Electromagnetic Processes In Strongly Magnetized Plasma

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Abstract

The electromagnetic processes of Compton scattering and photon splitting/merging are investigated in the presence of strongly magnetized electron-positron plasma. The influence of these processes on the radiation transfer in the astrophysical environment is studied. In particular, the contribution of the processes under consideration in coefficients of the transfer equation is calculated. We show the importance of photon splitting/merging contribution and taking into account of photon dispersion and wave function renormalization in strong magnetic field and plasma.

1 Introduction

Magnetars are extremely interesting objects both from the physical and astrophysical point of view. From one hand they are associated with SGR and AXP pulsars considered as isolated neutron stars with unusual spectral properties. From the other hand they allow one to investigate different phenomena taking place in super strong magnetic field condition not available elsewhere. Magnetic field strength of magnetar is believed to be $B \sim 10^{14} - 10^{16}$ G [5–7], i.e. $B \gg B_e$, where $B_e = m^2/e \simeq 4.41 \times 10^{13}$ G [1] is the critical magnetic field. The spectra analysis of these objects is also providing evidence for the presence of electron-positron plasma in magnetar environment. It is well-known that strong magnetic field and/or plasma could influence essentially on different quantum processes [8–11]. One of such phenomena is the radiation transfer in strongly magnetized plasma. This process is connected with the SGR and AXP spectral formation. Moreover it is the crucial ingredient of the models of SGR burst (see e.g. [6]) where the creation of magnetically trapped high temperature (~ 1 MeV) plasma fireball is assumed (see Fig. 1). It also defines the cooling rate of the outer crust of magnetar [12].

The various studies indicate that electromagnetic processes such as Compton scattering and photon splitting $\gamma \to \gamma\gamma$ (merging $\gamma\gamma \to \gamma$) could play a crucial role in these models.

In the present work the influence of these processes on radiation transfer is investigated in the presence of strong magnetic field and electron positron plasma, when the magnetic field strength B is the maximal physical parameter, namely $\sqrt{eB} \gg T,\ \mu\,\omega,\ E$. Here T is the plasma temperature, μ is the chemical potential, ω and E is the initial photon and electron (or positron) energies. In this case almost all electrons and positrons in plasma are on the ground Landau level.

The main goal of this talk is to demonstrate that the self-consistent accounting of strong magnetic field and dense plasma influence is necessary for the correct description of radiation transfer.

[1] We use natural units $c = \hbar = k = 1$, m is the electron mass, $e > 0$ is the elementary charge.

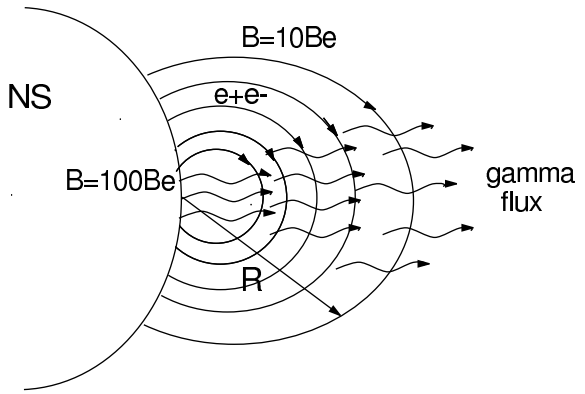

Figure 1: Radiation transfer in Tompson & Duncan model of SGR burst [6].

2 Photon dispersion properties

The propagation of the electromagnetic radiation in any active medium is convenient to describe in terms of normal modes (eigenmodes). In turn, the polarization and dispersion properties of normal modes are connected with eigenvectors and eigenvalues of polarization operator correspondingly. In the case of strongly magnetized plasma in the one loop approximation the eigenvalues of the polarization operator can be derived from the previously obtained results [23–25]:

$$\mathcal{P}^{(1)}(q) \simeq -\frac{\alpha}{6\pi}\left[q_\perp^2+\sqrt{q_\perp^4+\frac{(6N\omega)^2q^2}{q_\parallel^2}}\right]-q^2\,\Lambda(B), \tag{1}$$

$$\mathcal{P}^{(2)}(q) \simeq -\frac{2eB\alpha}{\pi}\left[H\left(\frac{q_\parallel^2}{4m^2}\right)+\mathcal{J}(q_\parallel)\right]-q^2\,\Lambda(B), \tag{2}$$

