# Conformal rapid-roll inflation

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**Abstract**

Usual inflation is realized with a slow rolling scalar field minimally coupled to gravity. In contrast, we consider dynamics of a scalar with a flat effective potential, conformally coupled to gravity. Surprisingly, it contains an attractor inflationary solution with the rapidly rolling inflaton field. We discuss the popular string-theoretic warped inflationary scenario, based on the interacting D3-anti D3 branes. The original warped brane inflation suffers a large inflaton mass due to conformal coupling to 4-dimensional gravity. Instead of considering this as a problem and trying to cure it with extra engineering, we show that warped inflation with the conformally coupled, rapidly rolling inflaton is yet possible. We consider modulate reheating for generation of density perturbations in this scenario.

## 1 Introduction

Realization of inflation is typically associated with a slow rolling scalar field minimally coupled to gravity [1]. Is it possible to have inflation with a non-minimally coupled scalar field, in particular with a conformally coupled field? This question is especially interesting in the context of superstring theory, where many scalars are conformally coupled to gravity; or in supergravity theory, where scalars typically have the mass of order of $H^2$ [2].

Indeed, among a number of inflationary scenarios, simple scenarios which come in the package with high energy physics draw an especial attention. Recently there is significant interest to package up inflation with the string theory. There are different models of string theory inflation (see, e.g. [3]). Often string inflation models require significant fine-tuning. The most typical problem however is the $\eta$-problem, manifested in heavy inflaton mass preventing slow roll of inflaton. For realization of slow-roll inflation the effective inflaton mass should be much smaller than the Hubble parameter during inflation, $m^2 \ll H^2$. An example of this problem is warped brane inflation (based on the configurations of interacting branes in warped geometry) where the inflaton is conformally coupled to the four-dimensional gravity [4, 5]. A scalar field $\phi$ conformally coupled to gravity acquires effective mass term $\frac{1}{2}\xi R\phi^2$ with $\xi = 1/6$. This gives the inflaton the effective mass $m^2 \simeq 12\xi H^2 = 2H^2$ which does not result in the slow-roll. This problem has been considered as a stumbling block on the way to build up successful warped brane-antibrane inflation.

In this presentation we demonstrate that in fact conformal coupling is not a problem for realization of inflation [6]. Inflationary regime in this case can be realized. However, it has new features which make it very different from other models. First, inflation with a conformally coupled inflaton with a potential $V(\phi)$, which is almost flat over the range of $\phi$, can be realized as the rapid-roll inflation, contrary to the customary slow roll inflation. To the best of our knowledge, this is the first model of inflation with a conformally coupled inflaton. Our model can be regarded also as a new model of the fast rolling inflaton. A model of fast roll inflaton based on a minimally coupled scalar field was suggested in [7].

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With conformal coupling, the only feature of the warped brane-antibrane inflation needed for conformal inflation is the form of its potential: very shallow for the most part of the inflaton rolling and changing sharply at the end point of inflation. Therefore, application of the conformal rapid-roll inflation is broader than the warped brane-antibrane inflation. It includes, for instance, field-theoretic hybrid models (with conformal inflaton) where the potential is almost flat during inflation.

Second, the string-theory realization of rapid-roll conformal inflation is a low energy inflation (close to the least possible energy of inflation at $1-100$ TeV scale). As a result it requires significantly less efoldings $N$ than the figure 62 typical for the GUT scale chaotic inflation. In the context of the warped geometry, as we will show, $N$ and the scale of inflation are directly related to the warp factor of the throat geometry of the internal manifold. Noticeably, lower bound for efolds of inflation $N$ coincides with the warping needed for the Randall-Sundrum solution of the hierarchy problem.

