Neutrino asymmetry and the growth of cosmological seed magnetic field

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Abstract

Primordial cosmological hypermagnetic fields polarize the early Universe plasma prior to the electroweak phase transition (EWPT). As a result of the long range parity violating gauge interaction present in the Standard Model their magnitude gets amplified, opening a new perturbative way of seeding the primordial Maxwellian magnetic field at EWPT.

1 Introduction

The electroweak phase transition (EWPT) has long been considered as a playing an important role in the generation of primordial magnetic fields [1, 2, 3].

In the paper [4] we showed how the interplay between the resulting polarization effects of the early Universe plasma and long range parity violating gauge interaction present the Standard Model (SM) subsequently amplifies the seed hypermagnetic field till the epoch close to the EWPT time, after which the evolution of the corresponding Maxwellian magnetic field is described by the standard MagnetoHydroDynamics (MHD) for relativistic plasma.

While the long-ranged non-Abelian magnetic fields (corresponding to the color $SU(3)$ or to the weak $SU(2)$) can not exist because at high temperatures the non-Abelian interactions induce a magnetic gap $\sim g^2T$ the Abelian hypercharge magnetic fields are never screened and can survive in the plasma for infinitely long times.

Consider the equations of motion for the hypercharge Y_μ-field in the hot plasma and in the presence of a pre-existing large scale hypermagnetic field $\mathbf{B}_0^Y$, regular on scales smaller than horizon size at $T_0 > T \gg T_{EW}$. For simplicity we neglect the Abelian anomaly and assume the Minkowski space.

We start from the SM Lagrangian for hypercharge field Y_μ:

$$\begin{aligned} L = \quad & -\frac{1}{4}Y_{\mu\nu}Y^{\mu\nu} + \sum_{l=e,\mu,\tau}\frac{g^{'}Y^\mu}{2}\left(-\bar{\nu}_{lL}\gamma_\mu\nu_{lL} - \bar{l}_L\gamma_\mu l_L - 2\bar{l}_R\gamma_\mu l_R\right) + \\ & + \sum_i^N \frac{g^{'}Y^\mu}{2}\left[\frac{1}{3}\bar{U}_{iL}\gamma_\mu U_{iL} + \frac{1}{3}\bar{D}_{iL}\gamma_\mu D_{iL} + \frac{4}{3}\bar{U}_{iR}\gamma_\mu U_{iR} - -\frac{2}{3}\bar{D}_{iR}\gamma_\mu D_{iR}\right] + \\ & + i\frac{g^{'}Y^\mu}{2}\left[\varphi^+ D_\mu\varphi - \left(D_\mu\varphi^+\right)\varphi\right], \qquad (1) \end{aligned}$$

where $l = e,\mu,\tau$, $U_i = u,c,t$, $D_i = d,s,b$ are leptons and quarks correspondingly, $\varphi = \left(\phi^+,\phi^{(0)}\right)^T$ - is the Higgs doublet.

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2 Equilibrium conditions above EWPT, $T \gg T_{EW}$

There are equilibrium relations among the chemical potentials implied by the corresponding conversions [5]:

$$
\begin{aligned}
&\mu_W = \mu_- + \mu_0 && (W^- \leftrightarrow \phi^- + \phi^0),\\
&\mu_{D_L} = \mu_{U_L} + \mu_W && (W^- \leftrightarrow \bar{U}_L + D_L),\\
&\mu_L^{(l)} = \mu_{\nu_L}^{(l)} + \mu_W && (W^- \leftrightarrow \bar{\nu}_L^{(l)} + l_L),\\
&\mu_{U_R} = \mu_0 + \mu_{U_L} && (\phi^0 \leftrightarrow \bar{U}_L + U_R),\\
&\mu_{D_R} = -\mu_0 + \mu_{D_L} && (\phi^0 \leftrightarrow D_L + \bar{D}_R),\\
&\mu_R^{(l)} = -\mu_0 + \mu_L^{(l)} && (\phi^0 \leftrightarrow l_L + \bar{l}_R).
\end{aligned} \tag{2}
$$

