Density matrix of the Universe reloaded: origin of inflation and cosmological acceleration

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Abstract

We present an overview of a recently suggested new model of quantum initial conditions for the Universe in the form of a cosmological density matrix. This density matrix originally suggested in the Euclidean quantum gravity framework turns out to describe the microcanonical ensemble in the Lorentzian quantum gravity of spatially closed cosmological models. This ensemble represents an equipartition in the physical phase space of the theory (sum over everything), but in terms of the observable spacetime geometry it is peaked about a set of cosmologies limited to a bounded range of the cosmological constant. This suggests a mechanism to constrain the landscape of string vacua and a possible solution to the dark energy problem in the form of the quasi-equilibrium decay of the microcanonical state of the Universe. The effective Friedmann equation governing this decay incorporates the effect of the conformal anomaly of quantum fields and features a new mechanism for a cosmological acceleration stage – big boost scenario. We also briefly discuss the relation between our model, the AdS/CFT correspondence and RS and DGP braneworlds.

1 Introduction

It is widely recognized that Euclidean quantum gravity (EQG) is a lame duck in modern particle physics and cosmology. After its summit in early and late eighties (in the form of the cosmological wavefunction proposals [1, 2] and baby universes boom [3]) the interest in this theory gradually declined, especially, in cosmological context, where the problem of quantum initial conditions was superseded by the concept of stochastic inflation [4]. EQG could not stand the burden of indefiniteness of the Euclidean gravitational action [5] and the cosmology debate of the tunneling vs no-boundary proposals [6].

Thus, a recently suggested EQG density matrix of the Universe [7] is hardly believed to be a viable candidate for the initial state of the Universe, even though it avoids the infrared catastrophe of small cosmological constant Λ, generates an ensemble of quasi-thermal universes in the limited range of Λ, and suggests a strong selection mechanism for the landscape of string vacua [7, 8]. Here we want to give a brief overview of these results and also justify them by deriving from first principles of Lorentzian quantum gravity applied to a microcanonical

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ensemble of closed cosmological models [9]. In view of the peculiarities of spatially closed cosmology this ensemble describes ultimate (unit weight) equipartition in the physical phase space of the theory. This can be interpreted as a sum over Everything, thus emphasizing a distinguished role of this candidate for the initial state of the Universe.

We analyze the cosmological evolution in this model with the initial conditions set by the instantons of [7, 8]. In particular, we derive the modified Friedmann equation incorporating the effect of the conformal anomaly at late radiation and matter domination stages [10]. This equation shows that the vacuum (Casimir) part of the energy density is "degravitated" via the effect of the conformal anomaly – the Casimir energy does not weigh. Moreover, together with the recovery of the general relativistic behavior, this equation can feature a stage of cosmological acceleration followed by what we call a *big boost* singularity [10]. At this singularity the scale factor acceleration grows in finite cosmic time up to infinity with a finite limiting value of the Hubble factor, when the Universe again enters a quantum phase demanding for its description an UV completion of the low-energy semiclassical theory. Then we discuss the hierarchy problem in this scenario which necessarily arises when trying to embrace within one model both the inflationary and acceleration (dark energy) stages of the cosmological evolution. The attempt to solve this problem via the (string-inspired) concept of evolving extra dimensions brings us to the AdS/CFT and braneworld setups [11, 12, 13, 14], including the Randall-Sundrum and DGP models tightly linked by duality relations to our anomaly driven cosmology.

2 Euclidean quantum gravity density matrix

A density matrix $\rho(q, q')$ in Euclidean quantum gravity [15] is related to a spacetime having two disjoint boundaries Σ and Σ' associated with its two entries q and q' (collecting both gravity and matter observables), see Fig.1. The metric and matter configuration on this spacetime $[g, \phi]$ interpolates between q and q', thus establishing mixing correlations.

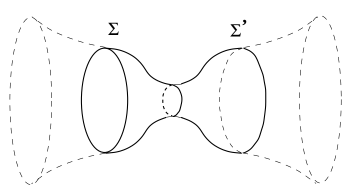

Figure 1: The picture of Euclidean spacetime underlying the EQG density matrix, whose two arguments are associated with the surfaces Σ and Σ'. Dashed lines depict the Lorentzian signature spacetime nucleating at Σ and Σ'.

This obviously differs from the pure Hartle-Hawking state $|\Psi_{HH}\rangle$ which can also be formulated in terms of a special density matrix $\hat\rho_{HH}$. For the latter the spacetime bridge between Σ and Σ' is broken, so that the spacetime is a union of two disjoint hemispheres which smoothly close up at their poles (Fig.2) — a picture illustrating the factorization of $\hat\rho_{HH} = |\Psi_{HH}\rangle\langle\Psi_{HH}|$.

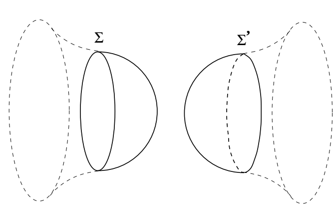

Figure 2: Density matrix of the pure Hartle-Hawking state represented by the union of two instantons of vacuum nature.

Analogously to the prescription for the Hartle-Hawking state [1], the EQG density matrix can be defined by the path integral [7, 8] over gravitational g and matter ϕ fields on the spacetime of the above type interpolating between the observables q and q' respectively at Σ and Σ',

$$\rho(\,q,q'\,) = e^{\Gamma} \int D[\,g,\phi\,]\;\exp\big(-S_E[\,g,\phi\,]\big), \tag{1}$$

where $S_E[\,g,\phi\,]$ is the classical Euclidean action of the system. In view of the density matrix normalization $\mathrm{tr}\hat\rho = 1$ the corresponding statistical sum $\exp(-\Gamma)$ is given by a similar path integral,

$$e^{-\Gamma} = \int\limits_{\rm periodic} D[\,g,\phi\,]\;\exp\big(-S_E[\,g,\phi\,]\big), \tag{2}$$

over periodic fields on the toroidal spacetime with identified boundaries Σ and Σ'.

For a closed cosmology with the S^3-topology of spatial sections this statistical sum can be represented by the path integral over the periodic scale factor $a(\tau)$ and lapse function $N(\tau)$ of the minisuperspace metric

$$ds^2 = N^2(\tau)\,d\tau^2 + a^2(\tau)\,d^2\Omega^{(3)} \tag{3}$$

on the toroidal $S^1\times S^3$ spacetime [7, 8]

$$e^{-\Gamma} = \int\limits_{\rm periodic} D[\,a,N\,]\;e^{-\Gamma_E[\,a,\,N\,]}, \tag{4}$$

$$e^{-\Gamma_E[\,a,\,N]} = \int\limits_{\rm periodic} D\Phi(x)\,e^{-S_E[\,a,\,N;\,\Phi(x)\,]}. \tag{5}$$

Here $\Gamma_E[\,a,\,N]$ is the Euclidean effective action of all inhomogeneous "matter" fields which include also metric perturbations on minisuperspace background $\Phi(x) = (\phi(x),\psi(x),A_\mu(x), h_{\mu\nu}(x),...)$, $S_E[a,N;\Phi(x)] \equiv S_E[\,g,\phi\,]$ is the original classical action of the theory under the decomposition of the full configuration space into the minisuperspace and perturbations sectors,

$$[\,g,\phi\,] = [\,a(\tau),N(\tau);\;\Phi(x)\,], \tag{6}$$

and the integration also runs over periodic fields $\Phi(x)$.

