ENERGY LOSS IN STOCHASTIC ABELIAN MEDIUM

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Abstract

Energy losses of fast charged particles in randomly spatially inhomogeneous stationary medium are considered. Analytical results for effective dielectric permittivity are presented.

Studies of energy losses of fast particles in a medium provide important information on it properties. Of special interest are situations in which the studied medium is not homogeneneous. A particular case that draws a lot of attention is that of a medium that is homogeneous only on average and characterized by random inhomogeneities on the event-by-event basis. This situation is common in radiophysics and acoustics, see e.g. [1]. In [2] it was shown that the parton system created at early stages of ultrarelativistic heavy ion collisions is characterized by violent event-by-event fluctuations of local partonic energy-momentum density. Progress in understanding the properties of energy loss of fast charged particles in such media generalizing the analysis referring to the homogeneous case, see e.g. [3, 4], is therefore very important in understanding the characteristics of dense matter created in ultrarelativistic heavy ion collisions. In the present talk we present estimates of the energy loss of fast particle in the random inhomogeneous medium in the abelian approximation based on the analytical calculation of the effective dielectric tensor. A detailed description of this calculation and its generalization to the case of quark-gluon plasma will appear in the forthcoming publication [5].

It is well known that the energy losses of charged particles in the medium can be computed by the work done on the particle by the electric field it creates in this medium. In the random medium the average losses per unit length dW/dz of a fast particle with charge e and velocity $\mathbf{v}$ are thus determined by the average electric field:

$$\frac{dW}{dz} = e\frac{\mathbf{v}}{v}\langle \mathbf{E}(\mathbf{r},t)\rangle_{\mathbf{r}=\mathbf{v}t} \tag{1}$$

In the case of inhomogeneous medium the local fluctuations of its properties are customarily parametrized by the coordinate-dependent dielectric permittivity $\varepsilon(\mathbf{r})$. For given configuration of $\varepsilon(\mathbf{r}$ the spectral component of the electric field is determined from

$$\triangle \mathbf{E} - \nabla\left(\nabla \mathbf{E}\right) + \omega^2\,\varepsilon(\mathbf{r}) = 4\pi i\omega j(\omega) \tag{2}$$

where $j(\omega)$ is a spectral component of the external current which in the considered case of uniformly moving fast particle reads $\mathbf{j}(\omega,\mathbf{k}) = 2\pi e\mathbf{v}\delta(\omega - \mathbf{k}\mathbf{v})$. Random inhomogeneities can

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conveniently be parametrized by explicitly identifying the non-random and random contributions to dielectric permittivity:

$$\varepsilon(\mathbf{r}) = \varepsilon_0 \left(1 + \xi(\mathbf{r})\right) \tag{3}$$

where $\xi(\mathbf{r})$ is a random contribution to permittivity having zero mean $\langle \xi(\mathbf{r}) \rangle = 0$. In what follows we shall consider the simplest case of Gaussian ensemble so that $\xi(\mathbf{r})$ are fully characterized by binary correlation function

$$\langle \xi\left(\mathbf{r}_1\right) \xi\left(\mathbf{r}_2\right) \rangle = g\left(|\mathbf{r}_1 - \mathbf{r}_2|\right) \tag{4}$$

It is well known that taking into account the random inhomogeneities leads to an equation for the average electric field containing a tensor of effective dielectric permittivity $\varepsilon_{ij}(\omega, \mathbf{k})$ that depends on the statistical properties of fluctuations $\xi(\mathbf{r})$:

$$\left[\varepsilon_{ij}(w, \mathbf{k}) - \frac{k^2}{w^2}\left(\delta_{i,j} - \frac{k_i k_j}{k^2}\right)\right] E_j(w, \mathbf{k}) = \frac{4\pi}{i\, w} j_i(w, \mathbf{k}) \tag{5}$$

where we have introduced a notation $w = \sqrt{\varepsilon_0}\omega$. It is convenient to explicitly introduce the transverse and longitudinal components ε_t and ε_l:

$$\varepsilon_{ij}(w, \mathbf{k}) \equiv \left(\delta_{ij} - \frac{k_i k_j}{k^2}\right) \varepsilon^t(w, \mathbf{k}) + \frac{k_i k_j}{k^2} \varepsilon^l(w, \mathbf{k}) \tag{6}$$

