Constraints on massive graviton dark matter from pulsar timing and astrometry

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Abstract

A narrow-band isotropic stochastic gravitational wave background (GWB) is one of observational consequences of theory of massive gravity with spontaneous Lorentz braking [1]. This background should have distinctive signatures in records of pulsar timing and astrometric measurements. The existing millisecond pulsar timing accuracy ($\sim 0.2\,\mu$s) is used to derive an upper limit on this sort of GWB. This limit essentially rules out any significant contribution of massive gravitons to the local dark halo density. The present-day accuracy of astrometrical measurements ($\sim 100\,\mu$as) sets less stringent constraints on this theory.

1 Introduction

Big progress in observational cosmology, especially in measurements of CMB radiation, has challenged our understanding of the Universe. The standard cosmological ΛCDM model based on GR is confirmed by observations with high accuracy [2, 3]. This model requires the present Universe to be dominated by dark matter and dark energy of unknown nature, so the modification of gravity at large distances could provide alternative description of the Universe. There are several theories with infrared modification of gravity based on quite different grounds (e.g. [4, 5, 6, 7, 8, 9, 10, 11, 12]). Among these possibilities, recently developed theories of massive gravity with violated Lorentz invariance [11, 12, 13] appear to be theoretically attractive and have interesting phenomenology (see the recent review [14]). In particular, in theory of massive gravity [13], the Lorentz invariance is spontaneously broken by the condensates of scalar fields, which allows to avoid problems of strong coupling and ghosts that are unavoidable in Lorentz-invariant theories with massive graviton.

Dubovsky [13] constructed a theory where gravitational waves (GWs) are massive while linearized equations for scalar and vector metric perturbations, as well as spatially flat cosmological solutions, are the same as in GR. In this theory an extra dark-energy term appears in the Friedmann equations suggesting an unusual explanation to the observed accelerated expansion. In addition, massive gravitons could be produced in the early Universe copiously enough to explain, in principle, all of the cold dark matter [1]. A distinctive feature of GWs produced by cold massive gravitons is a very narrow frequency range of the signal ($\Delta\nu/\nu \sim 10^{-6}$) as determined by virial motions of cold gravitons in the galactic halo. The central frequency itself is model-dependent, but GW emission from known relativistic binary systems place an upper limit on the frequency $\nu \leq 3 \times 10^{-5}$ Hz. At lower frequencies, the amplitude of the GWB could

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be of order $h \sim 10^{-10}\left(\frac{3\times10^{-5}\,\mathrm{Hz}}{\nu}\right)$ [1] assuming the density of gravitons matches the conservative estimate of the dark matter local density $\rho_{DM} = 0.3\,\mathrm{GeV\ cm}^{-3}$[17] . Clustering of GWs on $\sim$ kpc scales constrains de Brogile length of massive graviton accordingly thereby placing a lower limit of $\sim 10^{-8}\,\mathrm{Hz}$. This leaves out a region $\sim 10^{-8}\,\mathrm{Hz} < \nu <\sim 3\times10^{-5}\,\mathrm{Hz}$ for the allowed frequency of GW associated with massive gravitons. The amount of GW signal in the frequency range $\sim 10^{-5} - 10^{-6}\,\mathrm{Hz}$ is further constrained by the tracking data for the Cassini spacecraft [15] (see Fig. 1).

We show [16] that the amount of (almost) monochromatic GW in the entire allowed region can be strongly constrained from the existing pulsar timing data and astrometric measurements, essentially ruling out any significant contribution due to massive gravitons to the density of galactic dark matter. The propagation of electromagnetic waves from a remote astronomical source in the presence of a GW background causes an excessive noise in pulsar timing [18, 19] and alters stochastically the apparent position of the source [20, 21]. So, high-precision pulsar timing and astrometry of distant sources (for example, quasars) can be used to constrain the amplitude of the possible GW background.