From the viewpoint of observational cosmology, the possibility to motivate inflation which lasts just to provide homogeneity/isotropy only within the observable horizon (but not beyond it) is interesting with respect to the low multi-poles anomalies found in the COBE and WMAP CMB temperature anisotropies. There are attempts to build up the models with the spectra of primordial fluctuations truncated at the horizon scales (for instance see [8]), but not much motivation for the choice of the scale of truncation was found. The model of conformal inflation naturally gives truncation of fluctuations at the horizon scales.

The third feature of our model is different origin of its cosmological fluctuations. A conformally coupled inflaton cannot be responsible for generation of primordial fluctuations. As we shall show, fluctuations also cannot be generated from the angular degrees of freedom, associated with the angular positions of branes in the bulk Klebanov-Strassler throat geometry. We propose modulated (inhomogeneous reheating) fluctuations [9, 10] or curvaton fluctuations [11, 12] of the scalars of the SM sector or other moduli fields.

In the rest of this presentation we will address these features of the model.

## 2 Inflation from Conformally-Coupled Scalar with Flat Potential

Conventional inflationary models are based on minimally coupled scalar fields. Indeed, slow roll inflation requires relatively small effective inflaton mass $m^2 \ll H^2$, while conformal coupling violates this condition.

Let us nonetheless consider a conformally coupled scalar field with a very simple flat potential. This is relevant for a couple of interesting inflationary models. In the warped brane inflation the inflaton candidate is an open string modulus, brane-antibrane distance. Denote this scalar field in the four-dimensional effective theory as $\phi$. Brane-antibrane interaction results in the four-dimensional scalar field potential [4]

$$V = V_0 \left[ 1 - \left( \frac{M_p \Delta}{\phi} \right)^4 \right] , \qquad (1)$$

where $M_p = 1/\sqrt{8\pi G_N}$ ($\simeq 2 \times 10^{18} GeV$) is the reduced Planck mass, $\Delta$ is a dimensionless constant, $V_0$ is related to the warp factor $e^{-A}$ as $V_0 \sim T_3 e^{-4A}$ and $T_3$ is the $D3$-brane tension. Except for small values of $\phi$, this potential is very flat. Another example of almost flat potential is the usual field-theoretic hybrid inflation.

As the first step, consider dynamics of the FRW universe dominated by a conformally-coupled scalar field with a potential which can be approximated by a constant. Although the potential is flat, scalar field dynamics is significant due to the conformal coupling to the curvature, which can be interpreted as induced mass of the scalar field. Non-trivial observation is that de Sitter spacetime is self-consisted solution of this system. Moreover, inflation is an attractor solution for a FRW universe with a conformally-coupled scalar field with $V = const$.

For a scalar field $\phi$ described by the action

$$I = \int d^4x\sqrt{-g}\left[\frac{M_p^2 R}{2} - \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - V(\phi) - \frac{\xi}{2}R\phi^2\right] , \quad (2)$$

where $\xi$ is a coupling to gravity, the equations of motion are

$$\left(M_p^2 - \xi\phi^2\right) G_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - \frac{1}{2}\partial^\rho\phi\partial_\rho\phi g_{\mu\nu} - \xi\left[\nabla_\mu\nabla_\nu(\phi^2) - \nabla^\rho\nabla_\rho(\phi^2)g_{\mu\nu}\right] - V(\phi)g_{\mu\nu} , \quad (3)$$

and

$$\nabla^\mu\nabla_\mu\phi - V'(\phi) - \xi R\phi = 0 . \quad (4)$$

In the FRW background

$$ds^2 = -dt^2 + a(t)^2 d\mathbf{x}^2 \quad (5)$$

these equations are reduced to

$$3\left(M_p^2 - \xi\phi^2\right) H^2 = \frac{1}{2}\dot\phi^2 + 6\xi H\phi\dot\phi + V(\phi), \quad (6)$$

and

$$\ddot\phi + 3H\dot\phi + 6\xi(\dot H + 2H^2)\phi + V'(\phi) = 0. \quad (7)$$

With $\xi = 1/6$ and $V(\phi) = V_0$, it is convenient to introduce the new variable $\varphi$ by