$$\mathcal{P}^{(3)}(q) \simeq -\frac{\alpha}{6\pi}\left[q_\perp^2-\sqrt{q_\perp^4+\frac{(6N\omega)^2q^2}{q_\parallel^2}}\right]-q^2\,\Lambda(B), \tag{3}$$

where

$$\Lambda(B)=\frac{\alpha}{3\pi}\,\left[1.792-\ln(B/B_e)\right],\quad N=\int\limits_{-\infty}^{+\infty}dp_z\,\left[f_-(E)-f_+(E)\right],$$

$$\mathcal{J}(q_\parallel)=2q_\parallel^2m^2\int\frac{dp_z}{E}\,\frac{f_-(E)+f_+(E)}{(q_\parallel^2)^2-4(pq)_\parallel^2},\qquad E=\sqrt{p_z^2+m^2},$$

$f_\pm(E)=[e^{(E\pm\mu)/T}+1]^{-1}$ are the electron (positron) distribution functions,

$$H(z)=\frac{1}{\sqrt{z(1-z)}}\arctan\sqrt{\frac{z}{1-z}}-1,\quad 0\leqslant z\leqslant 1, \tag{4}$$

$$H(z)=-\frac{1}{2\sqrt{z(z-1)}}\ln\frac{\sqrt{z}+\sqrt{z-1}}{\sqrt{z}-\sqrt{z-1}}-1+\frac{i\pi}{2\sqrt{z(z-1)}},\quad z>1. \tag{5}$$

Here $z=q_\parallel^2/(4m^2)$, the four-vectors with indices $\perp$ and $\parallel$ belong to the Euclidean $\{1,\,2\}$-subspace and the Minkowski $\{0,\,3\}$-subspace correspondingly in the frame were the magnetic field is directed along z (third) axis; $(ab)_\perp=(a\Lambda b)=a_\alpha\Lambda_{\alpha\beta}b_\beta$, $(ab)_\parallel=(a\tilde{\Lambda}b)=a_\alpha\tilde{\Lambda}_{\alpha\beta}b_\beta$, where the tensors $\Lambda_{\alpha\beta}=(\varphi\varphi)_{\alpha\beta}$, $\widetilde{\Lambda}_{\alpha\beta}=(\tilde{\varphi}\tilde{\varphi})_{\alpha\beta}$, with equation $\widetilde{\Lambda}_{\alpha\beta}-\Lambda_{\alpha\beta}=g_{\alpha\beta}=diag(1,-1,-1,-1)$

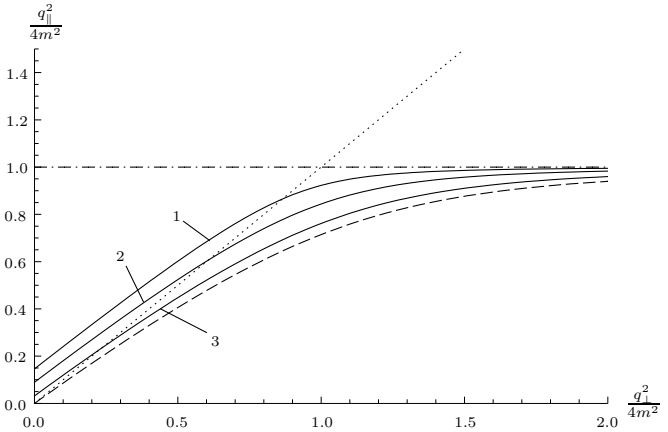

Figure 2: Photon dispersion laws in strong magnetic field $B/B_e = 200$ and neutral plasma vs. temperature: $T = 1$ MeV – 1, $T = 0.5$ MeV – 2 and $T = 0.25$ MeV – 3. Photon dispersion without plasma is depicted by dashed line. Dotted line corresponds to the vacuum dispersion law, $q^2 = 0$. The angle between the photon momentum and the magnetic field direction is $\pi/2$.

are introduced. $\varphi_{\alpha\beta} = F_{\alpha\beta}/B$ and $\tilde{\varphi}_{\alpha\beta} = \frac{1}{2}\varepsilon_{\alpha\beta\mu\nu}\varphi_{\mu\nu}$ are the dimensionless field tensor and dual field tensor correspondingly.