2 Constraints from pulsar timing

Pulsar timing was suggested in the late 1970s [18, 19] as a tool to detect or constrain the local GWB. A GW travelling through the Solar system affects the observed frequency of a pulsar resulting in anomalous residuals in the time of arrival (ToA) of pulses [22]. Because of unrivalled rotational stability, timing of millisecond pulsars is particularly well suited for detecting GWs [23]. The conventional technique of the stochastic background measurements using pulsar timing [24] assumes correlating ToA residuals of several pulsars. Using this method has yielded upper limits on the low-frequency broad-band stochastic GWBs [25].

A narrow-band GWB produces an excessive noise in pulsar timing at the corresponding frequency. The rms of timing residuals of even a single pulsar can put an upper limit on the GWB amplitude in the frequency range between the inverse of the pulsar timing data time span T (typically several years) and inverse time of the pulsar signal accumulation($\sim$ hours). The Parkes Pulsar Timing Array (PPTA) includes several pulsars with current rms residuals $r \le 0.2\,\mu\mathrm{s}$ ($0.12\,\mu$s, $0.19\,\mu$s and $0.17\,\mu$s for J0437-4715, J1713+0747 and J1939+2134, respectively) [23].

In the weak field limit of Dubovsky et al. theory, the equations of motion are the same as in Einstein's GR and we can therefore employ the results of GR calculations on the expected effect of a local GW on the observed frequency of a pulsar. For the GW power spectrum per logarithmic interval (as defined by Eq. (18) of [26]) we restrict ourselves to a δ-like function at some $k = 2\pi\nu/c$:

$$P_h(k') = \begin{cases} P_0, k < k' < k + \delta k \\ 0, \text{otherwise} \end{cases} \tag{1}$$

For this power spectrum, the mass density of GWs is [26]

$$\rho_{\mathrm{GW}} = (16\pi G)^{-1} c^2 k P_0 \delta k \tag{2}$$

providing necessary connection to the total power $P_0\delta k$.

The observed pulsar ToA rms variation r gives an upper limit on ToA dispersion due to GWs and therefore translates into the following upper limit on $P_0\delta k$ (see [16] for details):

$$P_0\delta k \le 3r^2k^3c^2, \tag{3}$$

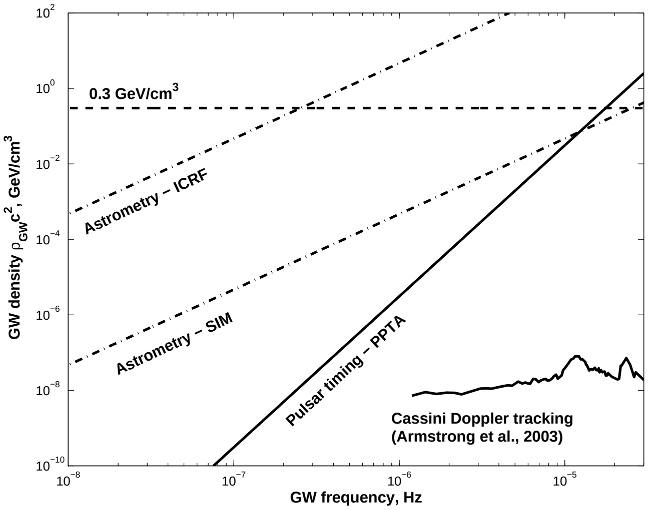

Figure 1: Astrometric (dot-dashed) and pulsar timing (solid) constraints on the overall energy density of a stationary isotropic background of monochromatic GWs as a function of the frequency ν in the range $\sim 10^{-8}\,\mathrm{Hz} < \nu <\sim 3\times10^{-5}\,\mathrm{Hz}$ allowed by binary pulsar GW emission and DM clustering constraints (see Introduction). The thick dashed line corresponds to the local DM energy density of $0.3\,\mathrm{GeV cm^{-3}}$. The constraint in the lower right corner of the graph is set by the Doppler tracking of the *Cassini* spacecraft [15].

