On the possibility of natural imitation of black holes

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1 Screening of the black hole temperature.

Let us consider the Shcwarzschild black hole surrounded by a thin spherical dust shell. Parameters of the system are m_{in} - the mass of the black hole, m_{out}- the mass of the whole system, $\Delta m = m_{out} - m_{in}$ - the total mass of the shell.

The temperature measured by a local observer obeys the law

$$T^{loc}\sqrt{g_{00}} = \text{const} \tag{1}$$

For a Schwarzschild space-time $g_{00} = 1 - \frac{2Gm}{r}$, where G is the Newtonian constant, and m is the mass of a gravitating source. Thus, an observer inside the shell knows that $(m = m_{in})$

$$T_{in}(r) = \frac{T_{BH}}{\sqrt{F_{in}}} \tag{2}$$

However, an observer outside the shell obtains $(m = m_{out})$

$$T_{out}(r) = \frac{T_{SYS}}{\sqrt{F_{out}}} \tag{3}$$

here, T_{BH} is the Hawking's temperature of the naked black hole. Evidently, on the shell $(r = r_0)$ one has $T_{in}(r_0) = T_{out}(r_0)$. Thus, from Eqns. (2) and (3) one obtains

$$T_{SYS} = T_{BH}\sqrt{\left(\frac{F_{out}}{F_{in}}\right)_{r=r_0}} \tag{4}$$

Since $m_{in} < m_{out}$, we can see that

$$T_{SYS} < T_{BH}$$

Such an effect is called *"screening of the black hole temperature"*.

2 Model for natural imitation of black hole

2.1 Imitation of the black holes. General definition

Since if an observer at spatial infinity is able to measure the temperature and mass, then, in general, he is able to distinct between the naked black hole and the system of the black hole and thin shell. Calculating. In this section we are interested in the situation when

$$T_{SYS} = \frac{1}{8\pi G(m + \Delta m)} \tag{5}$$

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i.e. observer armed with the knowledge of General Relativity and thermodynamics in curved space-time will see the temperature of the black hole with mass $m_{SYS} = m + \Delta m$. But in reality the system under consideration is not a black hole. We call this phenomenon *" imitation of the black hole"*.

2.2 Thin shell with orbiting constituents

It is known [1] that such an imitation is possible if we place the shell at certain radius around the black hole. This is however quite artifical model, since one has to hold the shell fixed by some extrinsic forces.

Therefore, it would be interesting to investigate a model where the shell is held at some fixed radius naturally, e.g. being at the bottom of a potential well. Such a situation could be realized, for instance, in globular clusters.

To construct such a model one can use a shell with orbiting constituents [2].

What does the expression "orbiting constituents mean"? Let us consider a point mass moving according to Newtonian gravity law (Kepler's problem in Celestial mechanics) in a field of gravitating center. Its orbit is an ellipse with such a center at one of the focuses. If we build an ensemble of such particles with the same angular momentum-to-mass ratio, they will have the same value of both pericentre and apocentre. Let us imagine that, initially, all these particles are smeared uniformly on a surface of a sphere whose radius is that of pericentre and let them start simultaneously (with equal absolute value of the velocity but in different directions, i.e. in different planes). Then such an ensemble will form a spherically symmetric thin shell oscillating between a pericentre and an apocentre. This is exactly what we call a thin shell with orbiting constituents. Such a construction can be applied equally to both non-relativistic and relativistic Coulomb problems. Of course,in relativistic case orbits are no longer closed ellipses, but it is all the same qualitatively.

For the shell with orbiting constituents one has the following surface stress-energy tensor [2]

$$S_0^0 = \frac{M}{4\pi\rho^2}\sqrt{1+\frac{a^2}{\rho^2}}. \tag{6}$$

$$S_2^2 = S_3^3 = -\frac{M\,a^2}{8\pi\rho^4\sqrt{1+\frac{a^2}{\rho^2}}}, \tag{7}$$

where M-bare mass of the shell, ρ-radius of the shell, $a = \frac{J}{\mu}$ and J, μ is the conserved angular momentum and the mass of each particle, correspondingly. Equations describing the shell motion are the following

$$\frac{\ddot{\rho}+\frac{1}{2}F'_{in}}{\sqrt{\dot{\rho}^2+F_{in}}} - \frac{\ddot{\rho}+\frac{1}{2}F'_{out}}{\sqrt{\dot{\rho}^2+F_{out}}} = 4\pi G(2S_2^2 - S_0^0), \tag{8}$$

$$\sqrt{\dot{\rho}^2+F_{\rm in}} - \sqrt{\dot{\rho}^2+F_{\rm out}} = 4\pi G\rho S_0^0, \tag{9}$$

$$F' = \frac{2Gm}{\rho^2}. \tag{10}$$

From Eqn. (9) one can get expression for the total mass of the shell

$$\Delta m \equiv m_{out} - m_{in} = E\sqrt{\dot{\rho}^2 + 1 - \frac{2Gm}{\rho}} - \frac{GE^2}{2\rho} \tag{11}$$

where $E = 4\pi\rho^2 S_0^0$.