Under the assumption that the system is dominated by free matter fields conformally coupled to gravity this action is exactly calculable by the conformal transformation taking the metric (3) into the static Einstein metric with $a = \mathrm{const}$ [7]. In units of the Planck mass $m_P = (3\pi/4G)^{1/2}$ the action reads

$$\Gamma_E[\,a,N\,] = m_P^2 \int d\tau\,N\left\{-aa'^2 - a + \frac{\Lambda}{3}a^3 + B\left(\frac{a'^2}{a} - \frac{a'^4}{6a}\right) + \frac{B}{2a}\right\} + F(\eta), \tag{7}$$

where

$$F(\eta) = \pm\sum_\omega \ln\big(1\mp e^{-\omega\eta}\big), \quad \eta = \int d\tau N/a, \tag{8}$$

and $a' \equiv da/Nd\tau$. The first three terms in curly brackets represent the classical Einstein action with a primordial cosmological constant Λ, the B-terms correspond to the contribution of the conformal anomaly and the contribution of the vacuum (Casimir) energy $(B/2a)$ of conformal fields on a static Einstein spacetime. $F(\eta)$ is the free energy of these fields — a typical boson or fermion sum over field oscillators with energies ω on a unit 3-sphere, η playing the role of the inverse temperature — an overall circumference of the toroidal instanton measured in units of the conformal time. The constant B,

$$B = \frac{3\beta}{4m_P^2} = \frac{\beta G}{\pi}, \tag{9}$$

is determined by the coefficient β of the topological Gauss-Bonnet invariant $E = R^2_{\mu\nu\alpha\gamma} - 4R^2_{\mu\nu} + R^2$ in the overall conformal anomaly of quantum fields

$$g_{\mu\nu}\frac{\delta\varGamma_E}{\delta g_{\mu\nu}} = \frac{1}{4(4\pi)^2} g^{1/2}\left(\alpha\Box R + \beta E + \gamma C^2_{\mu\nu\alpha\beta}\right) \tag{10}$$

($C^2_{\mu\nu\alpha\beta}$ is the Weyl tensor squared term). For a model with N_0 scalars, $N_{1/2}$ Weyl spinors and N_1 gauge vector fields it reads [17]

$$\beta = \frac{1}{360}\left(2N_0 + 11N_{1/2} + 124N_1\right). \tag{11}$$

The coefficient γ does not contribute to (7) because the Weyl tensor vanishes for any FRW metric. What concerns the coefficient α is more complicated. A nonvanishing α induces higher derivative terms $\sim \alpha(a'')^2$ in the action and, therefore, adds one extra degree of freedom to the minisuperspace sector of a and N and results in instabilities[1]. But α can be renormalized to zero by adding a finite *local* counterterm $\sim R^2$ admissible by the renormalization theory. We assume this *number of degrees of freedom preserving* renormalization to keep theory consistent both at the classical and quantum levels [7]. It is interesting that this finite renormalization changes the value of the Casimir energy of conformal fields in closed Einstein cosmology in such a way that for all spins this energy is universally expressed in terms of the same conformal anomaly coefficient B (corresponding to the $B/2a$ term in (7)) [7]. As we will see, this leads to the gravitational screening of the Casimir energy, mediated by the conformal anomaly of quantum fields.

Ultimately, the effective action (7) contains only two dimensional constants – the Planck mass squared (or the gravitational constant) $m_P^2 = 3\pi/4G$ and the cosmological constant Λ. They have to be considered as renormalized quantities. Indeed, the effective action of conformal fields contains divergences, the quartic and quadratic ones being absorbed by the renormalization of the initially singular bare cosmological and gravitational constants to yield finite renormalized m_P^2 and Λ [18]. Logarithmically divergent counterterms have the same structure as curvature invariants in the anomaly (10). When integrated over the spacetime closed toroidal FRW instantons they identically vanish because the $\Box R$ term is a total derivative, the Euler number E of $S^3 \times S^1$ is zero, $\int d^4x g^{1/2} E = 0$, and $C_{\mu\nu\alpha\beta} = 0$. There is however a finite tail of these vanishing logarithmic divergences in the form of the conformal anomaly action which incorporates the coefficient β of E in (10) and constitutes a major contribution to $\varGamma_E$ — the first two B-dependent terms of (8)[2]. Thus, in fact, this model when considered in the leading order of the $1/N$-expansion (therefore disregarding loop effects of the graviton and other non-conformal fields) is renormalizable in the minisuperspace sector of the theory.

The path integral (4) is dominated by the saddle points — solutions of the equation $\delta\varGamma_E/\delta N(\tau) = 0$ which reads as

$$-\frac{a'^2}{a^2} + \frac{1}{a^2} - B\left(\frac{1}{2}\frac{a'^4}{a^4} - \frac{a'^2}{a^4}\right) = \frac{\Lambda}{3} + \frac{C}{a^4}, \tag{12}$$

with C given by

$$C = \frac{B}{2} + \frac{dF(\eta)}{d\eta}, \quad \eta = 2k\int_{\tau_-}^{\tau_+}\frac{d\tau}{a}. \tag{13}$$

[1]In Einstein theory this sector does not contain physical degrees of freedom at all, which solves the problem of the formal ghost nature of a in the Einstein Lagrangian. Addition of higher derivative term for a does not formally lead to a ghost – the additional degree of freedom has a good sign of the kinetic term as it happens in $f(R)$-theories, but still leads to the instabilities discovered in [16].

[2]These terms can be derived from the metric-dependent Riegert action [19] or the action in terms of the conformal factor relating two metrics [20, 21, 22] and generalize the action of [23] to the case of a spatially closed cosmology with $\alpha = 0$.

Note that the usual (Euclidean) Friedmann equation is modified by the anomalous B-term and the radiation term C/a^4. The constant C sets the amount of radiation and satisfies the bootstrap equation (13), where $B/2$ is the contribution of the Casimir energy, and

$$\frac{dF(\eta)}{d\eta} = \sum_{\omega} \frac{\omega}{e^{\omega\eta} \mp 1} \tag{14}$$

is the energy of the gas of thermally excited particles with the inverse temperature η. The latter is given in (13) by the k-fold integral between the turning points of the scale factor history $a(\tau)$, $\dot{a}(\tau_\pm) = 0$. This k-fold nature implies that in the periodic solution the scale factor oscillates k times between its maximum and minimum values $a_\pm = a(\tau_\pm)$, see Fig.3

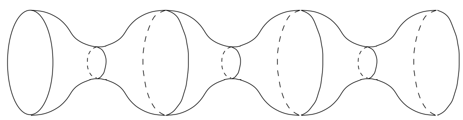

Figure 3: The garland segment consisting of three folds of a simple instanton glued at surfaces of a maximal scale factor.

As shown in [7], such solutions represent garland-type instantons which exist only in the limited range of values of the cosmological constant

$$0 < \Lambda_{\rm min} < \Lambda < \frac{3\pi}{2\beta G}, \tag{15}$$

and eliminate the vacuum Hartle-Hawking instantons corresponding to $a_- = 0$. Hartle-Hawking instantons are ruled out in the statistical sum by their infinite positive effective action which is due to the contribution of the conformal anomaly. Hence the tree-level predictions of the theory are drastically changed.

The upper bound of the range (15) is entirely caused by the quantum anomaly – it represents a new quantum gravity scale which tends to infinity when one switches the quantum effects off, $\beta \to 0$. The lower bound $\Lambda_{\rm min}$ can be attributed to both radiation and anomaly, and can be obtained numerically for any field content of the model. For a large number of conformal fields, and therefore a large β, the lower bound is of the order $\Lambda_{\rm min} \sim 1/\beta G$. Thus the restriction (15) can be regarded as a solution of the cosmological constant problem in the early Universe, because specifying a sufficiently high number of conformal fields one can achieve a primordial value of Λ well below the Planck scale where the effective theory applies, but high enough to generate a sufficiently long inflationary stage. Also this restriction can be potentially considered as a selection criterion for the landscape of string vacua [7, 9].

3 Lorentzian quantum gravity density matrix: sum over Everything

The period of the quasi-thermal instantons is not a freely specifiable parameter and can be obtained as a function of G and Λ from Eqs. (12)-(13). Therefore this model clearly does not describe a canonical ensemble, but rather a microcanonical ensemble [9] with only two freely specifiable dimensional parameters — the renormalized gravitational and renormalized cosmological constants as discussed above.