The dispersion properties of the normal modes could be defined from the dispersion equations

$$q^2 - \mathcal{P}^{(\lambda)}(q) = 0 \qquad (\lambda = 1, 2, 3). \tag{6}$$

Their analysis shows that 1 and 2 modes with polarization vectors

$$\varepsilon_\alpha^{(1)}(q) = \frac{(q\varphi)_\alpha}{\sqrt{q_\perp^2}}, \qquad \varepsilon_\alpha^{(2)}(q) = \frac{(q\tilde{\varphi})_\alpha}{\sqrt{q_\parallel^2}}. \tag{7}$$

are only physical ones in the case under consideration, just as it is in the pure magnetic field [2]. However, it should be emphasized that this coincidence is approximate to within $O(1/\beta)$ and $O(\alpha^2)$ accuracy.

Notice, that in plasma only the eigenvalue $\mathcal{P}^{(2)}(q)$ is modified in comparison with pure magnetic field case. It means that the dispersion law of the mode 1 is the same one as in the magnetized vacuum, where its deviation from the vacuum law, $q^2 = 0$, is negligibly small. From the other hand, the dispersion properties of the mode 2 essentially differ from the magnetized vacuum ones. In the Fig. 2 – 3 the photon dispersion in both strong magnetic field and magnetized plasma are depicted at various temperatures (for the charge-symmetric plasma) and chemical potential (for the degenerate plasma). One can see that in the presence of the magnetized plasma there exist the kinematical region, where $q^2 > 0$ contrary to the case of pure magnetic field. It is connected with the appearance of the plasma frequency in the present of the real electrons and positrons which can be defined from equation

$$\omega_{pl}^2 - \mathcal{P}^{(2)}(\omega_{pl}, \mathbf{k} \to 0) = 0. \tag{8}$$

This fact could lead to the modification of the kinematics of the different processes with photons. The analysis shows that the main channels of photon scattering and photon splitting/merging are

[2]Symbols 1 and 2 correspond to the $\parallel$ and $\perp$ polarizations in pure magnetic field [26] and E- and O- modes in magnetized plasma [6].

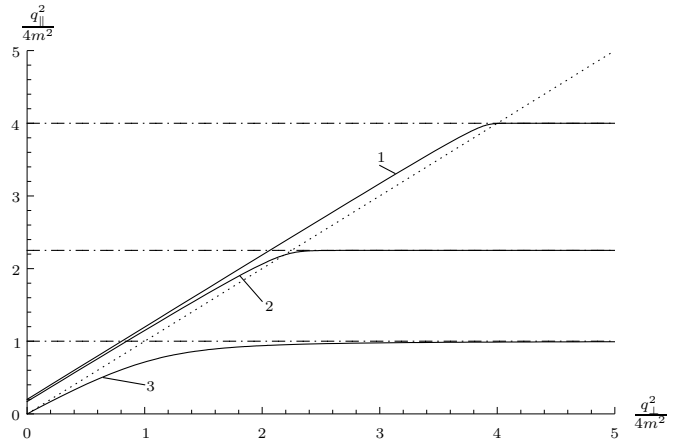

Figure 3: Photon dispersion in a strong magnetic field ($B/B_e = 200$) and degenerate plasma vs. chemical potential $\mu = 1$ MeV – 1, $\mu = 0.75$ MeV – 2 and without plasma – 3. Dotted line corresponds to the vacuum dispersion law, $q^2 = 0$. The angle between the photon momentum and the magnetic field direction is $\pi/2$.

- mode 1 (extraordinary photon):

$$\gamma_1 e^\pm \to \gamma_1 e^\pm, \gamma_1 e^\pm \to \gamma_2 e^\pm, \gamma_1 \to \gamma_1\gamma_2,$$
$$\gamma_1 \to \gamma_2\gamma_2, \gamma_1\gamma_2 \to \gamma_1, \gamma_1\gamma_1 \to \gamma_2;$$

- mode 2 (ordinary photon):

$$\gamma_2 e^\pm \to \gamma_2 e^\pm, \gamma_2 e^\pm \to \gamma_1 e^\pm, \gamma_2 \to \gamma_1\gamma_1,$$
$$\gamma_2\gamma_2 \to \gamma_1, \gamma_2\gamma_1 \to \gamma_1.$$

It follows from Eq. (2) that the eigenvalue of the polarization operator $\mathcal{P}^{(2)}$ becomes large near the electron-positron pair production threshold. This suggests that the renormalization of the wave function for a photon of this polarization should be taken into account:

$$\varepsilon_\alpha^{(2)}(q) \to \varepsilon_\alpha^{(2)}(q)\sqrt{Z_2}, \quad Z_2^{-1} = 1 - \frac{\partial \mathcal{P}^{(2)}(q)}{\partial \omega^2}. \tag{9}$$

