Cosmic censorship for phantom energy

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Abstract

The third law of black hole thermodynamics is violated in the test fluid approximation for the process of phantom energy accretion onto a rotating or an electrically charged black hole. The black hole mass is continuously decreasing but the angular momentum or electric charge is remaining constant in this process. As a result, a black hole reaches the extreme state during a finite time with a threat of black hole transformation into the naked singularity and violation of the cosmic censorship conjecture. We demonstrate this by using new analytical solutions for spherically symmetric stationary distribution of a test perfect fluid with an arbitrary equation of state in the Reissner-Nordström metric. Our speculative assumption, however, is that the cosmic cosmic censorship conjecture remains valid even for phantom energy case, if one takes into account the back reaction of an accreting fluid onto a near extreme black hole. Some hint for the validity of this hypothesis comes from the specific case of the ultra-hard fluid accretion onto the rotating black hole. In this case the energy density of an accreting fluid diverges at the event horizon of an extreme black hole, thus violating the test fluid approximation.

1 Introduction

The problem of matter accretion onto the compact objects in the Newtonian gravity was formulated in a self-similar manner by Bondi [1]. In the framework of General Relativity a steady-state accretion of test gas onto a Schwarzschild black hole was investigated by Michel [2]. The detailed studies of spherically symmetric accretion of different types of fluids onto black holes were further undertaken in a number of works, see e. g. [3]. In [4] (see also [5] for further discussion) it was shown that accretion of a *phantom* fluid onto the Schwarzschild black hole results in a diminishing of black hole mass due to a negative flux of energy through the event horizon. Usually in the General Relativity it is assumed that matter has a *suitable* form. By *suitable*, one can imply that the stress-energy tensor of matter satisfy particular energy conditions, i.e. the weak energy condition (see for details, e.g. [6]). Meanwhile the phantom energy violates by definition the weak energy condition $\rho+p<0$, where ρ is an energy density and p is a pressure. It is believed that phantoms generically contain ghosts [7] and thus should be denied as non-physical. It turns out, however, that it is possible to construct a physically reasonable model of phantom, which is stable in the ultraviolet limit, thus giving no catastrophic instabilities of vacuum [8]. Thus the study of phantoms seems to be not fully meaningless from the physical point of view. Violation of weak energy condition by phantom brings unusual consequences:

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in the cosmological context the Big Rip singularity can be formed [9]; while the accreting of phantom leads to the diminishing of a black hole mass [4, 5].

In this paper, we find a solution for the steady-state accretion of a test perfect fluid with an arbitrary equation of state, $p = p(\rho)$, onto the charged black hole, when the event horizon for the Reissner-Nordström metric exists, $m^2 > e^2$. A similar analysis was performed in [10]. We show that a phantom energy accretion leads to the decreasing of a black hole mass. This result is consistent with [4] for Schwarzschild case. On the other hand, we show that when the Reissner-Nordström metric describes a naked singularity with $m^2 < e^2$, a perfect fluid does not accrete at all onto the the naked singularity, instead, a static atmosphere is formed.

As we show below, when neglecting the gravitational back reaction of the accreting fluid on the background metric, the extreme state of electrically charged black hole is reached by phantom energy accretion during the finite time. It is natural to ask then, whether it is possible to convert a Reissner-Nordström black hole into a naked singularity by accretion of phantom fluid. When phantom accretes onto a Schwarzschild black hole, the latter becomes smaller and smaller with time and might completely disappear, i.e. in the Big Rip scenario. However, in the case of phantom energy accretion onto a charged black hole, when the mass becomes smaller than the charge, $m^2 < e^2$, we might naively think that black hole transforms to a naked singularity by phantom energy accretion. This would also imply that the third law of black hole thermodynamics breaks down as well [11]. Such a process of a Kerr-Newman black hole transformation into a naked singularity by accretion of phantom energy was first discussed in [12].

The key conjecture of general relativity is the cosmic censorship by R. Penrose [13] prohibiting the appearance of naked singularity in the gravitational collapse of suitable matter. At the same time, the inevitable formation of singularity inside a black hole horizon is guaranteed by singularity theorems [14, 15]. It is worthwhile to note that by "usual means" it seems to be impossible to make a naked singularity from a black hole. For example, the Kerr-Newman black hole cannot be transformed into a naked singularity by capturing test particles with an electrical charge or orbital angular momentum [11, 16] and can be only approached to the extreme state in a limiting process.