The interpretation of this equation is rather simple: the left hand side is the total energy (mass) of the shell, the first term in the right hand side is the kinetic energy together with the gravitational potential energy, while the second term is is the (negative) gravitational self-interaction energy of the shell. The absence of the potential rotation energy can be explained by zero value of the total angular momentum. The angular momenta of individual particles enter only through the bare energy

$$E = M\sqrt{1+\frac{a^2}{\rho^2}}$$

and this is natural in the spirit of General Relativity: "all kind of energy is gravitating", but the corresponding potential term reappeared in the non-relativistic limit. In the relativistic theory we can introduce an effective potential energy:

$$\begin{aligned} V_{eff} &= \Delta m(\dot\rho = 0) = E\sqrt{1-\frac{2Gm_{in}}{\rho}} - \frac{GE^2}{2\rho} \\ &= M\sqrt{1+\frac{a^2}{\rho^2}}\sqrt{1-\frac{2Gm_{in}}{\rho}} - \frac{GM^2}{2\rho}\left(1+\frac{a^2}{\rho^2}\right) \end{aligned} \tag{12}$$

If the angular momentum exceeds some critical value J_{cr} ($a_{cr} = \frac{J_{cr}}{\mu}$), then the effective potential (12) has the minimum. The critical value equals to (see [2])

$$a_{cr} = 2\sqrt{3}Gm_{in}$$

For the shell "sitting" at the bottom of the potential well (constituent particles moves along circular orbits) $\dot\rho = \ddot\rho = 0$. And the question arises: is it possible in such a case to imitate a black hole adjusting properly the intrinsic parameters of the shell, namely, M and a?

2.3 Natural imitation of black hole

To answer this question we need to solve Israel equations with $\dot\rho = \ddot\rho = 0$ together with the imitation equation (4) and the inequality $a > a_{cr}$. The system of our equations reads now as follows:

$$\frac{m_{in}}{\sqrt{F_{in}}} - \frac{m_{out}}{\sqrt{F_{out}}} = 4\pi\rho^2(2S_2^2 - S_0^0), \tag{13}$$

$$\sqrt{F_{in}} - \sqrt{F_{out}} = 4\pi G\rho S_0^0, \tag{14}$$

$$\frac{m_{in}}{m_{out}} = \frac{\sqrt{F_{out}}}{\sqrt{F_{in}}}. \tag{15}$$

Introducing new variables $\Pi = -4\pi\rho^2 S_2^2$, $y = \frac{m_{out}}{m_{in}} > 1$, $z = \frac{\rho}{Gm_{in}}$ and solving the third equation for z we obtain

$$\frac{m_{in}}{\sqrt{F_{in}}}(1-y^2) = -(2\Pi + E), \tag{16}$$

$$m_{in}\sqrt{F_{in}}\left(1-\frac{1}{y}\right) = \frac{E}{z}, \tag{17}$$

$$\frac{y+1}{y^2+y+1} = \frac{2}{z}. \tag{18}$$

After excluding z the remaining equations become

$$m_{in}\frac{\sqrt{y^2+y+1}(y^2-1)}{y} = 2\Pi + E, \tag{19}$$

$$2m_{in}\sqrt{y^2+y+1}\left(\frac{y-1}{y+1}\right) = E, \tag{20}$$

from which it follows that

$$\frac{2\Pi + E}{E} = \frac{(y+1)^2}{2y} > 2. \tag{21}$$

Therefore

$$E < 2\Pi. \tag{22}$$

It easy to check that for the shells with orbiting constituents

$$E > 2\Pi. \tag{23}$$

Hence in this particular case the natural imitation of black holes is impossible.

Nevertheless, we obtained the necessary condition for the natural black hole imitation: the equation of state of a two-dimensional perfect fluid of which the shell is made must obey the inequality

$$S_0^0 + 2S_2^2 < 0 \tag{24}$$

This means that the equation of state should be stiffer than that for a two-dimensional photon gas up to the stiffest one ($S_0^0 + S_2^2 = 0$) for which $y = y_{max} = 2 + \sqrt{3}$.

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References

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