To show this, contrary to the EQG construction of the above type, consider the density matrix as the canonical path integral in *Lorentzian* quantum gravity. Its kernel in the representation of 3-metrics and matter fields denoted below as q reads

$$\rho(q_+, q_-) = e^{\Gamma} \int\limits_{q(t_\pm) = q_\pm} D[\,q,p,N\,]\, e^{i\int_{t_-}^{t_+} dt\,(p\,\dot{q} - N^\mu H_\mu)}, \tag{16}$$

where the integration runs over histories of phase-space variables $(q(t), p(t))$ interpolating between $q_\pm$ at $t_\pm$ and the Lagrange multipliers of the gravitational constraints $H_\mu = H_\mu(q,p)$ — lapse and shift functions $N(t) = N^\mu(t)$. The measure $D[\,q,p,N\,]$ includes the gauge-fixing factor of the delta function $\delta\,[\,\chi\,] = \prod_t \prod_\mu \delta(\chi^\mu)$ of gauge conditions χ^μ and the relevant ghost factor [24, 25] (condensed index μ includes also continuous spatial labels). It is important that the integration range of N^μ,

$$-\infty < N < +\infty, \tag{17}$$

generates in the integrand the delta-functions of the constraints $\delta(H) = \prod_\mu \delta(H_\mu)$. As a consequence the kernel (16) satisfies the set of Wheeler-DeWitt equations

$$\hat{H}_\mu\big(q, \partial/i\partial q\big)\, \rho(\,q, q'\,) = 0, \tag{18}$$

and the density matrix (16) can be regarded as an operator delta-function of these constraints

$$\hat{\rho} \sim \text{“}\prod_\mu \delta(\hat{H}_\mu)\text{”}. \tag{19}$$

This expression should not be understood literally because the multiple delta-function here is not uniquely defined, for the operators $\hat{H}_\mu$ do not commute and form an open algebra. Moreover, exact operator realization $\hat{H}_\mu$ is not known except the first two orders of a semiclassical $\hbar$-expansion [26]. Fortunately, we do not need a precise form of these constraints, because we will proceed with their path-integral solutions adjusted to the semiclassical perturbation theory.

The very essence of our proposal is the interpretation of (16) and (19) as the density matrix of a *microcanonical* ensemble in spatially closed quantum cosmology. A simplest analogy is the density matrix of an unconstrained system having a conserved Hamiltonian $\hat{H}$ in the microcanonical state with a fixed energy E, $\hat{\rho} \sim \delta(\hat{H} - E)$. A major distinction of (19) from this case is that spatially closed cosmology does not have freely specifiable constants of motion like the energy or other global charges. Rather it has as constants of motion the Hamiltonian and momentum constraints H_μ, all having a particular value — zero. Therefore, the expression (19) can be considered as a most general and natural candidate for the quantum state of the *closed* Universe. Below we confirm this fact by showing that in the physical sector the corresponding statistical sum is a uniformly distributed (with a unit weight) integral over entire phase space of true physical degrees of freedom. Thus, this is the sum over Everything. However, in terms of the observable quantities, like spacetime geometry, this distribution turns out to be nontrivially peaked around a particular set of universes. Semiclassically this distribution is given by the EQG density matrix and the saddle-point instantons of the above type [7].

From the normalization of the density matrix in the physical Hilbert space we have

$$1 = \mathrm{Tr}_{\rm phys}\,\hat{\rho} = \int dq\, \mu\big(q, \partial/i\partial q\big)\, \rho(q, q')\Big|_{q'=q} = e^{\varGamma} \int\limits_{\rm periodic} D[\,q,p,N\,]\, e^{\,i\int dt (p\,\dot{q} - N^\mu H_\mu)}. \tag{20}$$

Here in view of the coincidence limit $q' = q$ the integration runs over periodic histories $q(t)$, and $\mu\big(q, \partial/i\partial q\big) = \hat{\mu}$ is the measure which distinguishes the physical inner product in the space of solutions of the Wheeler-DeWitt equations $(\psi_1|\psi_2) = \langle\psi_1|\hat{\mu}|\psi_2\rangle$ from that of the space of square-integrable functions, $\langle\psi_1|\psi_2\rangle = \int dq\, \psi_1^*\psi_2$. This measure includes the delta-function of unitary gauge conditions $\chi^\mu = \chi^\mu(q,p)$ and an operator factor incorporating the relevant ghost determinant [26].

On the other hand, in terms of the physical phase space variables the Faddeev-Popov path integral equals [24, 25]

$$\int\limits_{\rm periodic} D[\,q,p,N\,]\, e^{\,i\int dt\,(p\,\dot{q} - N^\mu H_\mu)} = \int\limits_{\rm periodic} Dq_{\rm phys}\, Dp_{\rm phys}\, e^{\,i\int dt\,\big(p_{\rm phys}\,\dot{q}_{\rm phys} - H_{\rm phys}(t)\big)}$$
$$= \mathrm{Tr}_{\rm phys}\Big(\mathbf{T}\, e^{-i\int dt\, \hat{H}_{\rm phys}(t)}\Big), \tag{21}$$

where $\mathbf{T}$ denotes the chronological ordering. The physical Hamiltonian and its operator realization $\hat{H}_{\rm phys}(t)$ are nonvanishing here only in unitary gauges explicitly depending on time [26], $\chi^\mu(q,p,t)$. In static gauges, $\partial_t\chi^\mu=0$, they vanish, because the full Hamiltonian in closed cosmology is a combination of constraints.

The path integral (21) is gauge-independent on-shell [24, 25] and coincides with that in the static gauge. Therefore, from Eqs.(20)-(21) with $\hat{H}_{\rm phys}=0$, the statistical sum of our microcanonical ensemble equals

$$e^{-\mathit{\Gamma}}={\rm Tr}_{\rm phys}\,\mathbf{I}_{\rm phys}=\int dq_{\rm phys}\,dp_{\rm phys}=\text{sum over Everything}. \tag{22}$$

Here $\mathbf{I}_{\rm phys}=\delta(q_{\rm phys}-q'_{\rm phys})$ is a unit operator in the physical Hilbert space, whose kernel when represented as a Fourier integral yields extra momentum integration (2π-factor included into $dp_{\rm phys}$). This sum over Everything (as a counterpart to the concept of creation from "anything" in [27]), not weighted by any nontrivial density of states, is a result of general covariance and closed nature of the Universe lacking any freely specifiable constants of motion. The Liouville integral over entire *physical* phase space, whose structure and topology is not known, is very nontrivial. However, below we show that semiclassically it is determined by EQG methods and supported by instantons of [7] spanning a bounded range of the cosmological constant.

Integration over momenta in (20) yields a Lagrangian path integral with a relevant measure and action

$$e^{-\mathit{\Gamma}}=\int D[\,q,N\,]\;e^{iS_L[\,q,\,N\,]}. \tag{23}$$

As in (20) integration runs over periodic fields (not indicated explicitly but assumed everywhere below) even for the case of an underlying spacetime with Lorentzian signature. Similarly to the decomposition (6) of [7, 8] leading to (4)-(5), we decompose $[\,q,N\,]$ into a minisuperspace $[\,a_L(t),N_L(t)\,]$ and the "matter" $\mathit{\Phi}_L(x)$ variables, the subscript L indicating their Lorentzian nature. With a relevant decomposition of the measure $D[\,q,N\,]=D[\,a_L,N_L\,]\times D\mathit{\Phi}_L(x)$, the microcanonical sum reads

$$e^{-\mathit{\Gamma}}=\int D[\,a_L,N_L\,]\;e^{i\mathit{\Gamma}_L[\,a_L,\,N_L\,]}, \tag{24}$$

$$e^{i\mathit{\Gamma}_L[\,a_L,\,N_L\,]}=\int D\mathit{\Phi}_L(x)\;e^{iS_L[\,a_L,\,N_L;\,\mathit{\Phi}_L(x)]}, \tag{25}$$

where $\mathit{\Gamma}_L[\,a_L,\,N_L\,]$ is a Lorentzian effective action. The stationary point of (24) is a solution of the effective equation $\delta\mathit{\Gamma}_L/\delta N_L(t)=0$. In the gauge $N_L=1$ it reads as a Lorentzian version of the Euclidean equation (12) and the bootstrap equation for the radiation constant C with the Wick rotated $\tau=it$, $a(\tau)=a_L(t)$, $\eta=i\int dt/a_L(t)$. However, with these identifications C turns out to be purely imaginary (in view of the complex nature of the free energy $F(i\int dt/a_L)$). Therefore, no periodic solutions exist in spacetime with a *real* Lorentzian metric.