We argue, however, that a test fluid approximation is inevitably violated when the Reissner-Nordström black hole or naked singularity is near to the extreme state. If this true, the back reaction of the accreting fluid on the background geometry may prevent black transformation into the naked singularity.

The paper is organized as follows. In Sec. 2 we construct a general formalism for the steady-state spherically symmetric accretion of a test perfect fluid onto the Reissner-Nordström black hole. In Sec. 3 we apply these results to the the particular examples of perfect fluids, the linear equation of state and the Chaplygin gas. The formation of fluid atmosphere around naked singularities is described in Sec. 4. The approaching to the extremal case by accretion of phantom and a possible violation of the third law of thermodynamics is studied in Sec. 5. We conclude and discuss the obtained results of the paper in Sec. 6.

2 Steady-state accretion of perfect fluid

In this Section we consider the steady-state accretion of a test perfect fluid with equation of state, $p = p(\rho)$, onto Reissner-Nordsröm black hole. We will closely follow the approach of Ref. [4] with the necessary modifications when needed.

The Reissner-Nordsröm metric is given by,

$$ds^2 = f dt^2 - f^{-1} dr^2 - r^2 (d\theta^2 + \sin^2\theta \, d\phi^2), \tag{1}$$

where

$$f(r) = 1 - \frac{2m}{r} + \frac{e^2}{r^2}. \tag{2}$$

In the case of $m^2 > e^2$ the are two roots of the equation $f = 0$:

$$r_\pm = m \pm \sqrt{m^2 - e^2}, \tag{3}$$

The larger root $r = r_+$ corresponds to the event horizon of the Reissner-Nordsröm black hole. The opposite case, $m < |e|$, corresponds to the naked singularity without the event horizon. The marginal case $m = |e|$ corresponds to the extreme black hole.

The energy-momentum of a perfect fluid is

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu - p g_{\mu\nu}, \tag{4}$$

where ρ and p are a fluid energy density and a pressure correspondingly, and $u^\mu = dx^\mu/ds$ is a fluid four-velocity with normalization, $u^\mu u_\mu = 1$. We assume that the pressure, is an arbitrary function of density only, $p = p(\rho)$. To find the integrals of motion we first use a projection of energy-momentum conservation on the four-velocity, $u_\mu T^{\mu\nu}{}_{;\nu} = 0$. This gives an equation for "particle" conservation,

$$u^\mu \rho_{,\mu} + (\rho + p)u^\mu_{;\mu} = 0. \tag{5}$$

Integrating Eq. (5) we find the first integral of motion (the energy flux conservation):

$$ur^2 \exp\left[\int\limits_{\rho_\infty}^{\rho} \frac{d\rho'}{\rho' + p(\rho')}\right] = -Am^2, \tag{6}$$

where $u < 0$ in the case of inflow motion, and a dimensionless constant $A > 0$ is an integration constant which is related to the energy flux.

Integration of the time component of energy-momentum conservation law $T^{\mu\nu}{}_{;\nu} = 0$ gives the second integral of motion for stationary spherically symmetric accretion in the Reissner-Nordsröm metric (the relativistic Bernoulli energy conservation equation):

$$(\rho + p)(f + u^2)^{1/2} r^2 u = C_1, \tag{7}$$

where $u = dr/ds$ and C_1 is an integration constant. From (6) and (7) one can easily obtain:

$$(\rho + p)(f + u^2)^{1/2} \exp\left[-\int\limits_{\rho_\infty}^{\rho} \frac{d\rho'}{\rho' + p(\rho')}\right] = C_2, \tag{8}$$

where $C_2 \equiv -C_1(Am^2)^{-1} = \rho_\infty + p(\rho_\infty)$.

Equations (6) and (8) along with an equation of state $p = p(\rho)$ describe a solution for accretion flow onto the Reissner-Nordström black hole. These equations are valid for a perfect fluid with an arbitrary equation of state $p = p(\rho)$ and may be applied, in particular, for the accretion of Chaplygin gas [17] or dark energy described by the generalized linear equation of state [18].