3 Transfer equation

In general case the propagation of photon modes through a magnetized plasma can be described by the following equations:

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2 D_1 \frac{dn_1}{dr}\right) + K_1(\bar{n} - n_1) + S_{12}(n_2 - n_1) = 0, \tag{10}$$

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2 D_2 \frac{dn_2}{dr}\right) + K_2(\bar{n} - n_2) + S_{21}(n_1 - n_2) = 0, \tag{11}$$

$$\bar{n} = \frac{\omega^3}{2\pi^2} f_\omega, \qquad f_\omega = [\exp(\omega/T) - 1]^{-1}.$$

where n_1, n_2 are photon occupation numbers for extraordinary and odinary modes, f_ω is photon distribution function, $D_\lambda, K_\lambda, S_{\lambda\lambda'}$ are diffusion, absorption and scattering coefficients for different photon modes correspondingly ($\lambda = 1, 2$) which can be obtained by the angle averaging

of the photon splitting/merging and photon scattering rates:

$$D_\lambda = \int \frac{d\Omega}{4\pi} \ell_\lambda(\theta, r) \cos^2\theta, \tag{12}$$

$$K_\lambda = \int \frac{d\Omega}{4\pi} \left[W_{\lambda\to\lambda'\lambda''}(\theta, r) + W_{\lambda\lambda'\to\lambda''}(\theta, r) \right], \tag{13}$$

$$S_{\lambda\lambda'} = \int \frac{d\Omega}{4\pi} W_{\lambda\to\lambda'}(\theta, r), \tag{14}$$

where

$$\ell_\lambda = \left[\sum_{\lambda'=1}^{2} W_{\lambda\to\lambda'} + \sum_{\lambda',\lambda''=1}^{2} \left(W_{\lambda\to\lambda'\lambda''} + W_{\lambda\lambda'\to\lambda''} \right) \right]^{-1}. \tag{15}$$

In turn, the rates of the processes under consideration are given by the following formulas:

$$W_{\lambda\to\lambda'} = \frac{eB}{16(2\pi)^4\omega_\lambda} \int \mid \mathcal{M}_{\lambda\lambda'} \mid^2 Z_\lambda Z_{\lambda'} \times \tag{16}$$

$$\times \quad f_E \left(1 - f_{E'}\right) \left(1 + f_{\omega'}\right) \delta(\omega_\lambda(\mathbf{k}) + E - \omega_{\lambda'}(\mathbf{k}') - E') \frac{dp_z\, d^3k'}{EE'\omega_{\lambda'}};$$

$$W_{\lambda\to\lambda'\lambda''} = \frac{1 - (1/2)\delta_{\lambda'\lambda''}}{32\pi^2\omega} \int \mid \mathcal{M}_{\lambda\lambda'\lambda''} \mid^2 Z_\lambda Z_{\lambda'} Z_{\lambda''} \times \tag{17}$$

$$\times \quad (1 + f_{\omega'})(1 + f_{\omega''}) \delta(\omega_\lambda(\mathbf{k}) - \omega_{\lambda'}(\mathbf{k} - \mathbf{k}'') - \omega_{\lambda''}(\mathbf{k}'')) \frac{d^3k''}{\omega_{\lambda'}\omega_{\lambda''}},$$

$$W_{\lambda\lambda'\to\lambda''} = \frac{1}{32\pi^2\omega} \int \mid \mathcal{M}_{\lambda\lambda'\lambda''} \mid^2 Z_\lambda Z_{\lambda'} Z_{\lambda''} \times \tag{18}$$

$$\times \quad f_{\omega'}(1 + f_{\omega''}) \delta(\omega_\lambda(\mathbf{k}) + \omega_{\lambda'}(\mathbf{k}') - \omega_{\lambda''}(\mathbf{k} + \mathbf{k}')) \frac{d^3k'}{\omega_{\lambda'}\omega_{\lambda''}}.$$