On the contrary, such solutions exist in the Euclidean spacetime. Alternatively, the latter can be obtained with the time variable unchanged $t=\tau$, $a_L(t)=a(\tau)$, but with the Wick rotated lapse function

$$N_L=-iN,\quad iS_L[\,a_L,N_L;\phi_L]=-S_E[\,a,N;\mathit{\Phi}\,]. \tag{26}$$

In the gauge $N=1$ ($N_L=-i$) these solutions exactly coincide with the instantons of [7]. The corresponding saddle points of (24) can be attained by deforming the integration contour (17) of N_L into the complex plane to pass through the point $N_L=-i$ and relabeling the real Lorentzian t with the Euclidean τ. In terms of the Euclidean $N(\tau)$, $a(\tau)$ and $\mathit{\Phi}(x)$ the integrals (24) and (25) take the form of the path integrals (4) and (5) in EQG,

$$i\mathit{\Gamma}_L[\,a_L,\,N_L]=-\mathit{\Gamma}_E[\,a,\,N\,]. \tag{27}$$

However, the integration contour for the Euclidean $N(\tau)$ runs from $-i\infty$ to $+i\infty$ through the saddle point $N = 1$. This is the source of the conformal rotation in Euclidean quantum gravity, which is called to resolve the problem of unboundedness of the gravitational action and effectively renders the instantons a thermal nature, even though they originate from the microcanonical ensemble. This mechanism implements the justification of EQG from the canonical quantization of gravity [28] (see also [29] for the black hole context).

4 Cosmological evolution from the initial microcanonical state: origin of inflation and standard GR scenario

The gravitational instantons of Sect.2 can be regarded as setting initial conditions for the cosmological evolution in the physical spacetime with the Lorentzian signature. Indeed, those initial conditions can be viewed as those at the nucleation of the Lorentzian spacetime from the Euclidean spacetime at the maximum value of the scale factor $a_+ = a(\tau_+)$ at the turning point of the Euclidean solution τ_+ — the minimal (zero extrinsic curvature) surface of the instanton. For the contribution of the one-fold instanton to the density matrix of the Universe this nucleation process is depicted in Fig. 1.

The Lorentzian evolution can be obtained by analytically continuing the Euclidean time into the complex plane by the rule $\tau = \tau_+ + it$. Correspondingly the Lorentzian effective equation follows from the Euclidean one (12) as

$$\frac{\dot a^2}{a^2} + \frac{1}{a^2} - \frac{B}{2}\left(\frac{\dot a^2}{a^2} + \frac{1}{a^2}\right)^2 = \frac{\Lambda}{3} + \frac{C - B/2}{a^4}, \tag{28}$$

where the dot, from now on, denotes the derivative with respect to the Lorentzian time t. This can be solved for the Hubble factor as

$$\frac{\dot a^2}{a^2} + \frac{1}{a^2} = \frac{1}{B}\left\{1 - \sqrt{1 - 2B\left(\frac{\Lambda}{3} + \frac{\mathcal{C}}{a^4}\right)}\right\}, \tag{29}$$

$$\mathcal{C} \equiv C - \frac{B}{2}. \tag{30}$$

We have thus obtained a modified Friedmann equation in which the overall energy density, including both the cosmological constant and radiation, very nonlinearly contributes to the square of the Hubble factor [10].

An interesting property of this equation is that the Casimir energy does not weigh. Indeed the term $B/2a^4$ is completely subtracted from the full radiation density C/a^4 in the right hand side of (28) and under the square root of (29). Only "real" thermally excited quanta contribute to the right-hand side of (29). Indeed, using (13), the radiation contribution $\mathcal{C}/a^4$ is seen to read simply as

$$\frac{\mathcal{C}}{a^4} = \frac{1}{a^4}\sum_\omega \frac{\omega}{e^{\omega\eta} \mp 1}. \tag{31}$$

This is an example of the gravitational screening which is now being intensively searched for the cosmological constant [30, 31]. As we see, in our case, this mechanism is mediated by the conformal anomaly action, but it applies not to the cosmological constant, but rather to the Casimir energy which has the equation of state of radiation $p = \varepsilon/3$. This gravitational screening is essentially based on the above mentioned renormalization that eradicates higher derivatives from the effective action and thus preserves the minisuperspace sector free from dynamical degrees of freedom.

After nucleation from the Euclidean instanton at the turning point with $a = a_+$ and $\dot a_+ = 0$ the Lorentzian Universe starts expanding, because $\ddot a_+ > 0$. Therefore, the radiation quickly dilutes, so that the primordial cosmological constant starts dominating and can generate an inflationary stage. It is natural to assume that the primordial Λ is not fundamental, but is due to some inflaton field. This effective Λ is nearly constant during the Euclidean stage and the inflation stage, and subsequently leads to a conventional exit from inflation by the slow roll mechanism[3].

During a sufficiently long inflationary stage, particle production of conformally non-invariant matter takes over the polarization effects of conformal fields. After being thermalized at the exit from inflation this matter gives rise to an energy density $\varepsilon(a)$ which should replace the energy density of the primordial cosmological constant and radiation. Therefore, at the end of inflation the combination $\Lambda/3 + \mathcal{C}/a^4$ should be replaced according to

$$\frac{\Lambda}{3} + \frac{\mathcal{C}}{a^4} \to \frac{8\pi G}{3}\,\varepsilon(a) \equiv \frac{8\pi G}{3}\,\rho(a) + \frac{\mathcal{C}}{a^4}. \tag{32}$$

Here $\varepsilon(a)$ denotes the full energy density including the component $\rho(a)$ resulting from the decay of Λ and the radiation density of the primordial conformal matter $\mathcal{C}/a^4$. The dependence of $\varepsilon(a)$ on a is of course determined by the equation of state via the stress tensor conservation, and $\rho(a)$ also includes its own radiation component emitted by and staying in (quasi)equilibrium with the baryonic part of the full $\varepsilon(a)$.

Thus the modified Friedmann equation finally takes the form [10]

$$\frac{\dot{a}^2}{a^2} + \frac{1}{a^2} = \frac{\pi}{\beta G}\left\{1 - \sqrt{1 - \frac{16G^2}{3}\,\beta\varepsilon}\,\right\}, \tag{33}$$

where we expressed B according to (9).

In the limit of small subplanckian energy density $\beta G^2\varepsilon \equiv \beta\varepsilon/\varepsilon_P \ll 1$ the modified equation goes over into the ordinary Friedmann equation in which the parameter β completely drops out

$$\frac{\dot{a}^2}{a^2} + \frac{1}{a^2} = \frac{8\pi G}{3}\,\varepsilon. \tag{34}$$

Therefore within this energy range the standard cosmology is recovered. Depending on the effective equation of state, a wide set of the standard scenarios of late cosmological evolution can be obtained, including those showing a cosmic acceleration, provided some kind of dark energy component is present [32, 33].