In terms of the decomposition (6) the expression for the energy losses takes, for some given $\varepsilon_{ij}(\omega, \mathbf{k})$, the form

$$\begin{aligned} \frac{dW}{dz} &= -\frac{e^2}{2\pi^2 v} \int d^3k \left\{ \frac{w}{k^2} \left[\mathrm{Im} \frac{1}{\varepsilon^l(w, \mathbf{k})} \right.\right. \\ &- \left.\left. \left(w^2 - v^2 k^2\right) \mathrm{Im} \frac{1}{w^2 \varepsilon^t(w, \mathbf{k}) - k^2} \right] \right\}_{w = \mathbf{kv}} \end{aligned} \tag{7}$$

Let us now consider the particular case of an exponential binary correlation function

$$g(r) = \sigma^2 e^{-\alpha r} \tag{8}$$

that allows an explicit analytical computation of $\varepsilon_{ij}(\omega, \mathbf{k})$ in the one-loop approximation corresponding to the regime $\sigma^2 w/\alpha \ll 1$ [5]. The corresponding calculation is naturally performed in terms of the polarization tensor $\Pi_{ij}(w, \mathbf{k})$ related to dielectric permittivity by the following relation:

$$\varepsilon_{ij}(w, \mathbf{k}) = \varepsilon_0 \left(1 - \frac{1}{w^2} \Pi_{ij}(w, \mathbf{k})\right) \tag{9}$$

The decomposition of dielectric permittivity (6) leads to the corresponding decomposition of the polarization tensor. Explicit expressions for it transverse and longitudinal components $\Pi^t_{ij}(w, \mathbf{k})$ and $\Pi^l_{ij}(w, \mathbf{k})$ read

$$\begin{aligned} \Pi^t_{ij}(w, \mathbf{k}) &= \sigma^2 w^2 \left[\frac{w^2}{(w + i\alpha)^2 - k^2} - \frac{\alpha(\alpha + iw)}{2k^2} \right. \\ &+ \left. \frac{\alpha^2 + w^2 + k^2}{k^2} \frac{\alpha}{k} \arctan\left(\frac{ik}{w + i\alpha}\right) \right] \end{aligned} \tag{10}$$

and

$$\begin{aligned} \Pi^l_{ij}(w, \mathbf{k}) &= \sigma^2 w^2 \left[1 + \frac{\alpha(\alpha + iw)}{2k^2} \right. \\ &+ \left. \frac{\alpha^2 + w^2 + k^2}{k^2} \frac{\alpha}{k} \arctan\left(\frac{ik}{w + i\alpha}\right) \right] \end{aligned} \tag{11}$$

Let us stress that even if the non-random delectric permittivity ε_0 does not have a significant imaginary part so that the corresponding energy losses determined by (7) are absent, the effective dielectric permittivity determined by (10,11) has a nontrivial imaginary part and, therefore, there arise specific energy losses directly related to the random fluctuations in the medium in which they propagate.

Let us present a few numerical calculations of this stochastic energy loss. In presenting the results it turns out convenient to rewrite the expression for the energy loss (7) in the form:

$$\frac{dW}{dz} = \frac{e^2\alpha^2}{\pi}\left[f_t(pa) + f_l(pa)\right] \tag{12}$$

where p is a momentum of the particle and $a = 1/\alpha$ is an inverse correlation radius of random fluctuations[1] Let us first consider the case $\varepsilon_0 = 0.7$ (i.e. without Cherenkov radiation) and $\sigma = 0.3$ and consider the stochastic energy loss as a function of the dimensionless variable pa. The result, broken into separate transverse and longitudinal contributions $f_t(pa)$ and $f_l(pa)$, is shown in Fig. 1. We see that with growing momentum transverse losses are rapidly becoming dominant. It is of interest to compare these results with the energy losses with the case in which Cherenkov losses exist already for the non-random case. To this aim let us consider the case $\varepsilon_0 = 1.1$ and the same value of $\sigma = 0.3$. The results are shown in Fig. 2. We see that the transverse losses are significantly amplified in comparison with the case of $\varepsilon_0 = 0.7$ while the longitudinal losses became smaller. It is also important to understand a dependence of the energy losses on the fluctuation magnitude σ. This dependence is shown, for the particular value of particle momentum $pa = 1$, in Fig. 3. We see that the dependence on σ is quite pronounced and the losses rapidly grow with growing σ.