and therefore

$$\rho_{GW}c^2 \le (16\pi G)^{-1}3r^2k^4c^4 = 3G^{-1}\pi^3r^2\nu^4 \qquad (4)$$
$$\approx 2.5\mathrm{GeV}\cdot\mathrm{cm}^{-3}.\left(\frac{\nu}{3\times10^{-5}\,\mathrm{Hz}}\right)^4\left(\frac{r}{0.2\,\mu\mathrm{s}}\right)^2$$

The upper limit corresponding to TOA residuals $r = 0.2\,\mu$s is plotted in Figure 1 as a function of ν and is clearly lower than what is needed for massive gravitons to be the dominant component of the local dark matter at frequency domains unconstrained by the Cassini data.

3 Astrometric constraints

The astrometric effect is different for the light propagating across a region of space with enhanced density of massive gravitons, e.g. a dark halo of a galaxy or galaxy cluster (the *en-route* effect), and for stochastic change of the position of the observer immersed in the massive graviton halo (the local effect). The former smears out the visible size of a distant source, while the latter changes stochastically the angular separation between different sources on the sky.

In astrophysically relevant cases the *en-route* effect is too small to be detected at the present level of accuracy of astrometric measurements. A very generous upper limit on the stochastically fluctuating change in the observed position of a distant source may be estimated as $\sigma_\Psi \sim h_{\max}\Psi_{\mathrm{L}}$, where Ψ_{L} is the angular size of the halo on the line of sight and $h_{\max}$ is the maximum amplitude of GWs comprising the halo (which, due to the Gaussian nature of these fluctuations, is essentially the same as the rms amplitude $h_c = \sqrt{P_h}$ that can be estimated from the dark matter density).

Contrary to naive expectations, in GR the light ray deflection does not execute a random walk and does not show the $\sim\sqrt{N}$ growth of the deflection. Instead, for traceless tensor perturbations travelling with the speed of light, only the gravitational wave field at emission and detection points matter [27, 20, 28]. The relative change $\Delta\Psi/\Psi$ in the angular separation between two sources due to the local effect is also of order of the GW background amplitude

h_c. At $h_c \sim 10^{-11} - 10^{-10}$, as Dubovsky et al. model suggests [1], this would yield $\sim \mu$as jitter in the angular separation for a couple of sources across the sky. Such jitter can be discovered in the future astrometric space experiments like SIM [29].

Present-day astrometric accuracy σ_Ψ can be estimated as that of the radio VLBI-based ICRF (International Cosmic Reference Frame) [30, 31], which involves more than 200 reference radio sources determining the celestial coordinate frame. The ICRF sources are observed for many years, and the accuracy of determination of source coordinates on the sky relative to this frame may be used as a measure of the angular separation stability. The best present-day accuracy of 100 μas (at $1\,\sigma$ level) [32] means $\Delta\Psi \leq 5 \times 10^{-10}$.

For the adopted power spectrum (1), this accuracy translates into the following upper limit on the GW mass density (see the derivation in [16]):

$$\rho_{\rm GW} \leq \frac{3\pi\nu^2\sigma_\Psi^2}{4G} \approx 4.2 \times 10^3\,{\rm GeV}\cdot{\rm cm}^{-3}\left(\frac{\nu}{3\times 10^{-5}\,{\rm Hz}}\right)^2\left(\frac{\sigma_\Psi}{100\,\mu{\rm as}}\right)^2. \quad (5)$$

Other limits for this value as a function of frequency ν are summarized in Fig. 1.

4 Conclusions

It is shown that the existing data on the millisecond pulsar timing stability set a tight upper limit on the narrow-band GWB amplitude at frequencies $\nu \leq 10^{-5}$Hz. This limit can be used to severely bound the amount of massive cold gravitons which can potentially produce a strong narrow-band GWB [1].

The present-day astrometric constraints are less restrictive than the timing ones at considered frequencies. However, both are still far above the tightest constraint set by the Doppler tracking of Solar system spacecrafts in the frequency range $\nu > 10^{-6}$ Hz [15].

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