Note that while the constant C_2 is fixed simply by the boundary conditions at the infinity, the numerical constant A in Eq. (6) is fixed by an additional physical requirement of the smooth transition through the critical sound point. This provides a continuous solution for an accretion from infinity down to the black hole horizon. Following to Michel [2], we finds relations at the critical point:

$$u_*^2 = \frac{mr_* - e^2}{2r_*^2}, \quad c_s^2(\rho_*) = \frac{mr_* - e^2}{2r_*^2 - 3mr_* + e^2}, \tag{9}$$

where $c_s(\rho) \equiv (\partial p/\partial \rho)^{1/2}$ is a sound velocity, and the subscript '$*$' means that the values are estimated at the critical point. From (9) one can find:

$$\frac{r_*^{(\pm)}}{m} = \frac{1 + 3c_{s*}^2}{4c_{s*}^2}\left\{1 \pm \left[1 - \frac{8c_{s*}^2(1 + c_{s*}^2)}{(1 + 3c_{s*}^2)^2}\frac{e^2}{m^2}\right]^{1/2}\right\}, \tag{10}$$

where $c_{s*} \equiv c_s(r_*)$. From this equation it follows that in general there are two critical (sound) surfaces. The critical points exist only if,

$$\frac{e^2}{m^2} \le \frac{\left(1+3c_s^2\right)^2}{8c_s^2\left(1+c_s^2\right)}. \tag{11}$$

It is worthwhile to note that in contrast to the case of accretion onto a Schwarzschild black hole, there are formally two different critical points, with plus and minus signs in Eq. (10). The limit $e \to 0$ suggests that the point inner critical point, $r_*^{(-)}$, is unphysical, since $r_*^{(-)} = 0$ in the limit $e = 0$. Note, that in a general case $e \le m$ the inner critical point is in between of two horizons, $r_+ \ge r_*^{(-)} \ge r_-$. The point r_* with the plus sign, $r_*^{(+)}$, is a physical one, since it corresponds to the critical point. It is easy to see that $r_*^{(+)} \ge r_+$ for $e \le m$.

One can see that for $c_s^2 < 1$ there is a range of parameter e, such that the solution $r_*^{(\pm)}$ is real even for a naked singularity, $e^2 > m^2$. However, as we will see below, the existence of this point does not mean that the steady-state accretion actually takes place. In fact, there is no steady-state accretion onto naked singularity.

For accretion of a "superluminal" fluid [19], with $c_s > 1$, the critical point is inside the black hole horizon. It is interesting to note that for the extreme black hole, $e = m$, a critical point for "superluminal" fluid is always coincides with the horizon, $r_*(c_s > 1) = m$. For a naked singularity, $e^2 > m^2$, a critical point exists only for the limiting range of c_{s*}.

Using (10) and (9) one can find ρ_* from Eq. (8). Then for any equation of state, $p = p(\rho)$, the energy density at critical point ρ_* can be found from (6) and (9), and finally the parameter A is fixed.

From (6) and (8) one can find relations for a fluid velocity $u_+ = u(r_+)$ and density $\rho_+ = \rho(r_+)$ at the event horizon:

$$A\,\frac{m^2}{r_+^2}\left[\frac{\rho_+ + p(\rho_+)}{\rho_\infty + p(\rho_\infty)}\right] = \exp\left[2\int\limits_{\rho_\infty}^{\rho_+}\frac{d\rho'}{\rho' + p(\rho')}\right]. \tag{12}$$

The black hole mass changes at a rate $\dot m = -4\pi r^2 T_0{}^r$ due to the fluid accretion. With the help of (6) and (8) this expression may be written as

$$\dot m = 4\pi A m^2[\rho_\infty + p(\rho_\infty)]. \tag{13}$$

From this equation it is clear that accretion of a phantom energy, defined by a condition $\rho_\infty + p(\rho_\infty) < 0$, is always accompanied with a diminishing of the black hole mass. This is in accordance with previous findings [4]. The result is valid for any equation of state $p = p(\rho)$ with $\rho + p(\rho) < 0$.

3 Analytic solutions

In this Section we present several analytic solutions for the steady-state accretion of a perfect fluid onto a charged black hole (for more details see [24]).