$$\phi = \frac{\varphi}{a}. \quad (8)$$

With this variable, the equations of motion are as simple as

$$a'^2 = H_0^2\left(a^4 + \frac{\varphi'^2}{2V_0}\right), \quad \varphi'' = 0, \quad (9)$$

where

$$H_0 \equiv \sqrt{\frac{V_0}{3M_p^2}}, \quad (10)$$

and a prime denotes the derivative w.r.t. the conformal time $\eta$ defined by $dt = a d\eta$. The second equation implies that

$$\varphi = \varphi_* + v\eta, \quad (11)$$

where $\varphi_*$ and $v$ are constants. Thus, the first equation becomes

$$a'^2 = H_0^2\left(a^4 + a_0^4\right) , \quad (12)$$

where $a_0^4 \equiv \frac{v^2}{2V_0}$. For the expanding branch ($a' > 0$), solution of Eq. (12) is

$$H_0(\eta_0 - \eta) = \int_a^\infty \frac{dx}{\sqrt{x^4 + a_0^4}} \equiv F(a) , \quad (13)$$

where $\eta_0 = const$.

For $a \gg a_0$, the function $F(a)$ is well approximated as $F(a) \simeq \frac{1}{a}$. Thus, from Eq. (13) we immediately obtain

$$a \simeq \frac{1}{H_0(\eta_0 - \eta)}, \quad \varphi \simeq \varphi_0 \quad \text{for } \eta \to \eta_0 - 0, \quad (14)$$

where $\varphi_0 \equiv \varphi_* + v\eta_0$. The scale factor (14) corresponds to the de Sitter solution. Therefore, we have shown that inflation with the Hubble expansion rate $H_0$ is an attractor solution of Eqs. (6), (7) with $\xi = 1/6$. Note that (almost) de Sitter expansion can be realized with a not small value of $\dot{\phi}^2/V_0$. For example, if $v = 0$ then we have an exact de Sitter expansion but $\dot{\phi}^2/V_0 = H_0^2\phi^2/V_0 = \phi^2/(3M_p^2) = \varphi_*^2/(3M_p^2 a^2)$ does not have to be small.

In Sec. 4, we shall illustrate this attractor behavior for the potential (1) by using the phase portrait method.

# 3 Condition for the Rapid Roll Inflation

As we have shown above, a conformally coupled scalar field can lead to inflation. During inflation, however, the inflaton evolves as

$$\phi \approx \frac{\varphi_0}{a}, \tag{15}$$

with the velocity

$$\dot{\phi} \approx -H\phi\ . \tag{16}$$

This is rapid roll inflation. The number of efolds $N = \int dt H$ for this rapid roll inflation is reduced to

$$N = -\int \frac{d\phi}{\phi} = \ln \frac{\phi_i}{\phi_f}\ , \tag{17}$$

where $\phi_i$ and $\phi_f$ are the values of the inflaton at the beginning and end of inflation. As we will show in the next section, low energy inflation can be realized for the warped geometry brane inflation.

In the previous section we have shown how conformal inflation can be realized with the flat potential. In this section we derive the conditions for a general potential $V(\phi)$ to realize conformal inflation.

For a minimally coupled inflaton, conditions for the potential $V(\phi)$ to provide inflation can be formulated in terms of smallness of the slow-roll parameters

$$\epsilon \equiv \frac{M_p^2}{2}\left(\frac{V'}{V}\right)^2\ ,\quad \eta \equiv \frac{M_p^2 V''}{V}. \tag{18}$$