5 Cosmological acceleration – Big Boost scenario

The range of applicability of the GR limit (34) depends on β. This makes possible a very interesting mechanism to happen for a very large β. Indeed, the value of the argument of the square root in (33) can be sufficiently far from 1 even for small ε provided $\beta \sim N_{\rm cdf} \gg 1$. Moreover, one can imagine a model with a variable number of conformal fields $N_{\rm cdf}(t)$ inducing a time-dependent, and implicitly a scale factor-dependent β, $\beta = \beta(a)$. If $\beta(a)$ grows with a faster than the rate of decrease of $\varepsilon(a)$, then the solution of (33) can reach a singular point, labeled below by ∞, at which the square root argument vanishes and the cosmological acceleration becomes infinite. This follows from the expression

$$\frac{\ddot{a}}{a} \sim \frac{4\pi}{3\beta G}\,\frac{a(G^2\beta\varepsilon)'}{\sqrt{1 - 16G^2\beta\varepsilon/3}}, \tag{35}$$

[3] In the Euclidean regime this field also stays in the slow roll approximation, but in view of the oscillating nature of a scale factor it does not monotonically decay. Rather it follows these oscillations with much lower amplitude and remains nearly constant during all Euclidean evolution, whatever long this evolution is (as it happens for garland instantons with the number of folds $k \to \infty$).

where prime denotes the derivative with respect to a. This expression becomes singular at $t = t_\infty$ even though the Hubble factor $H^2 \equiv (\dot{a}^2/a^2 + 1/a^2)$ remains finite when

$$(G^2\beta\varepsilon)_\infty = \frac{3}{16}, \;\; H^2_\infty = \frac{\pi}{\beta G}. \tag{36}$$

Assuming for simplicity that the matter density has a dust-like behavior and β grows as a power law in a

$$G\varepsilon \sim \frac{1}{a^3}, \;\; G\beta \sim a^n, \;\; n > 3, \tag{37}$$

one easily finds an inflection point $t = t_*$ when the cosmological acceleration starts after the deceleration stage when

$$(G^2\beta\varepsilon)_* = \frac{3}{4}\frac{n-2}{(n-1)^2}, \;\; H^2_* = \frac{2\pi}{n-1}\frac{1}{\beta G}. \tag{38}$$

The evolution ends in this model with the curvature singularity, $\ddot{a} \to \infty$, reachable in a finite proper time. Unlike the situation with a big brake singularity of [34] it cannot be extended beyond this singularity analytically even by smearing it out or taking into account its weak integrable nature. In contrast to [34] the acceleration at the singularity is positive. Hence, we called this type of singularity a *big boost* [10]. The effect of the conformal anomaly drives the expansion of the Universe to the maximum value of the Hubble constant, after which the solution becomes complex. This, of course, does not make the model a priori inconsistent, because for $t \to t_\infty$ an infinitely growing curvature invalidates the semiclassical and $1/N$ approximations. This is a new essentially quantum stage which requires a UV completion of the effective low-energy theory.

6 Hierarchy problem, strings and extra dimensions

As follows from (15) and (36)-(38) the inflation (which is run by the early primordial Λ) and cosmological acceleration stage both have Hubble factors given by the conformal anomaly coefficient β, $H^2 \sim 1/\beta G$. Therefore we have to reconcile the inflation and present Dark Energy scales in the hierarchy

$$H^2_{\rm inflation} = \#\frac{m_P^2}{\beta_{\rm inflation}} \sim ({\rm GUT\ scale})^2, \tag{39}$$

$$H^2_{\rm present} = \#\frac{m_P^2}{\beta_{\rm present}} \sim (10^{-33}{\rm eV})^2, \tag{40}$$

which can be done only by the assumption of the parameter β being a time dependent and tremendously growing variable, $\beta_{\rm present} \gg \beta_{\rm inflation}$. Of course, this can be considered as a rationale for the Big Boost mechanism of the previous section. This is not unusual now to suggest a solution to the hierarchy problem by introducing a gigantic number of quantum fields, like it has been done in [35] in the form of 10^{32} replicas of the Standard Model resolving the leap between the electroweak and Planck scales. Whatever speculative are such suggestions, we would consider a similar mechanism in our case even though we would have to consider a much bigger jump of about hundred twenty orders of magnitude.

So what can be the mechanism of a variable and indefinitely growing β? One such mechanism was suggested in [9]. It relies on the possible existence, motivated by string theory, of extra dimensions whose size is evolving in time. Theories with extra dimensions can promote β to the level of a modulus variable which can grow with the evolving size L of those dimensions, as we now explain. Indeed, the parameter β basically counts the number $N_{\rm cdf}$ of conformal degrees of freedom, $\beta \sim N_{\rm cdf}$ (see Eq.(11)). However, if one considers a string theory in a spacetime with more than four dimensions, the extra-dimension being compact with typical size L, the effective 4-dimensional fields arise as Kaluza-Klein (KK) and winding modes with masses (see e.g. [36])

$$m_{n,w}^2 = \frac{n^2}{L^2} + \frac{w^2}{\alpha'^2}\, L^2 \tag{41}$$

(where n and w are respectively the KK and winding numbers), which break their conformal symmetry. These modes remain approximately conformally invariant as long as their masses are much smaller than the spacetime curvature, $m_{n,w}^2 \ll H^2 \sim m_P^2/N_{\rm cdf}$. Therefore the number of conformally invariant modes changes with L. Simple estimates show that the number of pure KK modes ($w = 0$, $n \le N_{\rm cdf}$) grows with L as $N_{\rm cdf} \sim (m_P L)^{2/3}$, whereas the number of pure winding modes ($n = 0$, $w \le N_{\rm cdf}$) grows as L decreases as $N_{\rm cdf} \sim (m_P\alpha'/L)^{2/3}$. Thus, it is possible to find a growing β in both cases with expanding or contracting extra dimensions. In the first case it is the growing tower of superhorizon KK modes which *makes* the horizon scale

$$H \sim \frac{m_P}{\sqrt{N_{\rm cdf}}} \sim \frac{m_P}{(m_P L)^{1/3}} \tag{42}$$

decrease as L increases to infinity. In the second case it is the tower of superhorizon winding modes which makes this scale decrease with the decreasing L as

$$H \sim m_P \left(\frac{L}{m_P\alpha'}\right)^{1/3}. \tag{43}$$

At the qualitative level of this discussion, such a scenario is flexible enough to accommodate the present day acceleration scale (though, at the price of fine-tuning an enormous coefficient governing the expansion or contraction of L).

7 Dual description via the AdS/CFT correspondence

String (or rather string-inspired) models can offer a more explicit construction of these ideas within the AdS/CFT picture. Indeed, in this picture [11] a higher dimensional theory of gravity, namely type IIB supergravity compactified on $AdS_5 \times S^5$, is seen to be equivalent to a four dimensional conformal theory, namely $\mathcal{N} = 4$ $SU(N)$ SYM, thought to live on the boundary of AdS_5 space-time. This picture underlies the Randall-Sundrum model [12] where a 3-brane embedded into the AdS_5 space-time enjoys in a large distance limit a recovery of the 4D gravity theory without the need for compactification [13]. This model has a dual description. On the one hand it can be considered from a 5D gravity perspective, on the other hand it can also be described, thanks to the AdS/CFT correspondence, by a 4D conformal field theory coupled to gravity.