Let us note that chemical potentials are connected each other due to the global plasma neutrality. In particular, the electroneutrality condition $< Q >= 0$ and the absence of the isospin component $< Q_3 >= \mu_W = 0$ mean the hypercharge neutrality $< Y >= 0$,

$$
Y = 2(Q - Q_3) = 2\left[-2\sum_l \mu_L^{(l)} + 6\mu_{uL} + 14\mu_0\right] = 0, \tag{3}
$$

which in turn allows to get the chemical potential for the neutral Higgs boson:

$$
\mu_0 = \frac{\sum_l \mu_L^{(l)} - 3\mu_{uL}}{7}. \tag{4}
$$

Notice also the sphaleron equilibrium condition valid above EWPT,

$$
\sum_l \mu_L^{(l)} = -9\mu_{uL}, \tag{5}
$$

allows to express all chemical potentials through the lepton sum $\sum_l \mu_L^{(l)}$, or accounting for the third line in Eq. (2), through the sum of neutrino chemical potentials, $\sum_l \mu_{\nu_L}^{(l)}$, which we denote below as $\sum_l \mu_{\nu_L}^{(l)} = \mu_\nu$.

From the Dirac equation for massless fermions in a seed large-scale hypermagnetic field, $\mathbf{B}_0 = \nabla \times \mathbf{Y}^{(0)} = (0, 0, B_0^Y)$,

$$
\left[\hat{p} - f^{(a)}(g')\hat{Y}^{(0)}\right]\Psi^{(a)} = 0, \qquad f^{(a)}(g') = \frac{g' y_a}{2}, \tag{6}
$$

where y_a is the hypercharge, one finds the Landau spectrum for fermions (including neutrinos) in JWKB approximation, $g' B_0^Y \ll T^2$,

$$
\varepsilon(p_z, n, \lambda) = \sqrt{p_z^2 + \mid f^{(a)}(g') \mid B_0^Y (2n + 1 \mp \lambda)} \approx p \mp \mid f^{(a)}(g') \mid B_0^Y \frac{\lambda}{2p}\ . \tag{7}
$$

Here $p = \sqrt{p_z^2 + p_\perp^2}$ with the relation $p_\perp^2 = \mid f^{(a)}(g') \mid B_0^Y (2n + 1)$ is the fermion momentum, the upper sign applies to particles and the lower one to antiparticles and the last *paramagnetic* term in (7) is given by the spin projection on the seed hypermagnetic field $\mathbf{B}_0^Y$, $(\sigma_z)_{\lambda'\lambda} = \lambda\delta_{\lambda'\lambda}$. Together with chirality for leptons and quarks $\gamma_5\Psi_{(l,q)R,L} = \pm\Psi_{(l,q)R,L}$ it is a good quantum number since $[\gamma_5, \Sigma_z] = 0$.

As a result in JWKB approximation the equilibrium density matrix includes the spin distribution term:

$$
f_{\lambda'\lambda}^{(a,\bar{a})} = \frac{\delta_{\lambda'\lambda}}{\exp[(\varepsilon(p_z, n, \lambda) \mp \mu_a)/T] + 1} \approx \frac{\delta_{\lambda'\lambda}}{2} f_0^{(a,\bar{a})}(p) + \frac{\sigma^j_{\lambda'\lambda}}{2} S_0^{(a,\bar{a})j}(p), \tag{8}
$$

where $f_0^{(a,\bar{a})}(p) = [\exp(p \mp \mu_a)/T] + 1]^{-1}$ is the Fermi distribution, $S_0^{(a,\bar{a})j}(p)$ is the equilibrium spin distribution given by

$$\mathbf{S}_0^{(a,\bar{a})}(p) = -\mathbf{B}_0^Y \frac{\mid f_a(g^{'}) \mid}{2\mathrm{p}} \frac{\mathrm{d}f_0^{(a,\bar{a})}(\mathrm{p})}{\mathrm{dp}} = \frac{\mathbf{B}_0^Y}{B_0^Y} S_0^{(a,\bar{a})}(p). \tag{9}$$

Using the *equilibrium* spin distribution (9) we exclude in the statistically averaged four-pseudovector $J_{\mu 5}^Y \sim \sum_{\lambda,\lambda'} < \hat{\bar{\Psi}}_{\mathbf{p}\lambda}\gamma_\mu\gamma_5\hat{\Psi}_{\mathbf{p}'\lambda'} >$ its time component:

$$J_{05}^Y \sim \int \frac{d^3p}{(2\pi)^3} \frac{(\mathbf{p}\cdot\mathbf{S}^{(a,\bar{a})}(\mathbf{p},\mathbf{x},t))}{p} \to 0, \quad \text{if} \;\; \mathbf{S}^{(a,\bar{a})}(\mathbf{p},\mathbf{x},t) \to \mathbf{S}_0^{(a,\bar{a})}(p),$$

while the 3-pseudovector component differs from zero,

$$\mathbf{J}_5^Y \sim \int \frac{d^3p}{(2\pi)^3} \frac{\mathbf{p}(\mathbf{p}\cdot\mathbf{S}^{(a,\bar{a})}(\mathbf{p},\mathbf{x},t))}{p^2} \sim \mathbf{B}_0^Y \neq 0, \quad \text{if} \;\; \mathbf{S}^{(a,\bar{a})}(\mathbf{p},\mathbf{x},t) \to \mathbf{S}_0^{(a,\bar{a})}(p).$$