We will discuss their generalization for the rapid roll inflaton. For an arbitrary potential $V(\phi)$, the equation of motion and the Friedman equation for the conformal inflaton can be re-written as

$$H^2 = \frac{1}{3M_p^2}\left(\frac{1}{2}\pi^2 + V\right),\qquad \dot{\pi} + 2H\pi + V' = 0, \tag{19}$$

where

$$\pi \equiv \dot{\phi} + H\phi. \tag{20}$$

Inflation corresponds to the slow variation of the Hubble parameter $H$. Therefore it is instructive to construct

$$-\frac{\dot{H}}{H^2} = \frac{\pi^2 + V'\phi/2}{\pi^2/2 + V}. \tag{21}$$

It is convenient to introduce the following combinations

$$\tilde{\epsilon} \equiv \frac{V'\phi}{2V},\qquad \eta_c \equiv \eta + \frac{c+2}{3}\left[\frac{V''\phi}{V'} + c\right], \tag{22}$$

where $c$ is a dimensionless constant chosen so that $|\eta_c|$ is minimized in a range of $\phi$.

In Appendix A we derive the conditions for the conformal inflation:

$$\epsilon \ll 1, \tag{23}$$
$$|\tilde{\epsilon}| \ll 1, \tag{24}$$
$$|\eta_c| \ll 1. \tag{25}$$

Under these conditions the equations motion are approximated as

$$\begin{aligned} H^2 &\simeq \frac{1}{3M_p^2}V, \\ (2+c)H\pi &\simeq -V', \end{aligned} \tag{26}$$

and the Hubble expansion rate $H$ is slowly varying with time, i.e. $|\dot{H}/H^2| \ll 1$.

## 4 Phase Portrait for Conformal Inflation

In the KKLMMT setup for the warped brane inflation [4], an inflaton emerges as a conformal field. It has been believed that the conformal coupling should spoil the flatness of the inflaton potential and that additional severe fine-tuning should be necessary. In this section we shall show that, contrary to the folklore, inflation in this model can be realized without further fine-tuning.

The inflaton in the warped setup is a conformally coupled scalar field with the potential (1). For this potential, the condition (25) with $c = 5$ is reduced just to $|\eta| \ll 1$. Thus, the conditions for inflation (23), (24) and (25) can be written as

$$\left|\frac{\phi}{M_p}\right| \gg Max[\Delta, \Delta^{2/3}]. \tag{27}$$

In this section we will analyze generic solutions of the Eqs. (19) with the potential (1). We can study scalar field/gravity dynamics in great details using the powerful phase portrait method, which shows the character of *all* solutions for $a(t)$ and $\phi(t)$ as trajectories in the two-dimensional phase space. For the minimally coupled scalar field, convenient coordinates for the phase portrait are $(\phi, \dot{\phi})$. Phase portraits for monomic potentials contain separatrices which attract most of the trajectories [13, 14]. These separatrices correspond to the inflationary regime of slow rolling.

In the case of a conformal scalar field, the qualitative behavior of the scalar field/gravity system can be also studied by drawing the phase portrait. For this purpose, we introduce dimensionless variables $q$ and $p$ and the dimensionless time coordinate $\tau$ as

$$q \equiv \frac{\phi}{M_p}, \quad p \equiv \frac{\pi}{\sqrt{V_0}}, \quad \tau \equiv \frac{\sqrt{V_0}}{M_p}t. \tag{28}$$

We will deal with the two-dimensional phase portrait in terms of $(\phi, \pi)$, or with its dimensionless copy in terms of $(q, p)$.

The equations of motion (19) can be written in terms of $(q, p)$ as

$$\begin{aligned} \partial_\tau q &= p - \frac{1}{\sqrt{3}}hq, \\ \partial_\tau p &= -\frac{2}{\sqrt{3}}hp - \frac{4\Delta^4}{q^5}\ , \end{aligned} \tag{29}$$

where $h \equiv \sqrt{3M_p^2/V_0}H$. The constraint equation is written in terms of $q$, $p$ and $h$ as