3.1 Generalized linear equation of state

As a first example we consider the case of the generalized linear equation of state,

$$p = \alpha(\rho - \rho_0), \tag{14}$$

where α and ρ_0 are constants. This equation was introduced in [4] (see also [18]) to avoid hydrodynamical instability for a perfect fluid with the negative pressure. One can easily see that the constant α in (14) is the square of the sound speed of small perturbations, $\alpha = c_s^2$, and it must be positive to avoid catastrophic hydrodynamical instability. Using (9) and (10), one can calculate from (6) the dimensionless constant A for the linear equation of state

$$A = \alpha^{1/2} \frac{r_*^2}{m^2} \left(\frac{2\alpha r_*^2}{m r_* - e^2} \right)^{\frac{1-\alpha}{2\alpha}} . \tag{15}$$

To find the velocity and energy density profile versus r in the model (14) we need the joint solution of equations (6) and (8):

$$f + u^2 = \left(-\frac{ux^2}{A} \right)^{2\alpha}, \quad \frac{\rho + p}{\rho_\infty + p(\rho_\infty)} = \left(-\frac{A}{ux^2} \right)^{1+\alpha} . \tag{16}$$

It is possible to find analytical solutions of these equations for specific values of α, namely, $\alpha = 1/4,\ 1/3,\ 1/2,\ 2/3,\ 1$ and 2. For example, for $\alpha = 1/3$ we find for a radial distribution of energy density

$$\rho = \frac{\rho_0}{4} + \left(\rho_\infty - \frac{\rho_0}{4} \right) \left(\frac{1+2z}{3f} \right)^2 , \tag{17}$$

where

$$z = \begin{cases} \cos \dfrac{2\pi - \beta}{3}, & r_+ \le x \le r_*; \\ \\ \cos \dfrac{\beta}{3}, & x > r_* \end{cases} \tag{18}$$

and

$$\beta = \arccos \left(1 - \frac{27}{2} A^2 \frac{f^2}{x^4} \right) . \tag{19}$$

Phantom energy in this particular case corresponds to $\rho_0 > 4\rho_\infty$. At the horizon $x = x_+ = r_+/m$ we have

$$\rho_+ = \frac{\rho_0}{4} + \left(\rho_\infty - \frac{\rho_0}{4} \right) \frac{A^2}{x_+^4} . \tag{20}$$

Analogously, in the case of ultra-hard equation of state with $\alpha = 1$ we have

$$u^2 = \frac{(x - x_-) x_+^4}{(x + x_+)(x^2 + x_+^2) x^2} ; \tag{21}$$

$$\rho = \frac{\rho_0}{2} + \left(\rho_\infty - \frac{\rho_0}{2} \right) \frac{(x + x_+)(x^2 + x_+^2)}{(x - x_-) x^2} , \tag{22}$$

where $x = r/m$. Now at the horizon we find

$$\rho_+ = \frac{\rho_0}{2} + \left(\rho_\infty - \frac{\rho_0}{2} \right) \frac{2x_+}{\sqrt{m^2 - e^2}} . \tag{23}$$

From this equation it is seen that in the case of ultra-hard fluid an energy density at the horizon ρ_+ is diverging at $e \to m$. As a result, the test fluid approximation is violated in the limit $e \to m$.

3.2 Chaplygin gas

As an other solvable example we consider the Chaplygin gas with an equation of state,