3 Maxwell equations for hypercharge fields

Maxwell equations for hypercharge fields $\mathbf{E}_Y$ and $\mathbf{B}_Y$ given by the SM Lagrangian Eq. (1) take the form

$$\begin{aligned}
&\nabla\cdot\mathbf{B}_Y = 0,\\
&\nabla\cdot\mathbf{E}_Y = 4\pi\left[J_0^Y(\mathbf{x},t) + J_{05}^Y(\mathbf{x},t)\right],\\
&\frac{\partial\mathbf{B}_Y}{\partial t} = -\nabla\times\mathbf{E}_Y,\\
&-\frac{\partial\mathbf{E}_Y}{\partial t} + \nabla\times\mathbf{B}_Y = 4\pi\left[\mathbf{J}^Y(\mathbf{x},t) + \mathbf{J}_5^Y(\mathbf{x},t)\right],
\end{aligned} \tag{10}$$

which differs from the Maxwell equations for QED plasma due to the presence of pseudovector currents $J_{\mu 5}^Y$ originated by the parity violation in SM.

The total vector (pseudovector) currents in Maxwell equations (10) are the following. The total vector current $J_\mu^Y = \sum_l J_{l\mu}^Y + 3NJ_{(q)\mu}^Y + J_{(\phi)\mu}^Y$ includes lepton, quark and Higgs contributions. In particular, the lepton vector current

$$J_{l\mu}^Y(\mathbf{x},t) = -\frac{g^{'}}{4}\left[2\delta j_\mu^{l_R}(\mathbf{x},t) + \delta j_\mu^{l_L}(\mathbf{x},t) + \delta j_\mu^{\nu_{lL}}(\mathbf{x},t)\right]$$

is given by the lepton asymmetries,

$$\delta j_\mu^{(a)} = j_\mu^{(a)} - j_\mu^{(\bar{a})} = \int \frac{d^3p}{(2\pi)^3}\frac{p_\mu}{p}\left[f^{(a)}(\mathbf{p},\mathbf{x},t) - f^{(\bar{a})}(\mathbf{p},\mathbf{x},t)\right].$$

Accounting for the fermion (antifermion) particle number densities for small asymmetries $\mu_a/T \ll 1$,

$$n^{(a,\bar{a})} = \int \frac{d^3p}{(2\pi)^3}\frac{1}{\exp([p\mp\mu_a]/T)+1} \approx n_{eq}\left[1 \pm \frac{\pi^2}{9\zeta(3)}\left(\frac{\mu_a}{T}\right) + O\left(\left(\frac{\mu_a}{T}\right)^2\right)\right],$$

where $n_{eq} = 3\zeta(3)T^3/4\pi^2$ is the equilibrium fermion density when $\mu_a = 0$, one can show that the hypercharge density vanishes in correspondence with Eq. (3),

$$J_0^Y = -\gamma n_{eq}\left(\frac{2\pi^2}{9\zeta(3)}\right)\left(\frac{g^{'}}{4T}\right)\left[-2\sum_l \mu_L^l + 6\mu_{uL} + 14\mu_0\right] = 0 \qquad \text{since} \qquad < Y >= 0,$$

while the hypercharge 3-current differs from zero,

$$\mathbf{J}^Y(\mathbf{x},t)=\sum_a\frac{f^{(a)}(g^{'})}{2}\gamma n_{eq}[\mathbf{V}^{(a)}-\mathbf{V}^{\bar{a}}]\neq 0.$$

