = 1/b'(0) > 0$. (The case $b'(0) = 0$ is possible as well. Because of (11) it follows that $a'(0) = 0$ also. This case will be elaborated further.) Taking into account (15) and (16) and applying the considerations we used after (15) one can see that in order to keep the functions $b' - a'$ and $b'$ finite in the bulk the following integrals

$$\int\limits_0^\infty e^{4a(z)}\, dz < \infty, \quad \text{and} \quad \int\limits_0^\infty e^{-a(z)}\, dz < \infty$$

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[5]We are going to utilize this trick in several places in this proof. The main idea is to show that under imposed conditions a denominator inevitably becomes zero at some point in the bulk thereby turning the whole expression to infinity. It shows that no background without bulk singularities is allowed under imposed bulk and brane NECs.

have to converge which cannot be provided simultaneously.

Now we treat the case with $b'(0) = a'(0)$ (it also includes the case $b'(0) = 0$). One can have $b'(z) = a'(z)$ for all positive $z$ or $b'(z) \neq a'(z)$ in some domain. The first opportunity will be considered separately in the Section 4 and corresponds to the Lorentz invariant case. As for the second one it appears that only $b'(z) \geq a'(z)$ case can be realized. Indeed let us perform Taylor expansion of both functions

$$a'(z) = a_0 + a_1 z + O(z^2), \quad b'(z) = a_0 + b_1 z + O(z^2). \tag{17}$$

The constant $a_0$ appears in both expansions and provides the condition $b'(0) = a'(0)$. Then up to the first order in $z$ the inequalities (12) and (13) become

$$b_1 - a_1 - 4a_0(b_1 - a_1)z + O(z^2) \geq 0, \quad b_1 - a_0(b_1 - a_1)z + O(z^2) \geq 0$$

We assume that $a_0 \neq 0$, $a_1 \neq 0$, $b_1 \neq 0$. It follows that $b_1 \geq 0$ and $b_1 \geq a_1$ in order to provide (12) and (13) at small $z$. Hence there should exists some point $z_n > 0$ where $b'(z_n) > a'(z_n)$ as the functions $b'$ and $a'$ are assumed to be different.[6] Starting from this point we can solve the same Cauchy problem as (14) but on the interval $(z_n, +\infty)$ with initial conditions $b'(z_n) - a'(z_n) = \text{const} > 0$. All considerations can be repeated and one can show that $b' - a'$ will diverge at some point in the bulk.

So far we have elaborated all possible cases except the one with $a(z) = b(z)$ corresponding to the Lorentz invariant background. If this is not the case then the statement from the last passage of the Section 2 is proved. But it appears that for $a = b$ there exists a class of solutions which respect NEC. We shall consider it separately in the next Section.

**Are the found singularities physical?** Now we show that the singularities we have found above are indeed physical, i.e. the energy momentum tensor has to become infinite at these points. The verification should be done as it happens sometimes that a singularity is just coordinate one (like horizon point in Schwarzschild-type metrics).

Let us consider $T_5^5$ which by corresponding Einstein equation is equal to $G_5^5$ from (3)

$$T_5^5 = -3b'(b' + a')$$

If $b'$ has a pole and $a'$ is finite or vice versa then $T_5^5$ has a pole as well. As we have shown the functions $a'$ and $b'$ cannot have singularities at the same point otherwise one of the expressions (15) or (16) becomes infinite. The remaining case is when $a' = b'$ identically and it can be also ruled out as $T_5^5$ diverges. Thus we have shown that all singularities are essentially physical and correspond to poles in the energy-momentum tensor.

## 4 Examples, Special Cases and How to Evade the Theorem

In this Section we discuss various deviations from the statement we have just proved. The first one refers to the Lorentz invariant case when the metric can be rewritten via one warp factor

$$ds^2 = e^{-2b(z)}(dt^2 - d\mathbf{x}^2) - dz^2. \tag{18}$$

We recall that the case with $b(z) = kz$ for $k = \sqrt{-\Lambda/6}$ corresponds to the well known RS models [5, 6]. The null energy condition for this case is satisfied as $\Lambda$ term does not affect the inequality (6). We are going to show that one can satisfy NEC as well with some media in the bulk apart from cosmological constant for the background (18) under particular choice of the function $b$.

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[6]If all derivatives in the function $b$ (or $a$) up to n'th order vanish one should start the expansion in (17) with $b_n + b_{n+1}z^{n+1} + \dots$ and proceed analogously.