$$p = -\frac{\alpha}{\rho} , \tag{24}$$

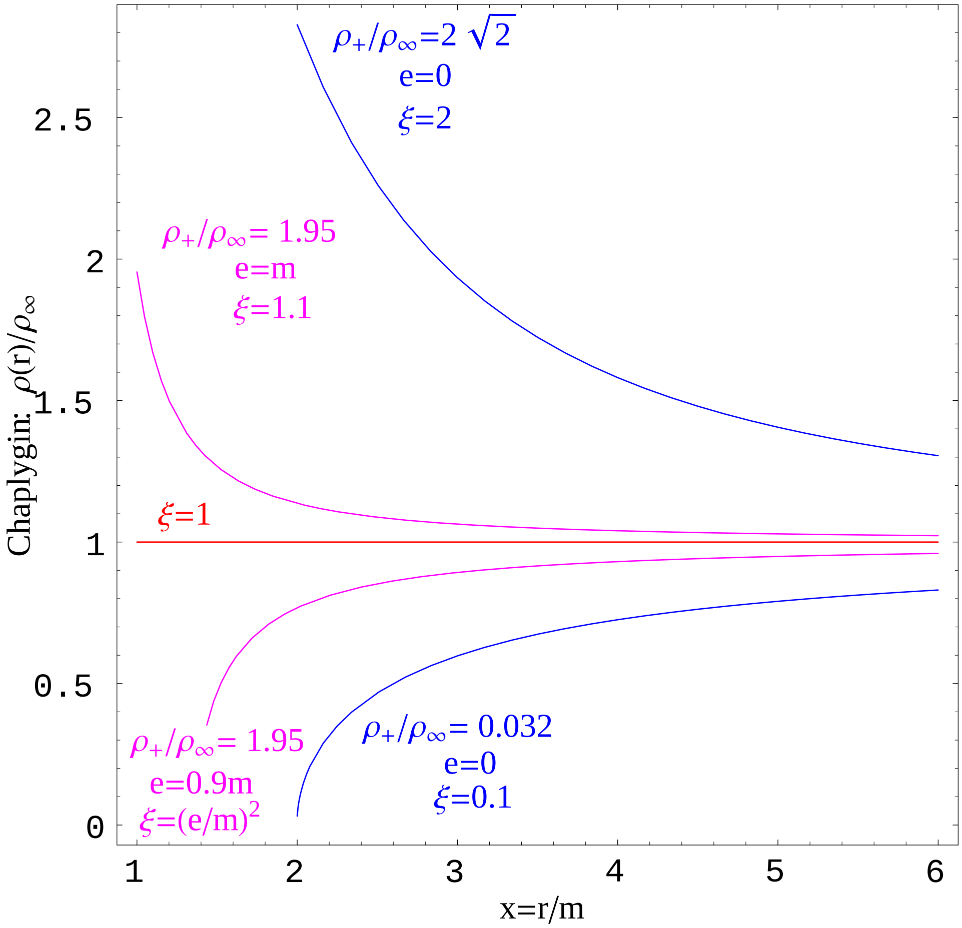

Figure 1: Examples of the radial energy density distribution of the Chaplygin gas accreting onto the Schwarzschild ($e = 0$) extreme Reissner-Nordström black hole ($e = m$). The horizontal lines $\rho(r) = \rho_\infty$ corresponds to the vacuum energy density ($\xi = 1$).

where constant $\alpha > 0$ corresponds to the hydrodynamically stable fluid. The Chaplygin gas with $\rho^2 < \alpha$ represents a phantom energy with superluminal speed of sound. Respectively, the case of $\rho^2 > \alpha$ corresponds to a dark energy with $\rho + p > 0$ and $0 < c_s^2 < 1$.

In the Reissner-Nordström metric with an equation of state (24) we find for the critical point:

$$f_* = \frac{\xi - 1}{\xi}, \quad r_* = \xi \left[1 \pm \left(1 - \frac{1}{\xi} \frac{e^2}{m^2} \right)^{1/2} \right], \quad A = \frac{x_*^2}{\xi^{1/2}}, \tag{25}$$

where $\xi = \rho_\infty^2/\alpha$. The sonic point exists if $\xi \geq \xi_{\min} = (e/m)^2$. At $\xi < \xi_{\min}$ the accretion is subsonic and the value of A is indefinite. The minimum value of A for a supersonic accretion is $A_{\min} = (e/m)^3$, corresponding to $r_* = \xi_{\min}$. For a radial dependence of dimensional energy density $y = \rho/\rho_\infty$ and radial 4-velocity u we find

$$\frac{\rho}{\rho_\infty} = \left[\frac{f + A(1-\xi)x^{-4}}{(1-\xi) + \xi f} \right]^{1/2}, \quad u = -\frac{A}{x^2} \left[\frac{1-\xi}{1 - \xi(\rho/\rho_\infty)^2} \right]^{1/2}. \tag{26}$$

The value of energy density at the event horizon is $\rho(r_+)/\rho_\infty = Am^2/r_+^2$. Solution (26) in the specific case $\xi = 1$ corresponds to the vacuum state with $p = -\rho = -\rho_\infty$ and $u = 0$. See in Fig. 1 some examples of radial energy density distribution of accreting Chaplygin gas around black hole.

In the case $e^2 > m^2$, by putting $u = A = 0$ in the equation (26), we obtain the static Chaplygin gas energy density distribution around the naked singularity (without the influx).

4 Lightweight atmosphere around naked singularity

In contrast to the case of the Reissner-Nordström black hole, there is no stationary accretion of a perfect fluid onto a naked singularity with $e^2 > m^2$. Formally this happens because no stationary solution for the accretion onto the naked singularity exists. Instead, a static atmosphere of lightweight perfect fluid around the Reissner-Nordström naked singularity is established with a zero influx. Assuming $u = 0$ from Eq. (8) we find a static distribution of a test perfect fluid with an arbitrary equation of state $p = p(\rho)$ around the Reissner-Nordström naked singularity

$$\frac{\rho + p}{\rho_\infty + p(\rho_\infty)} \exp\left[-\int\limits_{\rho_\infty}^{\rho} \frac{d\rho'}{\rho' + p(\rho')}\right] = f^{-1/2}. \tag{27}$$