To be more precise, the 5D SUGRA — a field-theoretic limit of compactified type IIB string theory — induces on the brane of the underlying AdS background the quantum effective action of the conformally invariant 4D $\mathcal{N} = 4$ $SU(N)$ SYM theory coupled to the 4D geometry of the boundary. The multiplets of this CFT contributing according to (11) to the total conformal anomaly coefficient β are given by $(N_0, N_{1/2}, N_1) = (6N^2, 4N^2, N^2)$ [37], so that

$$\beta = \frac{1}{2}\, N^2. \tag{44}$$

The parameters of the two theories are related by the equation [11, 13, 14]

$$\frac{L^3}{2G_5} = \frac{N^2}{\pi}, \tag{45}$$

where L is the radius of the 5D AdS space-time with the negative cosmological constant $\Lambda_5 = -6/L^2$ and G_5 is the 5D gravitational constant. The radius L is also related to the 't Hooft parameter of the SYM coupling $\lambda = g_{SYM}^2 N$ and the string length scale $l_s = \sqrt{\alpha'}$, $L = \lambda^{1/4} l_s$. The generation of the 4D CFT from the local 5D supergravity holds in the limit when both N and λ are large. This guarantees the smallness of string corrections and establishes the relation between the weakly coupled tree-level gravity theory in the bulk ($G_5 \to 0$, $L \to \infty$) and the strongly coupled 4D CFT ($g_{SYM}^2 \gg 1$). Moreover, as said above, the AdS/CFT correspondence explains the mechanism of recovering general relativity theory on the 4D brane of the Randall-Sundrum model [13, 14]. The 4D gravity theory is induced on the brane from the 5D theory with the negative cosmological constant $\Lambda_5 = -6/L^2$. In the one-sided version of this model the brane has a tension $\sigma = 3/8\pi G_5 L$ (the 4D cosmological constant is given by $\Lambda_4 = 8\pi G_4 \sigma$), and the 4D gravitational constant $G_4 \equiv G$ turns out to be

$$G = \frac{2G_5}{L}. \tag{46}$$

One recovers 4D General Relativity at low energies and for distances larger than the radius of the AdS bulk, L. Thus, the CFT dual description of the 5D Randall-Sundrum model is very similar to the model considered above. Moreover, even though the CFT effective action is not exactly calculable for $g_{SYM}^2 \gg 1$ it is generally believed that its conformal anomaly is protected by extended SUSY [39] and is exactly given by the one-loop result (10). Therefore it generates the exact effective action of the anomalous (conformal) degree of freedom given by (8), which guarantees a good $1/N_{\rm cdf}$-approximation for the gravitational dynamics.

Applying further the above relations it follows a relation between our β coefficient and the radius L of the AdS space-time, given by $\beta G = \pi L^2/2$. Introducing this in the modified Friedmann equation (33), the latter becomes explicitly depending on the size of the 5D AdS spacetime as given by

$$\frac{\dot{a}^2}{a^2} + \frac{1}{a^2} = \frac{2}{L^2}\left\{1 - \sqrt{1 - L^2\left(\frac{8\pi G}{3}\rho + \frac{\mathcal{C}}{a^4}\right)}\right\}, \tag{47}$$

where we have reintroduced the decomposition (32) of the full matter density into the decay product of the inflationary and matter domination stages, with energy density ρ, and the thermal excitations of the primordial CFT (31).

For low energy density, $GL^2\rho \ll 1$ and $L^2\mathcal{C}/a^4 \ll 1$, in the approximation beyond the leading order, cf. Eq.(34), the modified Friedmann equation coincides with the modified Friedmann equation in the Randall-Sundrum model [38]

$$\frac{\dot{a}^2}{a^2} + \frac{1}{a^2} = \frac{8\pi G}{3}\rho\left(1 + \frac{\rho}{2\sigma}\right) + \frac{\mathcal{C}}{a^4}, \tag{48}$$

where $\sigma = 3/8\pi G_5 L = 3/4\pi G L^2$ is the Randall-Sundrum brane tension and $\mathcal{C}$ is the braneworld constant of motion [38, 41].[4] Note that the thermal radiation on the brane (of non-Casimir energy nature) is equivalent to the mass of the bulk black hole associated with this constant. This fact can be regarded as another manifestation of the AdS/CFT correspondence in view of the known duality between the bulk black hole and the thermal CFT on the brane [41].

Interestingly, this comparison between our model and the Randall-Sundrum framework also allows one to have some insight on the phenomenologically allowed physical scales. Indeed, it is well known that the presence of an extra-dimension in the Randall-Sundrum model, or in the dual language, that of the CFT, manifests itself typically at distances lower than the AdS radius L. Hence, it is perfectly possible to have a large number of conformal fields in the Universe, *à la*

[4] We assume that the dark radiation term is redshifted, as a grows, faster than the matter term and expand to the second order in ρ, but the first order in $\mathcal{C}$.

Randall-Sundrum, without noticing their presence in the everyday experiments, provided L is small enough. Moreover, if one uses the scenario of [7] to set the initial conditions for inflation, it provides an interesting connection between the Hubble radius of inflation, given by eq. (15), and the distance at which the presence of the CFT would manifest itself in gravity experiments, both being given by L. Last, it seems natural in a string theory setting, to imagine that the AdS radius L can depend on time, and hence on the scale factor.

In this case, assuming that the AdS/CFT picture still holds when L is adiabatically evolving, one can consider the possibility that $GL^2\varepsilon$ is large, and that $L^2(t)$ grows faster than $G\varepsilon(t)$ decreases during the cosmological expansion. One would then get the cosmological acceleration scenario of the above type followed by the big boost singularity.

In this case, however, should this acceleration scenario correspond to the present day accelerated expansion, L should be of the order of the present size of the Universe, i.e. $L^{-2} \sim H^2_{\rm present}$. Since the Randall-Sundrum mechanism recovers 4D GR only at distances beyond the curvature radius of the AdS bulk, $r \gg L$, it means that local gravitational physics of our model (47) at the acceleration stage is very different from the 4D general relativity. Thus this mechanism can hardly be a good candidate for generating dark energy in real cosmology.

8 Anomaly driven cosmology and the DGP model

It is interesting that there exists an even more striking example of a braneworld setup dual to our anomaly driven model. This is the generalized DGP model [40] including together with the 4D and 5D Einstein-Hilbert terms also the 5D cosmological constant, Λ_5, in the special case of the *vacuum* state on the brane with a vanishing matter density $\rho = 0$. In contrast to the Randall-Sundrum model, for which this duality holds only in the low energy limit — small ρ and small $\mathcal{C}/a^4$, vacuum DGP cosmology *exactly* corresponds to the model of [7] with the 4D cosmological constant Λ simulated by the 5D cosmological constant Λ_5.

Indeed, in this model (provided one neglects the bulk curvature), gravity interpolates between a 4D behaviour at small distances and a 5D behaviour at large distances, with the crossover scale between the two regimes being given by r_c,

$$\frac{G_5}{2G} = r_c, \tag{49}$$

and in the absence of stress-energy exchange between the brane and the bulk, the modified Friedmann equation takes the form [42]

$$\frac{\dot a^2}{a^2} + \frac{1}{a^2} - r_c^2\left(\frac{\dot a^2}{a^2} + \frac{1}{a^2} - \frac{8\pi G}{3}\rho\right)^2 = \frac{\Lambda_5}{6} + \frac{\mathcal{C}}{a^4}. \tag{50}$$

Here $\mathcal{C}$ is the same as above constant of integration of the bulk Einstein's equation, which corresponds to a nonvanishing Weyl tensor in the bulk (or a mass for a Schwarzschild geometry in the bulk) [38, 41]. It is remarkable that this equation with $\rho = 0$ exactly coincides with the modified Friedmann equation of the anomaly driven cosmology (28) under the identifications

$$B \equiv \frac{\beta G}{\pi} = 2r_c^2, \tag{51}$$

$$\Lambda = \frac{\Lambda_5}{2}. \tag{52}$$

These identifications imply that in the DGP limit $G \ll r_c^2$, the anomaly coefficient β is much larger than 1.

This looks very much like the generation of the vacuum DGP model for any value of the dark radiation $\mathcal{C}/a^4$ from the anomaly driven cosmology with a very large $\beta \sim m_P^2 r_c^2 \gg 1$.

However, there are several differences. A first important difference between the conventional DGP model and the anomaly driven DGP is that the former does not incorporate the self-accelerating branch [42, 43] of the latter. This corresponds to the fact that only one sign of the square root is admissible in Eq.(29) — a property dictated by the instanton initial conditions at the nucleation of the Lorentzian spacetime from the Euclidean one. So, one does not have to worry about possible instabilities associated with the self-accelerating branch.