**Lorentz invariant case.** For the Lorentz invariant case bulk and brane NECs simplify dramatically. The inequality (9) becomes trivial and (10) (which comes from the condition $T_0^0 - T_5^5 \geq 0$) reads simply as $b'' \geq 0$. Thus if we do not require anything else here then the NEC in the bulk can be satisfied with ease as well as all the other four conditions in the theorem. Due to physical reasons let us consider a bulk matter with nonnegative energy density $T_0^0 \geq 0$ (note that in the case with broken Lorentz invariance this condition is redundant). Using (2) and (3) one has

$$3b'' - 6b'^2 - \Lambda = T_0^0 \geq 0$$

We are going to apply the same trick here and rewrite the inequality via the following equation with some function $\phi$

$$b''\left(1 - \frac{\Lambda}{3b''}\right) - 2b'^2 - \phi b' = 0$$

where we have identified $3\phi b' = T_0^0$ and assumed that there exists a region where $b''$ does not vanishes. The assumption is reasonable as due to (13) which for the Lorentz invariant case turns merely into $b'' > 0$ at least in some vicinity of the origin or otherwise $b'' = 0$ for all $z$ (RS2 solution).[7] We have

$$b' = \frac{\exp\left(\int\limits_0^z dy\, \frac{\phi(y)}{1-\Lambda/3b''}\right)}{-2\int\limits_0^z dy\, \frac{\exp\left(\int\limits_0^y dt\, \frac{\phi(t)}{1-\Lambda/3b''}\right)}{1-\Lambda/3b''} + C'}$$

One can see that if the integral in the denominator of the above expression converges as $z \to \infty$ then the appropriate initial condition $C'$ in a way to make the denominator nonzero thereby providing finiteness of $b'$ in the bulk can be chosen.

For instance the following function can be considered

$$b'' = \begin{cases} (z - z_0)^2, & 0 \leq z < z_0, \\ 0, & z \geq z_0. \end{cases}$$

For $z \geq z_0$ the solution corresponds to the RS2 case and $T_B^A$ vanishes but for $0 \leq z < z_0$ one has different solution with nonzero bulk matter energy-momentum tensor satisfying the NEC.

**Example with perfect fluid.** Here we shall discuss one parametric deformation of the RS2 model with broken Lorentz invariance. The following energy-momentum tensor for the bulk matter is used

$$\begin{aligned}
T_0^0 &= ((1+\omega)u_0 u^0 - \omega)\rho \\
T_1^1 &= ((1+w)u_1 u^1 - w)\rho \\
T_5^5 &= ((1+\omega)u_5 u^5 - \omega)\rho \\
T_5^0 &= (1+\omega)\rho u^0 u_5
\end{aligned} \tag{19}$$

Here $u^A$ is a covariant velocity vector and $u^A u_A = 1$. The energy-momentum tensor (19) corresponds to an anisotropic perfect fluid with the isotropy parameter $w/\omega$. For $w = \omega$ the above tensor describes isotropic perfect fluid with state equation $p = w\rho$ where $p$ is the pressure and $\rho$ is the energy density.

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[7]The same considerations as in the previous passage may be applied to show that if $b''(0) = 0$ then it will either be identically zero or it will be positive at least in small vicinity of the origin.

One can show that bulk Einstein equations (2) in the generic background (1) with (19) in the r.h.s. can be solved as follows

$$a(z) = \xi k|z|, \quad b(z) = \zeta k|z|$$

$$\rho = -\Lambda - 6k^2\zeta^2$$

$$w = -1 + \frac{3\zeta^2 - 2\zeta\xi - \xi^2}{\Lambda/k^2 + 6\zeta^2} = -1 + k^2\frac{(3\zeta + \xi)(\xi - \zeta)}{\rho} \tag{20}$$

$$\omega = -1 + \frac{3\zeta(\zeta - \xi)}{\Lambda/k^2 + 6\zeta^2} = -1 + k^2\frac{3\zeta(\xi - \zeta)}{\rho}$$

with $u_1 = u_5 = 0$. We see that for the Lorentz invariant case ($\xi = \zeta$) only the bulk cosmological constant remains and it also corresponds to $w = \omega = -1$. This is indeed the RS2 model but with shifted cosmological constant $\tilde{\Lambda} = \Lambda + 6k^2\zeta^2$ and appropriate matching of this constant and $k$. Therefore anisotropy of the fluid controls the deviation from the RS2 configuration.

Israel junction conditions (5) for the brane energy-momentum tensor (4) for this particular case read

$$-2\xi k = p_b + \frac{2}{3}\rho_b - \frac{\sigma}{3}$$

$$-2\zeta k = -\frac{\rho_b}{3} - \frac{\sigma}{3}$$

Combining these equations we obtain $\rho_b = 6\zeta k - \sigma$ and

$$\omega_b = -\frac{2}{3} - \frac{1}{3}\frac{6\xi k - \sigma}{6\zeta k - \sigma} = -1 + \frac{2k(\zeta - \xi)}{\rho_b} \tag{21}$$

Let us now discuss the NECs for this configuration. One can show that for (19) in the bulk and for (4) on the brane they read

$$\omega \geq -1, \quad w \geq -1, \quad \omega_b \geq -1$$

We easily see that these conditions for (20) and (21) can be satisfied only for the Lorentz invariant case that is for the RS2 model.