In a particular case of linear equation of state (14) we obtain for the static atmosphere

$$\rho(r) = \frac{\alpha\rho_0}{1+\alpha} + \left(\rho_\infty - \frac{\alpha\rho_0}{1+\alpha}\right) f^{-\frac{1+\alpha}{2\alpha}}. \tag{28}$$

In Fig. 2 the distribution of energy density for the thermal radiation ($\alpha = 1/3$, $\rho_0 = 0$) and the phantom energy ($\alpha = 1/3$, $\rho_0 = 6\rho_\infty$) around the Reissner-Nordström naked singularity with $e = 2m$ is shown. For an ordinary matter with $\rho_0 = 0$ and $\alpha > 0$ the energy density tends to zero at the singularity, $\rho \propto r^{1+1/\alpha}$ at $r \to 0$. In the case of phantom, the energy density is finite at $r = 0$, and so phantom fluid overcomes the naked singularity repulsiveness.

5 Approaching to extreme state

An approaching to the extreme black hole state by capturing of particles with an electric charge and/or angular momentum is possible only in the limiting process during an infinite time [11, 20, 21]. At the same time, during accretion of a neutral phantom energy the electric charge of the Reissner-Nordström BH is unchanged, $e = const$, while a black hole mass diminishes. As a result the black hole is approaching to the near extreme state due to the growing of ratio $e/m(t)$. In the test fluid approximation, a black hole reaches the extreme state after the finite time $t = t_{\rm NS}$, defined by $e = m(t_{\rm NS})$. Using Eq. (13), the time $t_{\rm NS}$ for a black hole BH with initial mass $m = m(0)$ and electric charge $e = const$ may be calculated from relation:

$$\int_0^{t_{\rm NS}} \dot{m}\, dt = e - m(0). \tag{29}$$

If we neglect the cosmological evolution of ρ_∞, then from (13), (15) and (29) for a particular case of a stationary phantom energy with the ultra-hard equation of state (with $c_s = 1$) we obtain

$$t_{\rm NS} = \frac{q^3 - 3q^2 + 2 - 2(1-q^2)^{3/2}}{3q^4}\,\tau, \tag{30}$$

where $q = e/m(0)$ and $\tau = -\{4\pi[\rho_\infty + p(\rho_\infty)]m(0)\}^{-1}$ is a characteristic accretion time. A corresponding relation for time $t_{\rm NS}$ needed to bring the Kerr black hole with an angular momentum $J = const$ to the near extreme state by accretion of phantom energy in the test fluid approximation is