where f_E is the electron distribution function, $\mathcal{M}_{\lambda\lambda'}$ and $\mathcal{M}_{\lambda\lambda'\lambda''}$ are the partial amplitudes of the photon scattering and photon splitting processes. Using the expressions for Z_λ (9) and taking account for photon dispersion properties in energy conservation law inside δ-function in (16)-(18) one could obtained the self-consistent result for the coefficients in transfer equations (10), (11). To calculate the corresponding amplitudes in the presence of strong magnetic field one should use the Dirac equation solutions at the ground Landau level. For the electron propagator it is relevant to use its asymptotic form [10]. It is possible to present them in the covariant form. For Compton scattering one has [27]:

$$\mathcal{M}_{11} = -\frac{8\pi\alpha m}{eB} \frac{(q\varphi q')(q\tilde{\varphi}q')}{\sqrt{q_\perp^2 q_\perp'^2 (-Q_\parallel^2)}}, \tag{19}$$

$$\mathcal{M}_{12} = -\frac{8\pi\alpha m}{eB} \frac{(q\Lambda q')(q'\tilde{\Lambda}Q)}{\sqrt{q_\perp^2 q_\parallel'^2 (-Q_\parallel^2)}}, \tag{20}$$

$$\mathcal{M}_{21} = \frac{8\pi\alpha m}{eB} \frac{(q\Lambda q')(q\tilde{\Lambda}Q)}{\sqrt{q_\parallel^2 q_\perp'^2 (-Q_\parallel^2)}}, \tag{21}$$

$$\mathcal{M}_{22} = 16i\pi\alpha m \frac{\sqrt{q_\parallel^2 q_\parallel'^2}\, \sqrt{(-Q_\parallel^2)}\, \varkappa}{(q\tilde{\Lambda}q')^2 - \varkappa^2 (q\tilde{\varphi}q')^2}, \tag{22}$$

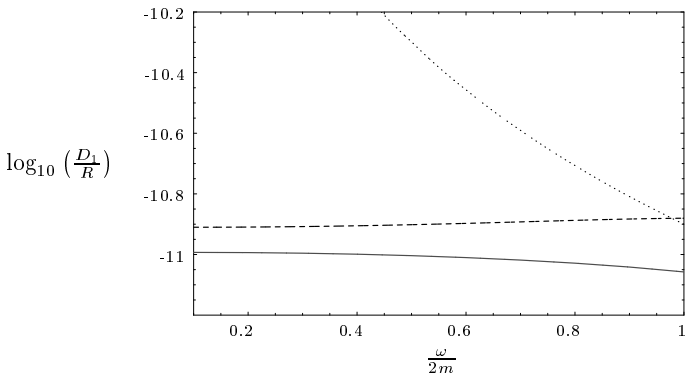

Figure 4: Diffusion coefficient for the 1-mode photon calculated at $B = 100B_e, T = 1MeV$ (solid line) and $B = 60B_e, T = 0.5MeV$ (chain line). Dotted line corresponds to the diffusion coefficient calculated using approximation (26).

where $\varkappa = \sqrt{1 - 4m^2/Q_\parallel^2}$ and $Q_\parallel^2 = (q - q')_\parallel^2 < 0$.

The amplitudes of photon splitting are given by the following equations [28]:

$$\mathcal{M}_{112} = i4\pi \left(\frac{\alpha}{\pi}\right)^{3/2} \frac{(q^{'}\varphi q^{''})(q^{'}\tilde{\varphi} q^{''})}{[q_\perp^{'2} q_\parallel^{''2} q_\perp^2]^{1/2}} \mathcal{G}(q_\parallel^{''}), \tag{23}$$

$$\mathcal{M}_{122} = i4\pi \left(\frac{\alpha}{\pi}\right)^{3/2} \frac{(q'q'')_\parallel}{[q_\parallel^{'2} q_\parallel^{''2} q_\perp^2]^{1/2}} \tag{24}$$

$$\times \left\{(qq'')_\perp \mathcal{G}(q'_\parallel) + (qq')_\perp \mathcal{G}(q''_\parallel)\right\},$$

$$\mathcal{M}_{211} = \mathcal{M}_{112}(q \leftrightarrow q''), \tag{25}$$

where $\mathcal{G}(q) = H(q_\parallel^2/(4m^2)) + \mathcal{J}(q_\parallel)$.

The analysis of the last equations shows that all amplitudes much smaller than $\mathcal{M}_{22}$. Therefore one could expect that mode 2 has the largest scattering absorption rate. It means that the radiation transfer in the magnetically trapped plasma may be described as s diffusion of the 1-mode photons whereas 2-mode photons are locked [6, 29].