$$h = \sqrt{1 + \frac{1}{2}p^2 - \left(\frac{\Delta}{q}\right)^4}. \tag{30}$$

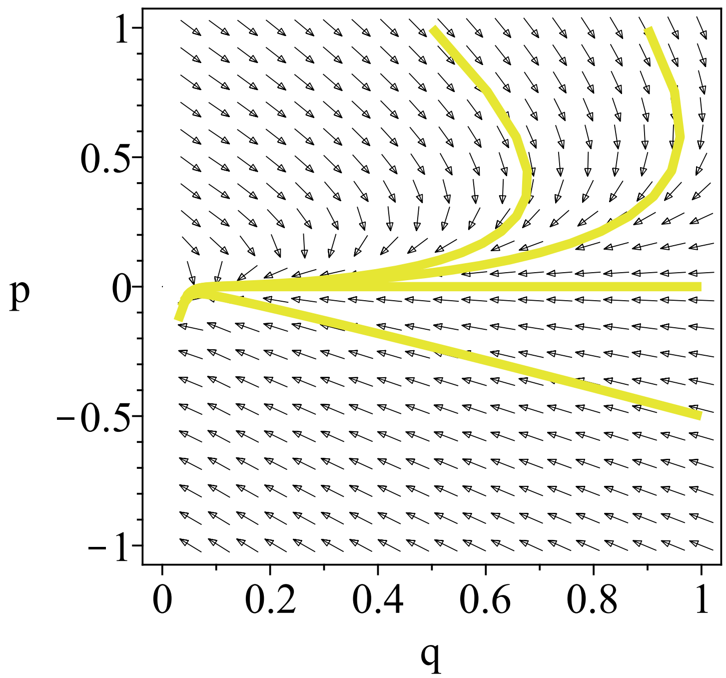


Figure 1: Phase portrait for the system of equations (29) with $\Delta = 0.01$. The horizontal axis is $q$ and the vertical axis is $p$.

Here, we have chosen the expanding branch $h > 0$. Figure 1 shows the phase portrait of the system (29) with (30). There is a separatric with $|p| \ll 1$. One can easily see that most of the solutions quickly approaches this inflationary separatrices.

## 5 Efoldings of inflation and Mass Hierarchy

At first glance there is no common ground for the mass hierarchy and duration of inflation. We give a surprising example where both of those emerge from the same underlying physics.

Consider warped brane inflation. Suppose inflation begins at the initial field value $\phi_i$, and ends at the end point of inflation $\phi_f$. According to Eq. (17), number of efolds of the rapid roll inflation is $\ln(\phi_i/\phi_e)$. In the context of the warped geometry the modulus field $\phi$ is related to the radial coordinate of the throat $y$ as $\phi = e^{-y/R}$, where $R$ is a radius of the AdS which well approximates the KS throat geometry. Thus the number of efolds is

$$N = (y_i - y_e)/R \tag{31}$$

and is bounded from above as

$$N \lesssim A, \tag{32}$$

where $A \equiv (y_{top} - y_{tip})/R$, and $y_{top}$ and $y_{tip}$ are values of the radial coordinate at the top and the tip of the throat, respectively. Note that $e^{-A}$ is the warp factor at the tip of the throat geometry. The value of the warp factor $e^{-A}$ depends on the energy scale associated with the throat. For example, the warp factor $\sim e^{-37}$ is related to the Standard Model, TeV scale throat. (See (35) below with $M \simeq TeV$.) Thus, in single-throat scenarios where the brane-antibrane pair is also in the TeV scale throat, the inequality (32) says that the number of efolding $N$ is less than 37.

We shall see below that even in multi-throat scenarios where the scale of inflation and the electroweak scale are not necessarily related [1], the number of efoldings $N_*$ corresponding to the wavelength of the present observable horizon is bounded from above. Actually, we shall obtain the bound $N_* \lesssim 34$ applicable to both single- and multi-throat scenarios. Moreover, we shall show that the scale of inflation $M = V^{1/4}$ is also bounded from above by $\sim 10^6 GeV$. This bound also applies to both single- and multi-throat scenarios.