$$\int_0^{t_{\rm NS}} \dot{m}\, dt = \sqrt{J} - m(0). \tag{31}$$

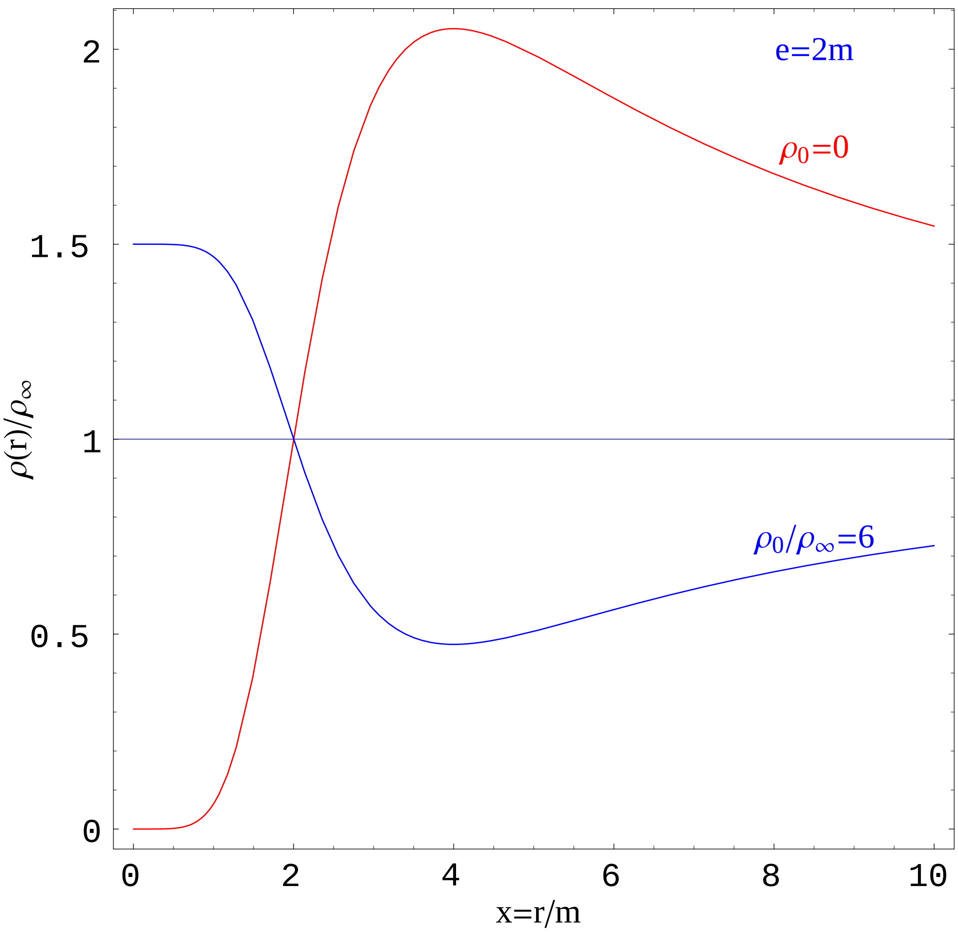

Figure 2: An example of energy density distribution for the thermal radiation ($\alpha = 1/3$, $\rho_0 = 0$) and the phantom energy ($\alpha = 1/3$, $\rho_0 = 6\rho_\infty$) in a static atmosphere around the Reissner-Nordström naked singularity with $e = 2m$. The inverse energy density profile of thermal radiation near the singularity is a manifestation of the repulsive character of naked singularity.

From this relation in a case of accretion of phantom energy with the ultra-hard equation of state ($c_s = 1$) by using (13) and (31) with $A = 2r_+/m$ from [22] we find

$$t_{\rm NS} = \frac{1}{6\tilde{a}^{1/2}}\left[1 - \frac{1-\sqrt{1-\tilde{a}^2}}{\tilde{a}^{3/2}} + 2F(\frac{1}{2}\arccos\tilde{a}, 2)\right], \qquad (32)$$

where $\tilde{a} = J/m^2(0)$ and $F(\phi, k)$ is an elliptic integral of the first kind.

According to [22] an energy density of the accreting fluid with $c_s = 1$ at the event horizon of the Kerr black hole is

$$\left[\frac{\rho_+ - \rho_0/2}{\rho_\infty - \rho_0/2}\right] = \frac{1}{r_+^2 + a^2\cos^2\theta}\left(\frac{4r_+^2 m}{\sqrt{m^2-a^2}} - a^2\sin^2\theta\right), \qquad (33)$$

where $\rho_+ = \rho(r_+)$. This energy density is diverging at $a \to m$. The similar energy density in the case of Reissner-Norsdström:

$$\rho = \frac{\rho_0}{2} + \left(\rho_\infty - \frac{\rho_0}{2}\right)\frac{r^4 - B^2 m^4}{r^2(r^2 - 2mr + e^2)}, \qquad (34)$$

where $B = r_+$. The energy density at the horizon of near extreme Reissner-Norsdström black hole is

$$\rho_+ = \frac{\rho_0}{2} + \left(\rho_\infty - \frac{\rho_0}{2}\right)\frac{2r_+}{\sqrt{m^2-e^2}} \to \pm\infty \quad \text{at} \quad m \to e. \qquad (35)$$

Respectively, it can be easily verified from (27), (28) and (33) that energy density of light atmosphere around the near extreme electrically charged or rotating naked singularity with $\epsilon \ll 1$ is also diverging at $r = m$. Thus the test fluid approximation breaks down in the case $\alpha = 1$ (i.e., $c_s = 1$). This divergent behavior of the energy density is remained also in a more general case $\alpha > 1$. Indeed, the corresponding values of u and ρ at the horizon in the case of a linear equation of state are according to (16) are

$$u_+ = \left(\frac{A}{x_+^2}\right)^{\alpha/(\alpha-1)}, \quad \rho_+ = \frac{\alpha}{\alpha+1}\rho_0 + \left[\rho_\infty - \frac{\alpha}{\alpha+1}\rho_0\right]\left(\frac{x_+^2}{A}\right)^{(\alpha+1)/(\alpha-1)}. \tag{36}$$

It can be seen from (15) that $A \to (4\epsilon)^{1/4}$ at $\epsilon \to 0$, where the extreme parameter $\epsilon = (m^2 - e^2)/m^2$. As a result at the horizon $u \to 0$ and $\rho \to \infty$ in the limit $e \to m$.