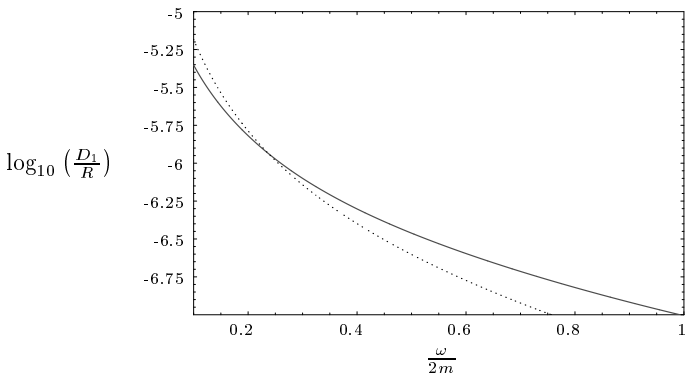

Figure 5: Diffusion coefficient for the 1-mode photon calculated at $B = 10B_e, T = 0.05MeV$ (solid line). Dotted line corresponds ton the diffusion coefficient calculated using approximation (26).

In general case, the calculation of the reactions rates (16)-(18) is rather complicated mathematical problem. However in some limiting cases it is possible to obtain the simple expression for the rates. For example, in low temperature ($T \ll m$) and low energy ($\omega \ll m$) limits the

mean free path (15) for the 1-mode photon in the charge symmetric plasma ($\mu = 0$) can be presented in the following form:

$$\ell_1^{-1} = n_e \sigma_T (B_e \omega / Bm)^2 + (\alpha^3 \sin^6\theta / 2160\pi^2)(\omega/m)^5 m, \tag{26}$$

where $\sigma_T = \frac{8\pi}{3}\frac{\alpha}{m^2}$ is the Thompson cross section and the number of electron (positron) density in a strongly magnetized, charge-symmetric rarefied plasma can be estimated as

$$n_e \simeq eB\sqrt{\frac{mT}{2\pi^3}}\, e^{-m/T}. \tag{27}$$

In formula (26) the fist term corresponds to the Compton scattering process and the second

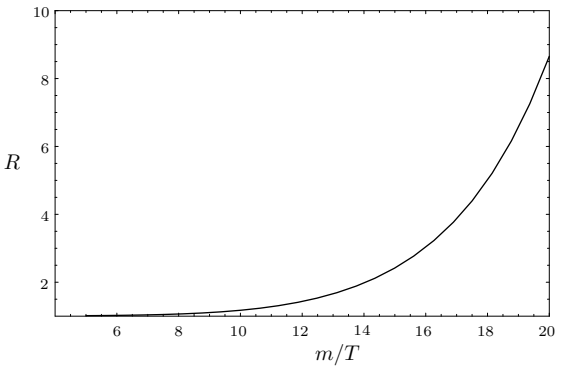

Figure 6: The ratio of the diffusion coefficients without and with taking into account of photon splitting process as a function of the inverse temperature at $B = 200B_e$.

one comes from the photon splitting contribution. It is the estimation for the photon mean free path that usually is used in the radiation transfer analysis in strongly magnetized plasma. Moreover, the process of photon splitting/merging is not taking into account. We would like to show that even in low energy limit this approximation is not appropriate.

4 Discussion

We have made the numerical calculation of the coefficients in (10) and (11) in charge symmetric plasma. Our results are represented in figures 4-7.

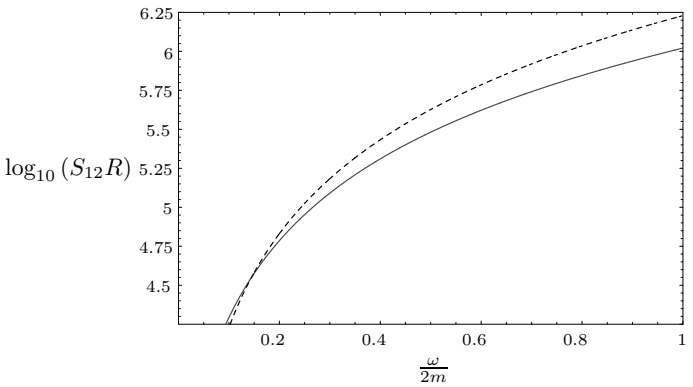

Figure 7: Scattering coefficient for the 1-mode photon calculated at $B = 10B_e, T = 0.05MeV$ (solid line). Dashed line corresponds to the diffusion coefficient calculated using approximation (26).