Another important difference concerns the way the matter energy density manifests itself in the Friedmann equation for the non-vacuum case. In our 4D anomaly driven model it enters the right hand side of the equation as a result of the decay (32) of the effective 4D cosmological constant Λ, while in the DGP model it appears inside the parenthesis of the left hand side of equation (50). Therefore, the DGP Hubble factor reads as

$$\frac{\dot{a}^2}{a^2} + \frac{1}{a^2} = \frac{8\pi G}{3}\,\rho + \frac{1}{2r_c^2}\left\{1 - \sqrt{1 - 4r_c^2\left(\frac{\Lambda_5}{6} + \frac{\mathcal{C}}{a^4} - \frac{8\pi G}{3}\,\rho\right)}\,\right\} \tag{53}$$

(note the negative sign of ρ under the square root and the extra first term on the right hand side). In the limit of small ρ, $\mathcal{C}/a^4$ and Λ_5, the above equation yields a very different behavior from the GR limit of the anomaly driven model (34),

$$\frac{\dot{a}^2}{a^2} + \frac{1}{a^2} \simeq \frac{\Lambda_5}{6} + \frac{\mathcal{C}}{a^4} + r_c^2\left(\frac{\Lambda_5}{6} + \frac{\mathcal{C}}{a^4} - \frac{8\pi G}{3}\,\rho\right)^2. \tag{54}$$

For vanishing Λ_5 and $\mathcal{C}/a^4$ this behavior corresponds to the 5D dynamical phase [42, 43] which is realized in the DGP model for a very small matter energy density on the brane $\rho \ll 3/32\pi G r_c^2 \sim m_P^2/r_c^2$.

Of course, in this range the DGP braneworld reduces to a vacuum brane, but one can also imagine that the 5D cosmological constant decays into matter constituents similar to (32) and thus simulates the effect of ρ in Eq.(33). This can perhaps provide us with a closer correspondence between the anomaly driven cosmology and the non-vacuum DGP case. But here we would prefer to postpone discussions of such scenarios to future analyses and, instead, focus on the generalized *single-branch* DGP model to show that it also admits the cosmological acceleration epoch followed by the big boost singularity.

Indeed, for positive Λ_5 satisfying a very weak bound

$$\Lambda_5 > \frac{3}{2r_c^2} \tag{55}$$

Eq.(53) has a solution for which, during the cosmological expansion with $\rho \to 0$, the argument of the square root vanishes and the acceleration tends to $\pm\infty$. For the effective a-dependence of r_c^2 and $G\rho$ analogous to (37), $r_c^2(a) \sim a^n$ and $G\rho(a) \sim 1/a^3$, the acceleration becomes positive at least for $n \geq 0$,

$$\frac{\ddot{a}}{a} \simeq \frac{n + 32\pi G\, r_c^2\rho}{4r_c^2\sqrt{1 + 4r_c^2\left(\frac{8\pi G}{3}\,\rho - \frac{\Lambda_5}{6} - \frac{\mathcal{C}}{a^4}\right)}} \to +\infty. \tag{56}$$

This is the big boost singularity labeled by ∞ and having a finite Hubble factor $(\dot{a}^2/a^2 + 1/a^2)_\infty = \Lambda_5/6 + 1/4r_c^2$.

Thus, the *single-branch* DGP cosmology can also lead to a big boost version of acceleration. For that to happen, one does not actually need a growing r_c (which can be achieved at the price of having a time dependent G_5 — itself some kind of a modulus, in a string inspired picture). The DGP crossover scale r_c can be constant, $n = 0$, and the big boost singularity will still occur provided the lower bound (55) is satisfied [5]. When Λ_5 violates this bound, the acceleration stage is eternal with an asymptotic value of the Hubble factor squared $H^2 = \dot{a}^2/a^2$ given by $\big(1-\sqrt{1-2r_c^2\Lambda_5/3}\big)/2r_c^2$.

9 Conclusions

To summarize, within a minimum set of assumptions (the equipartition in the physical phase space (22)), we have a mechanism to generate a limited range of a positive cosmological constant which is likely to constrain the landscape of string vacua and get the full evolution of the Universe as a quasi-equilibrium decay of its initial microcanonical state. Thus, contrary to anticipations of Sidney Coleman that "there is nothing rather than something" regarding the actual value of the cosmological constant [3], one can say that something (rather than nothing) comes from everything.

We have obtained the modified Friedmann equation for this evolution in the anomaly dominated cosmology. This equation exhibits a gravitational screening of the quantum Casimir energy of conformal fields — this part of the total energy density does not weigh, being degravitated due to the contribution of the conformal anomaly. Also, in the low-density limit this equation does not only show a recovery of the standard general relativistic behavior, but also coincides with the dynamics of the Randall-Sundrum cosmology within the AdS/CFT duality relations. Moreover, for a very large and rapidly growing value of the Gauss-Bonnet coefficient β in the conformal anomaly this equation features a regime of cosmological acceleration followed by a big boost singularity. At this singularity the acceleration factor grows in finite proper time up to infinity with a finite limiting value of the Hubble factor. A proper description of the late phase of this evolution, when the Universe enters again a quantum phase, would require a UV completion of the low-energy semiclassical theory.

A natural mechanism for a growing β can be based on the idea of an adiabatically evolving scale associated with extra dimensions [9] and realized within the picture of AdS/CFT duality, according to which a conformal field theory is induced on the 4D brane from the 5D nonconformal theory in the bulk. As is well known, this duality sheds light on the 4D general relativistic limit in the Randall-Sundrum model [13, 14]. Here we observed an extended status of this duality from the cosmological perspective — the generalized Randall Sundrum model with the Schwarzschild-AdS bulk is equivalent to the anomaly driven cosmology for small energy density. In particular, the radiation content of the latter is equivalent to the dark radiation term $\mathcal{C}/a^4$ pertinent to the Randall-Sundrum braneworld with a bulk black hole of mass $\mathcal{C}$.

Another intriguing observation concerns the *exact* correspondence between the anomaly driven cosmology and the vacuum DGP model generalized to the case of a nonvanishing bulk cosmological constant Λ_5. In this case a large β is responsible for the large crossover scale r_c, (49). For positive Λ_5 satisfying the lower bound (55) this model also features a big boost scenario even for stabilized β. Below this bound (but still for positive $\Lambda_5 > 0$, because a negative Λ_5 would imply a time of maximal expansion from which the Universe would start recollapsing) the cosmological evolution eventually enters an eternal acceleration phase. However, the DGP model with matter on the brane can hardly be equivalent to the 4D anomaly driven cosmology, unless one has some mechanism for Λ_5 to decay and to build up matter density on the brane.

Unfortunately, our scenario put in the framework of the AdS/CFT correspondence with adiabatically evolving scale of extra dimension cannot agree with the observed dark energy, because, for the required values of the parameters, the local gravitational physics of this model would become very different from the 4D general relativity.

In general, the idea of a very large central charge of CFT algebra, underlying the solution of the hierarchy problem in the dark energy sector and particle phenomenology, seems hovering

[5]More precisely, one should also take into account here the modification due the dark radiation contribution $\mathcal{C}/a^4$. However, the latter is very small at late stages of expansion.

in current literature [44, 35]. Our idea of a big growing β belongs to the same scope, but its realization seems missing a phenomenologically satisfactory framework. In essence, it can be considered as an attempt to cross a canyon in two endeavors — the leap of 32 decimal orders of magnitude in $N_{\rm cdf}$ of [35], separating the electroweak and Planckian scales, versus our 120 orders of magnitude needed to transcend separation between the Hubble and Planckian scales. Both look equally speculative from the viewpoint of local phenomenology.

Probably some other modification of this idea can be more productive. In particular, an alternative mechanism of running β could be based on the winding modes. These modes do not seem to play essential role in the AdS/CFT picture with a big scale of extra dimensions L, because they are heavy in this limit. On the contrary, this mechanism should work in the opposite case of contracting extra dimensions, for which the restrictions from local gravitational physics do not apply (as long as for $L \to 0$ the short-distance correction go deeper and deeper into UV domain).