**The Way Out – Positive 3d Curvature.** Here we want to show that analogously with [9] it is possible to have both NECs on the brane and in the bulk for a generic background if the spatial 3D curvature is positive. The following ansatz is employed

$$ds^2 = e^{-2\xi k|z|}dt^2 - \frac{e^{-2\zeta k|z|}}{\left(1 + \frac{1}{4}\kappa^2\mathbf{x}^2\right)^2}d\mathbf{x}^2 - dz^2,$$

where $\kappa$ in the Fubini–Study part of the metric is the inverted radius of the 3-sphere. It is straightforward to plug this expression in the Einstein equations to obtain the Einstein tensor ($z \neq 0$)

$$G_0^0 = 12\kappa^2 e^{2\zeta kz} - 6k^2\zeta^2$$

$$G_1^1 = 4\kappa^2 e^{2\zeta kz} - k^2(\xi^2 + 2\xi\zeta + 3\zeta^2) \tag{22}$$

$$G_5^5 = 3\kappa^2 e^{2\zeta kz} - 3k^2(\xi\zeta + \zeta^2)$$

Let us omit the cosmological constant as it does not play a role for the NEC. The Einstein equations read

$$G_0^0 = T_0^0 = \rho$$

$$G_1^1 = T_1^1 = -w\rho \tag{23}$$

$$G_5^5 = T_5^5 = -\omega\rho$$

we have

$$w = -\frac{(\xi^2 + 2\xi\zeta + 3\zeta^2)e^{-2\zeta kz} - 4\kappa^2/k^2}{6\zeta^2 e^{-2\zeta kz} - 12\kappa^2/k^2}, \quad \omega = -\frac{(\xi\zeta + \zeta^2)e^{-2\zeta kz} - 4\kappa^2/k^2}{2\zeta^2 e^{-2\zeta kz} - 4\kappa^2/k^2}$$

Now $\omega$ and $w$ become functions of $z$. We recall that for $\kappa = 0$ the violation of the bulk NEC for $\xi < \zeta$ took place. One can show that choosing appropriate ratio $\kappa/k$ one can adjust the parameters in order to make $w, \omega > -1$. Indeed we have $w \to -\frac{1}{3}$ and $\omega \to -1$ as $z \to +\infty$ and the choice $\kappa \sim k$ provides desirable values of $w$ and $\omega$ at small $z$. At the same time the brane NEC holds since in this case we need merely (11) which indeed holds.

## Conclusions

In this paper we proved the strong version of the no-go theorem thereby approving the result of [9]. Our statement does not allow generic smooth flat braneworld backgrounds with positively defined energy density satisfying NECs in the bulk and on the brane to exist. It is noteworthy that the statement of the theorem does not depend on finiteness of the volume of the extra dimension which claims that the integral

$$\int\limits_{-\infty}^{+\infty} dz \sqrt{g} < \infty.$$

converges. It appears that for a generic choice of the function $a$ and $b$ from (1) corresponding to the background with broken bulk Lorentz invariance the theorem holds. But it fails for the Lorentz invariant setup and one can obtain any number (depending on a particular choice of matter in the bulk) of the corresponding solutions.

Then we elaborated the one parametric Lorentz violating braneworld solution with anisotropic perfect fluid and negative cosmological constant in the bulk. The anisotropy parameter corresponds to the parameter controlling Lorentz invariance violation. The well known RS2 model is a particular case of the setup considered in this work and corresponds to the unbroken Lorentz invariance. It was also shown that if the brane curvature is positive then the theorem can be evaded provided that three dimensional spatial curvature is of the same order as the Anti-de-Sitter scale. It would mean, however, that warping effects in the extra dimension are of the same order as curvature effects in our Universe what is unacceptable from a phenomenological point of view.

One may expect that for time dependent ansatz with a broken Lorentz invariance the above pattern can be dramatically improved. From qualitative reasons one may think about fast enough changing of a scale factor with time in order to adjust Einstein tensor to provide $T_B^A$ satisfying null energy and positiveness of energy density conditions. This aspects will be elaborated in future works.

## Acknowledgments

## References

[1] N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, Phys. Lett. B **429**, 263 (1998) [arXiv:hep-ph/9803315].

[2] V. A. Rubakov. Phys. Usp. **44**, 871 (2001) [Usp. Fiz. Nauk **171**, 913 (2001)] [arXiv:hep-ph/0104152].

[3] V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. B **125**, 136 (1983).

[4] I. Antoniadis, Phys. Lett. B **246**, 377 (1990).

[5] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999) [arXiv:hep-ph/9905221].

[6] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 4690 (1999) [arXiv:hep-th/9906064].

[7] P. Brax and C. van de Bruck, Class. Quant. Grav. **20**, R201 (2003) [arXiv:hep-th/0303095].

[8] E. Kiritsis, Fortsch. Phys. **52**, 200 (2004) [Phys. Rept. **421**, 105 (2005 ERRAT,429,121-122.2006)] [arXiv:hep-th/0310001].

[9] J. M. Cline and H. Firouzjahi, Phys. Rev. D **65**, 043501 (2002) [arXiv:hep-th/0107198].

[10] M. V. Libanov and V. A. Rubakov. JCAP **0509**, 005 (2005) [arXiv:astro-ph/0504249].

[11] M. V. Libanov and V. A. Rubakov. Phys. Rev. D **72**, 123503 (2005) [arXiv:hep-ph/0509148].

[12] W. Israel, Nuovo Cim. B **44S10**, 1 (1966) [Erratum-ibid. **48**, 463 (1967 NU-CIA,B44,1.1966)].