Analogous violation of a test fluid approximation occurs at a radius $r = m$ in a static atmosphere around the near extreme naked singularity with $-\epsilon \ll 1$ due to divergence of the energy density. Namely, from (28) it can be verified that $\rho(m) \propto [\rho_\infty - \alpha\rho_0/(1+\alpha)]\,|\epsilon|^{-\frac{1+2\alpha}{4\alpha}}$ at $-\epsilon \ll 1$. Additionally, for phantom energy case, when $[\rho_\infty - \alpha\rho_0/(1+\alpha)] < 0$, the strong energy domination condition is violated, $\rho(m) \to -\infty$ at $-\epsilon \to 0$. A similar divergence of energy density occurs at radius $r = m$ in a stationary atmosphere around the near extreme Kerr naked singularity [23].

Meanwhile, in the case of $0 < \alpha < 1$ the energy density of the accreting fluid remains finite even for the extreme black hole. Nevertheless, the validity of the test fluid approximation is still questionable. We assume, however, that back reaction of the accretion flow will prevent the transformation of black hole into the naked singularity. Similar idea of the importance of back reaction was proposed in [25] for absorption of scalar particles with large angular momentum by a near extreme black hole.

6 Conclusion and discussion

In this paper we considered the stationary distribution of the test perfect fluid with an arbitrary equation of state, $p = p(\rho)$, in the Reissner-Nordström metric. Similarly to the well-known case of the stationary accretion of perfect fluid onto the Schwarzschild black hole, the corresponding solution for the accretion exists also in the case of Reissner-Nordström black hole. On the contrary, there is no stationary accretion of the perfect fluid onto the Reissner-Nordström naked singularity, $e > m$. Instead, the static atmosphere of the fluid is formed around the naked singularity. In both cases of the black hole and the naked singularity we found the analytical solution to the problem of the steady state configurations of the perfect fluids with an arbitrary equation of state, $p = p(\rho)$. As the particular cases, we considered fluid with the linear equation of state, $p = \alpha(\rho - \rho_0)$ and the Chaplygin gas, $p = \alpha/\rho$.

When the accreting fluid is phantom, $\rho + p < 0$, the mass of the Reissner-Nordström black hole decreases. This result is in the agreement with the previous findings [4, 5]. This immediately leads us to the question, whether it is possible to transform the Reissner-Nordström black hole into the naked singularity by by accretion of phantom. Formally it seems so, since the accreting phantom decreases the black hole mass, while the electric charge of the black hole remains the same. Thus, one can expect that at some finite moment of time a black hole will turn into the naked singularity. Indeed, as we have shown in Sec. 5, it takes the finite time for the Reissner-Nordström black hole to reach the extreme case. The similar result also holds for the Kerr black hole.

However, this naive picture, taken out in the test fluid approximation, does not seem realized if one takes into account the back reaction of perfect fluid onto the background metric. First of all, in the case of ultra-hard equation of state, $p = \rho$, the fluid density diverges at the horizon, $r = r_+$, when black hole is approaching to the extremal state, $m \to e$. Thus, the test fluid approximation breaks down and the results is not applicable. In other cases, when $c_s^2 \neq 1$, the situation is more subtle and beyond the scope of this paper. Our preliminary investigations show, however, that in a general case, the test fluid approximation breaks down at the extreme case. We expect that black hole cannot be converted into the naked singularity even by the accretion of phantom energy, and thus the third law of thermodynamics is not violated in this case.

We would like to stress, that although the test fluid approximation seems to break down for the near-extreme state of the black hole/naked singularity, for the far-from-the-extreme state black hole (in particular, for the Schwarzschild solution), the parameters of the perfect fluid and the boundary condition at the infinity can be tuned so, that the test fluid approximation describes well the problem under consideration. In particular, we stress, that the phantom energy accretion indeed leads to decreasing of the black hole mass.

If the back reaction does not prevent the process of phantom accretion onto a charged black hole or rotating black hole, then it is a way to violate the cosmic censorship. Otherwise, the phantom energy must be totally forbidden on the more fundamental basis, as for instance, a quantum instability.

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