Conclusions

Let us formulate the main results of the present talk:

1. Energy losses of charged particle in randomly inhomogeneous medium based on an analytical calculation of the effective dielectric permittivity were considered.

2. A significant difference between transverse and longitudinal components was demonstrated.

3. A rapid growth of energy loss with growing fluctuation magnitude σ was found.

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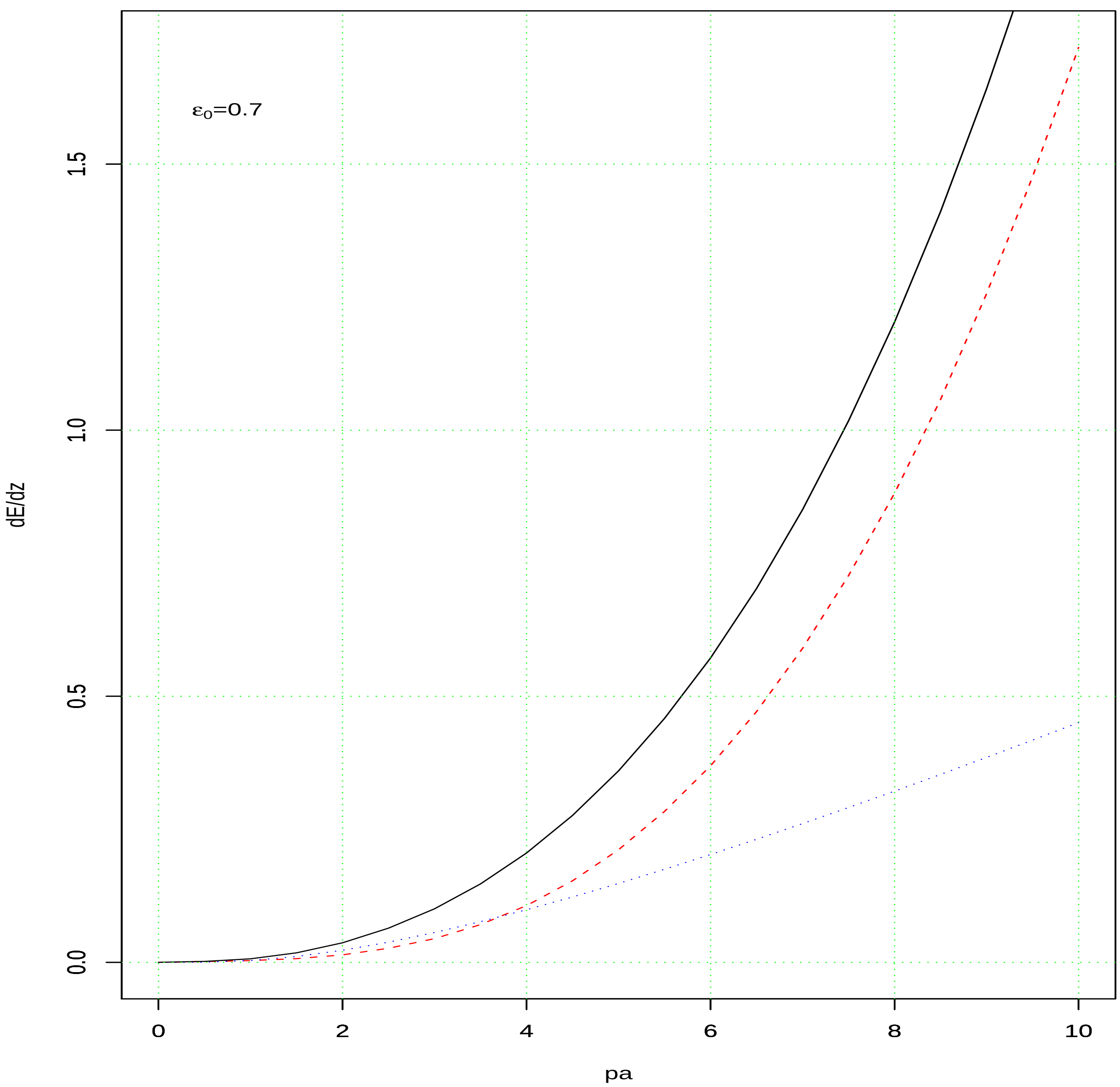

Figure 1: Stochastic energy losses for $\varepsilon_0 = 0.7$ as a function of momentum scale pa: transverse losses $f_t(pa)$- dashed line; longitudinal losses $f_l(pa)$ - dotted line; total losses - solid line.