In figures 4, 5 and 7 one can see that diffusion coefficient calculated with taking account of photon dispersion and large radiative correction strongly deviate from the coefficient obtained by using approximation (26). In addition, in Fig. 6 the ratio of the diffusion coefficient for only Compton scattering to the diffusion coefficient with photon splitting is depicted. One can see that at low temperatures the additional absorption process of photon splitting leads to the significant decreasing of the diffusion coefficient.

We have also analysed the problem of the radiation transfer in the cold degenerate plasma. In this case the main channel of photon splitting is $\gamma_2 \to \gamma_1\gamma_1$. In the figures 8 and 9 the contributions of photon splitting and Compton scattering to photon mean free path are depicted. One can see that in cold plasma the photon splitting contribution is negligibly small in comparison with Compton scattering. It means that the only process of photon scattering on electrons defines the radiation transfer in cold plasma.

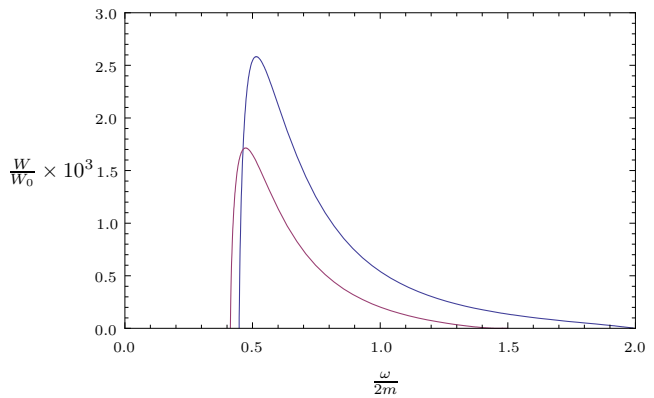

Figure 8: Absorption rate of the process $\gamma_2 \to \gamma_1\gamma_1$ at $B = 200B_e, \mu = 1.5m$ (lower line), $\mu = 2m$ (lower line) , $W_0 = (\alpha/\pi)^3\, m \simeq 3.25 \cdot 10^2 cm^{-1}$.

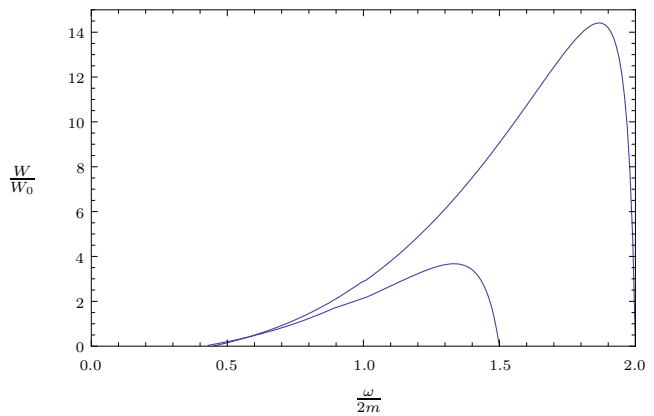

Figure 9: Absorption rate of the process $\gamma_2 e^{\pm} \to \gamma_1 e^{\pm}$ at $B = 200B_e, \mu = 1.5m$ (lower line), $\mu = 2m$ (upper line), $W_0 = (\alpha/\pi)^3\, m \simeq 3.25 \cdot 10^2 cm^{-1}$.

5 Conclusion

We have investigated the influence of strongly magnetized plasma on the radiation transfer with taking into account of photon dispersion and large radiative corrections. We have studied the processes of Compton scattering and photon splitting into two photons and calculated their contribution in transfer equation coefficient. The main conclusions of this work are

- Photon dispersion and radiative correction in strongly magnetized plasma could essentially influence on the radiation transfer process.
- In charge symmetric plasma ($\mu = 0$) it is necessary to take into account the processes of photon splitting and photon merging.
- In strongly degenerate plasma the influence of the photon splitting and photon merging processes on radiation transfer is negligibly small.

Acknowledgments

We express our deep gratitude to the organizers of the Seminar "Quarks-2008" for warm hospitality. This work was supported in part by the Russian Foundation for Basic Research under the Grant No. 07-02-00285-a, and by the Council on Grants by the President of the Russian Federation for the Support of Young Russian Scientists and Leading Scientific Schools of Russian Federation under the Grant No. NSh-497.2008.2 and No. MK-732.2008.2 (MC).

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