Acknowledgements

The work of A.O.B. was supported by the Russian Foundation for Basic Research under the grant No 08-01-00737 and the grant LSS-1615.2008.2. The collaboration of A.B. and C.D. was made in part possible via the ANR grant "MODGRAV". A.K. was partially supported by the RFBR grant 08-02-00923, the grant LSS-4899.2008.2 and by the Research Programme "Elementary Particles" of the Russian Academy of Sciences.

References

[1] J.B.Hartle and S.W.Hawking, Phys.Rev. **D28**, 2960 (1983); S.W.Hawking, Nucl. Phys. **B 239**, 257 (1984).

[2] A.D. Linde, JETP **60**, 211 (1984); A.Vilenkin, Phys. Rev. **D 30**, 509 (1984).

[3] S.R.Coleman, Nucl. Phys. **B 310**, 643 (1988).

[4] A.A.Starobinsky, in *Field Theory, Quantum Gravity and Strings*, 107 (eds. H.De Vega and N.Sanchez, Springer, 1986); A.D.Linde, *Particle physics and inflationary cosmology* (Harwood, Chur, Switzerland, 1990).

[5] G.W.Gibbons, S.W.Hawking and M.Perry, Nucl. Phys. **B 138**, 141 (1978).

[6] A.Vilenkin, Phys. Rev. **D58**, 067301 (1998), gr-qc/9804051; gr-qc/9812027.

[7] A.O.Barvinsky and A.Yu.Kamenshchik, J. Cosmol. Astropart. Phys. **09**, 014 (2006), hep-th/0605132.

[8] A.O.Barvinsky and A.Yu.Kamenshchik, Phys. Rev. **D74**, 121502 (2006), hep-th/0611206.

[9] A.O.Barvinsky, Phys. Rev. Lett. **99** (2007) 071301, hep-th/0704.0083

[10] A.O.Barvinsky, C.Deffayet and A.Yu.Kamenshchik, JCAP **05** (2008) 020, arXiv:0801.2063.

[11] J.Maldacena, Adv. Theor. Math. Phys. **2** 231 (1998); Int. J. Theor.Phys. **38** 1113 (1999), hep-th/9711200; E.Witten, Adv. Theor. Math. Phys. **2**, 253 (1998); S.S.Gubser, I.R.Klebanov and A.M.Polyakov, Phys. Lett. **B428**, 105 (1998), hep-th/9802109.

[12] L. Randall and R. Sundrum, Phys. Rev. Lett. **83** (1999) 4690 [arXiv:hep-th/9906064].

[13] S.Gubser, Phys. Rev. **D63**, 084017 (2001), hep-th/9912001.

[14] S.W. Hawking, T. Hertog and H.S. Reall, Phys.Rev. **D62** (2000) 043501.

[15] D.N.Page, Phys. Rev. **D 34**, 2267 (1986).

[16] A.A.Starobinsky, Phys. Lett. **91B**, 99 (1980).

[17] M.J.Duff, Class. Quant. Grav **11**, 1387 (1994), hep-th/9308075.

[18] I.L.Buchbinder, Fortsch. Phys. **34**, 605 (1986).

[19] R.J.Riegert, Phys. Lett. **134 B**, 56 (1984); P.O.Mazur and E.Mottola, Phys. Rev. **D 64**, 104022 (2001).

[20] E.S.Fradkin and A.A.Tseytlin, Phys. Lett. **134 B**, 187 (1984).

[21] A.O.Barvinsky, A.G.Mirzabekian and V.V.Zhytnikov, "Conformal decomposition of the effective action and covariant curvature expansion", gr-qc/9510037.

[22] I.L.Buchbinder, V.P.Gusynin and P.I.Fomin, Sov. J. Nucl. Phys. **44**, 534 (1986); I.L.Buchbinder and S.M.Kuzenko, Nucl. Phys. **B274**, 653 (1986).

[23] M.V.Fischetti, J.B.Hartle and B.L.Hu, Phys. Rev. **D 20**, 1757 (1979).

[24] L.D.Faddeev, Theor. Math. Phys. **1**, 1 (1969).

[25] A.O.Barvinsky, Phys. Rep. **230**, 237 (1993); Nucl. Phys. **B 520**, 533 (1998).

[26] A.O.Barvinsky and V.Krykhtin, Class. Quantum Grav. **10**, 1957 (1993); A.O.Barvinsky, gr-qc/9612003; M.Henneaux and C.Teitelboim, *Quantization of Gauge Sytems* (Princeton University Press, Princeton, 1992).

[27] A.A.Starobinsky, Gravit. Cosmol. **6**, 157 (2000), astro-ph/9912054.

[28] J.B. Hartle and K. Schleich, in *Quantum field theory and quantum statistics*, 67 (eds. I.Batalin et al, Hilger, Bristol, 1988); K. Schleich, Phys.Rev. **D 36**, 2342 (1987).

[29] D. Brown and J.W. York, Jr., Phys. Rev. **D 47**, 1420 (1993), gr-qc/9209014.

[30] N.C.Tsamis and R.P.Woodard, Nucl. Phys. **B 474**, 235 (1996).

[31] G.Dvali, S.Hofmann and J.Khoury, "Degravitation of the Cosmological Constant and Graviton Width", hep-th/0703027.

[32] R.R. Caldwell, R. Dave and P.J. Steinhardt, Phys. Rev. Lett. **80**, 1582-1585 (1998); L.-M. Wang, R.R. Caldwell, J.P. Ostriker and P.J. Steinhardt, Astrophys. J. **530**, 17 (2000).

[33] A.Yu. Kamenshchik, U. Moschella and V. Pasquier, Phys. Lett. **B511**, 265 (2001); V. Sahni and A.A. Starobinsky, Int. J. Mod. Phys. **D9**, 373 (2000); **D15**, 2105 (2006).

[34] V. Gorini, A.Yu. Kamenshchik, U. Moschella and V. Pasquier, Phys. Rev. **D69**, 123512 (2004).

[35] G.Dvali, "Black Holes and Large N Species Solution to the Hierarchy Problem", arXiv:0706.2050; G.Dvali and M.Redi, "Black Hole Bound on the Number of Species and Quantum Gravity at LHC", arXiv:0710.4344 [hep-th].

[36] J.Polchinski, *String Theory* (Cambridge University Press, Cambridge, 1998).

[37] M.J.Duff and J.T.Liu, Phys. Rev. Lett. **85**, 2052 (2000), hep-th/0003237.

[38] P.Binetruy, C.Deffayet and D.Langlois, Phys. Lett. **477** 275 (2000), hep-th/9910219.P. Binetruy, C. Deffayet, U. Ellwanger and D. Langlois, Phys. Lett. B **477** (2000) 285 [arXiv:hep-th/9910219].

[39] Hong Liu and A.A. Tseytlin, Nucl.Phys. **B533**, 88 (1998), hep-th/9804083.

[40] G. R. Dvali, G. Gabadadze and M. Porrati, Phys. Lett. B **485** (2000) 208 [arXiv:hep-th/0005016].

[41] T. Shiromizu, K. i. Maeda and M. Sasaki, Phys. Rev. D **62**, 024012 (2000) [arXiv:gr-qc/9910076]. P. Kraus, JHEP **9912**, 011 (1999) [arXiv:hep-th/9910149].

[42] C.Deffayet, Phys. Lett. **B 502**, 199 (2001), hep-th/0010186.

[43] C.Deffayet, G.Dvali and G.Gabadadze, Phys. Rev. **D 65**, 044023 (2002), astro-ph/0105068.

[44] N.Arkani-Hamed, S.Dimopoulos, G.Dvali and G.Gabadadze, "Non-Local Modification of Gravity and the Cosmological Constant Problem", hep-th/0209227.