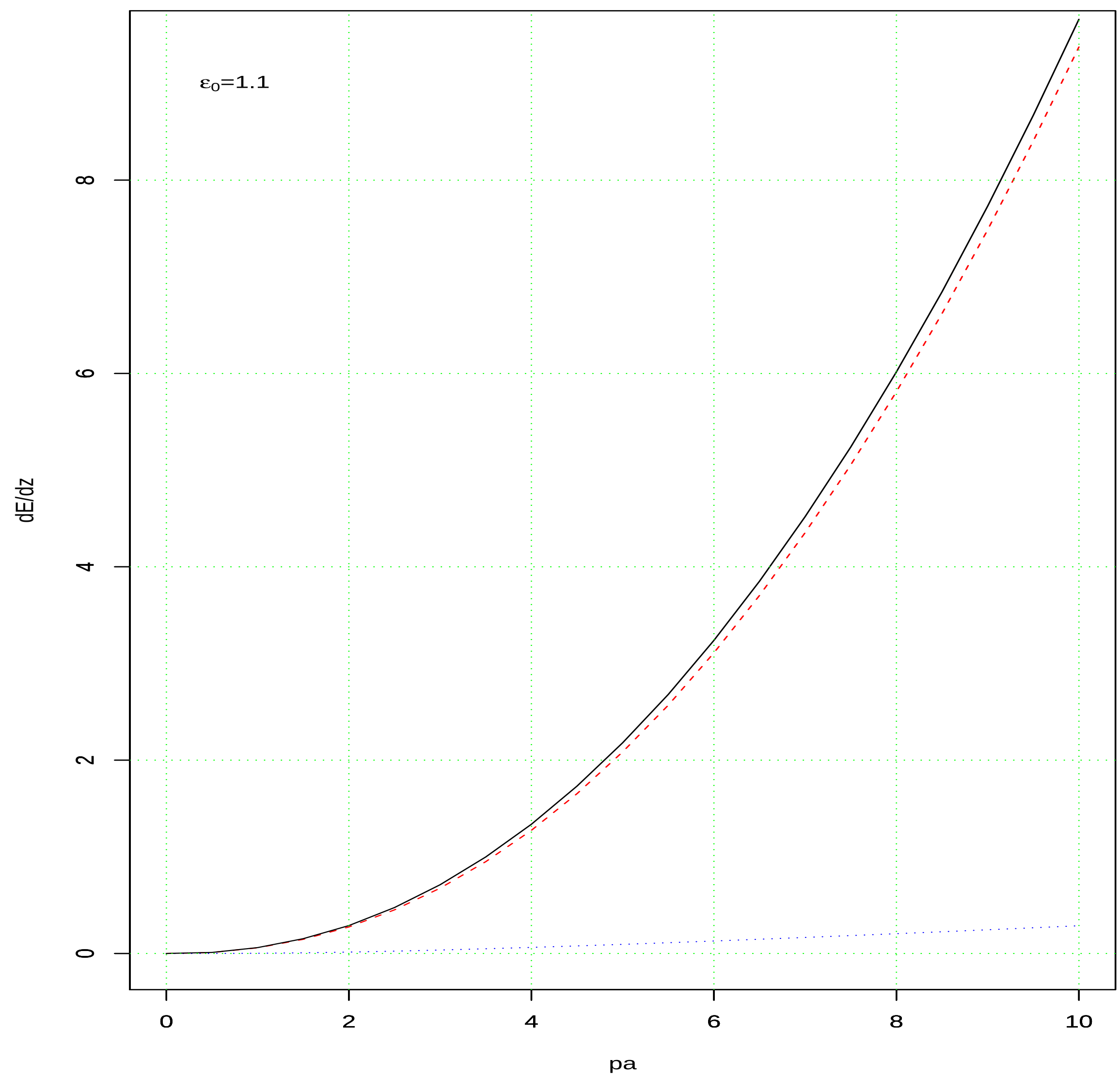

Figure 2: Stochastic energy losses for $\varepsilon_0 = 1.1$ as a function of momentum scale pa: transverse losses $f_t(pa)$- dashed line; longitudinal losses $f_l(pa)$ - dotted line; total losses - solid line.