The total pseudovector current in Maxwell Eq. (10) consists of the fermion (antifermion) terms only, $J^Y_{\mu 5}=\sum_l J^Y_{l\mu 5}(\mathbf{x},t)+3NJ^Y_{(q)\mu 5}(\mathbf{x},t)$. In particular, the lepton pseudovector current

$$J^Y_{l\mu 5}(\mathbf{x},t)=-\frac{g^{'}}{2}\int\frac{d^3p}{p(2\pi)^3}\delta A_\mu^{(l_R)}(\mathbf{p},\mathbf{x},t)+\frac{g^{'}}{4}\int\frac{d^3p}{p(2\pi)^3}\delta A_\mu^{(l_L)}(\mathbf{p},\mathbf{x},t)+$$
$$+\frac{g^{'}}{4}\int\frac{d^3p}{p(2\pi)^3}\delta A_\mu^{(\nu_{lL})}(\mathbf{p},\mathbf{x},t),$$

is given by the spin distribution asymmetries $\delta A_\mu^{(a)}(\mathbf{p},\mathbf{x},t)=A_\mu^{(a)}(\mathbf{p},\mathbf{x},t)-A_\mu^{(\bar{a})}(\mathbf{p},\mathbf{x},t)$ through the four component pseudovector functions:

$$A_\mu^{(a)}(\mathbf{p},\mathbf{x},t)=\left[(\mathbf{p}\cdot\mathbf{S}^{(a)}(\mathbf{p},\mathbf{x},t));\ \frac{\mathbf{p}(\mathbf{p}\cdot\mathbf{S}^{(a)}(\mathbf{p},\mathbf{x},t))}{p}\right].$$

Substituting the equilibrium spin distributions Eq. (9) one gets

$$J^Y_{05}=0\quad \text{since odd integrand}:\quad \int\frac{\mathrm{d^3p}}{(2\pi)^3}\frac{(\mathbf{p}\cdot\mathbf{S}_0^{(\mathrm{a},\bar{\mathrm{a}})}(\mathrm{p}))}{\mathrm{p}}=0,$$

while 3-vector part differs from zero in equilibrium plasma,

$$(\mathbf{J}_5^Y)_{eq}=\frac{g^{'2}}{96\pi^2}\left[-2\sum_l\mu_L^l+10\mu_{uL}+32\mu_0\right]\mathbf{B}_0^Y=\frac{47}{1512\pi^2}g^{'2}\mu_\nu\mathbf{B}_0^Y\neq 0\ . \tag{11}$$

Now instead of general Eq. (10) we get finally Maxwell equations for hypercharge fields $\mathbf{E}_Y$, $\mathbf{B}_Y$ in hot *equilibrium* plasma at $T\gg T_{EW}$ as

$$\begin{aligned}&\nabla\cdot\mathbf{B}_Y=0,\qquad \nabla\cdot\mathbf{E}_Y=0,\\ &\frac{\partial\mathbf{B}_Y}{\partial t}=-\nabla\times\mathbf{E}_Y,\\ &-\frac{\partial\mathbf{E}_Y}{\partial t}+\nabla\times\mathbf{B}_Y=4\pi\left[\mathbf{J}^Y(\mathbf{x},t)+\frac{47}{378}\times\frac{g^{'2}\mu_\nu}{\pi}\mathbf{B}_Y\right],\end{aligned} \tag{12}$$

where $\mu_\nu=\sum_l\mu_{\nu_L}^{(l)}$ and $l=e,\mu,\tau$.

4 Faraday equation and α^2 -dynamo

In the rest frame $\mathbf{V}=0$ of the isotropic early Universe plasma combining last Maxwell-like equations in Eq. (12) and the Ohm law $\mathbf{J}^Y=\sigma_{cond}\mathbf{E}_Y$ we can write the Faraday equation describing so-called α^2-dynamo [6] of hypermagnetic field:

$$\frac{\partial\mathbf{B}_Y}{\partial t}=\nabla\times\alpha\mathbf{B}_Y+\eta\nabla^2\mathbf{B}_Y\ . \tag{13}$$

Here $\eta=(4\pi\sigma_{cond})^{-1}$ is the magnetic diffusion coefficent, the parameter α is the hypermagnetic helicity coefficent given as

$$\alpha=\frac{47g^{'2}\mu_\nu}{1512\pi^2\sigma_{cond}} \tag{14}$$

plays crucial role in the evolution of hypermagnetic field [4]. We can solve Eq. (13) through Foirier harmonics as $\mathbf{B}_Y(\mathbf{x},t) = \int (d^3k/(2\pi)^3)\mathbf{B}_Y(\mathbf{k},t)e^{i\mathbf{kx}}$ where $B_Y(k,t)$ is expressed as

$$B_Y(k,t) = B_0^Y \exp\left[\int_{t_0}^{t} [\alpha(t^{'})k - \eta(t^{'})k^2]dt^{'}\right] . \tag{15}$$