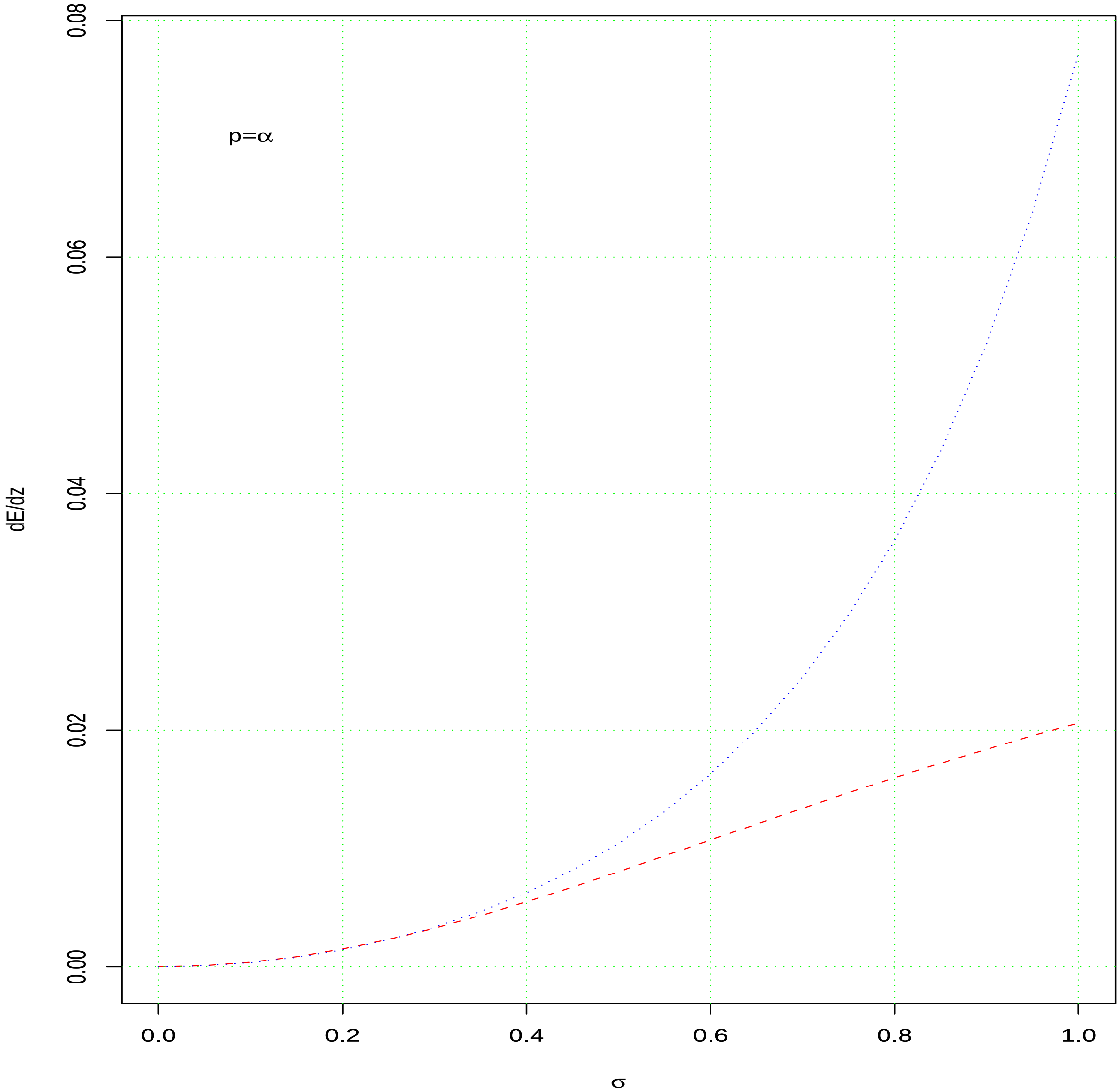

Figure 3: Dependence of stochastic energy losses on the fluctuation magnitude σ at $p = \alpha$: transverse losses - dashed line; longitudinal losses - dotted line.