For $0 < k < \alpha/\eta$, or correspondigly correlation length scales $\eta/\alpha < \Lambda < \infty$ such field gets exponentially amplified, but differently for different scales Λ. E.g. for the Fourier mode $k = \alpha/2\eta$ (or $\Lambda \simeq 2\eta/\alpha$) one gets the maximum amplification $\gamma = \alpha k - \eta k^2 = \alpha^2/4\eta$ [6, 7]

$$B_Y(t) = B_0^Y \exp\left[\int_{t_0}^{t} \frac{\alpha^2(t^{'})}{4\eta(t^{'})}dt^{'}\right] = B_0^Y \exp\left[32\int_{x}^{x_0} \frac{dx^{'}}{x^{'2}}\left(\frac{\xi_\nu(x^{'})}{0.001}\right)^2\right] , \tag{16}$$

where we introduced the new variables $x = T/T_{EW}$, $\xi_\nu = \mu_\nu/T$ and B_0^Y is the assumed initial amplitude of the hypermagnetic field at $T \gg T_{Ew}$.

For larger scales, $\Lambda > 2\eta/\alpha$, the amplification factor in the exponent (16) becomes less than ~ 32, nevertheless, it is enough both for a strong enhancement of the initial hypermagnetic field and to survive against ohmic dissipation (magnetic field diffusion) if $\Lambda > l_{diff} = \sqrt{\eta l_H}$.

5 Discussion

In contrast to the mechanism suggested in refs. [2] and [8] ours [4] does not rely on the Chern-Simons anomaly term added to the SM Lagrangian [9]. The presence of the anomaly acting at the later EWPT epoch could play an important role in the subsequent evolution of the lepton asymmetry produced by the parity violating hypercharge interaction. Here we do not study the EWPT conversion of the hypercharge field to the Maxwellian magnetic field $\mathbf{B}$. However we note that the seed value $B_Y \sim 0.3T^2 < T_{EW}^2 \sim 10^{24}$ *Gauss* can be easily reached through our Eq. (16). This provides a strongly first order EWPT that, in turn, allows to avoid the sphaleron constraint for the baryogenesis within the Standard Model [10].

There is a difference between the amplification (exponential) factors for α^2 -dynamo mechanisms acting before ($T \gg T_{EW}$ in Eq. (16) here) and after EWPT, $T \ll T_{EW}$ (see Eq. (14) in paper [7]). Such difference with the dependence $\sim x^{'10}$ in the integrand of Eq. (14) [7] follows from the low energy approximation, $q^2 \ll M_W^2$, leading to the point-like Fermi interaction of neutrinos with plasma that, in turn, corresponds to short-range weak forces. While in the first case for $T \gg T_{EW}$ the massless hypercharge field Y_μ provides long-range forces for neutrino -plasma interaction leading to the dependence $\sim 1/x^{'2}$ in the integrand of Eq. (16).

Let us comment on the physical interpretation of the new magnetic helicity term. The original seed field B_0^Y polarizes the fermions and antifermions (including neutrinos) propagating along the field in the main Landau level, $n = 0$. This polarization effect causes fermions and antifermions to move in opposite directions with a relative drift velocity proportional to the neutrino asymmetry $\mu_\nu = \sum_l \mu_{\nu_L}^{(l)}$. The existence of a basic parity violating hypercharge interaction in the SM induces a new term in the hypermagnetic field in Eq. (13) $\nabla \times \alpha \mathbf{B}_Y$ which winds around the rectilinear pseudovector hypercharge current $\mathbf{J}_5$ parallel to $\mathbf{B}_Y$. This term amplifies the seed hypermagnetic field B_0^Y according to Eq. (16).

In summary, we described a simple (polarization) mechanism [4] for amplification of hypermagnetic field based on the SM in particle physics and plasma physics that is different from the mechanism based on Abelian anomaly for hypercharge field [2, 8]. The lepton number violation through hypermagnetic helicity change $\sim \mathbf{E}_Y \cdot \mathbf{B}_Y$ given by the Abelian anomaly [2] can be completed by the standard evolution equation for the hypermagnetic helicity $H = \int \mathbf{Y} \cdot \mathbf{B}_Y d^3x$ that will be the subject of a future work. It is interesting also fo follow conversion of hypermagnetic helicity to the Maxwellian one [11] at the EWPT time, $T \sim T_{EW}$.

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