One might think that $N_* \lesssim 34$ would be too small for successful inflation. GUT scale inflation requires at the minimum about 60 efolds or so. However, low energy inflation needs significantly smaller $N_*$. Indeed, recall that number of efolds which corresponds to the wavelength of the present-day observable horizon is related to the energy scale of inflation $M = V^{1/4}$ as

$$N_* \approx 62 - \ln \frac{10^{16} GeV}{M} - \Delta_r \ , \tag{33}$$

where $\Delta_r = \frac{4}{3} \ln \frac{M}{T_r}$ and $T_r$ is reheat temperature. Here, for simplicity we have assumed that the energy scale of inflation $M = V^{1/4}$ is not significantly changing during inflation. Depending on the (p)reheating scenario, $\Delta_r$ varies in the range

$$1 \lesssim \Delta_r \lesssim 10. \tag{34}$$

For example, let us take TeV as the scale of inflation, then number of efolds in (33) for the TeV scale inflation can be pushed to the lowest value $N_* \sim 30$.

In order to derive the upper bounds on $N_*$ and $M$, let us recall the warping (Randall-Sundrum) solution of the hierarchy problem [17] between the Planck scale $M_{Pl}$ $(= \sqrt{8\pi} M_p \simeq 10^{19} GeV)$ and low energy scale $M$, based on the warped AdS geometry

$$M \simeq M_{Pl} e^{-A}. \tag{35}$$

Suppose that the scale of inflation $M = V^{1/4}$ is determined in this way. Then, by substituting this to (33) we obtain

$$A + N_* \simeq 69 - \Delta_r. \tag{36}$$

Thus, (34) implies that

$$59 \lesssim A + N_* \lesssim 68. \tag{37}$$

Combining this with (32) and using (35) again, we obtain the bounds

$$N_* \lesssim 34, \quad M \lesssim 10^6 GeV. \tag{38}$$

These bounds apply to both single- and multi-throat scenarios and tell that the conformal inflation in warped geometry is low-energy inflation.

Let us now see that the conformal inflation is compatible with $M \simeq TeV$. For this purpose, note that the inequality (32) is not saturated in general. Inflation does not have to start exactly at the top of the throat and ends before the tip. Moreover, the efoldings $N_*$ corresponding to the wavelength of the present horizon be smaller than the total efoldings $N$. Thus, let us introduce the offset $\Delta_A$ as

$$\Delta_A \equiv A - N_* \quad (> 0). \tag{39}$$

Hence, we obtain

$$29.5 + \frac{1}{2}\Delta_A \lesssim A \lesssim 34 + \frac{1}{2}\Delta_A, \tag{40}$$

where the first inequality is saturated with $\Delta_r \simeq 10$ and the second with $\Delta_r \simeq 1$. Thus, if $6 \lesssim \Delta_A \lesssim 15$ then $M \simeq TeV$ is possible with $A \simeq 37$. Note that the formula (33) relates the number of efolds and the horizon-size wavelength of cosmological fluctuation. It assumes

[1] For problems with multi-throat scenarios and a possible resolution, see [15].

that the universe is at the inflationary stage at $N_*$ efolds from the end of inflation. However, it is more comfortable to have the total number of efolds $N$ slightly bigger, to dilute potential inhomogeneities which could exit prior inflation. Thus, from this point of view, it is plausible to have $\Delta_A \gtrsim 6$. On this other hand, $\Delta_A \lesssim 15$ implies that the total number of efolds $N$ is not significantly larger than the number of efolds $N_*$ corresponding to the wavelength of the present horizon scale: $N - N_* \lesssim 15$.

## 6 Other Examples of Conformal Inflation

Besides the warped brane inflation of the string theory, we can give other, phenomenological examples in which the conformal inflation is realized. Consider the potential $V(\phi)$, which is flat for large values of $\phi$, $V = M^4 = constant$, but abruptly drops to zero at some value $\phi_f$. Let us estimate energy scale $M$ and correspondingly the number of efolds in this model. Number of efolds $N$ is given by the formula (17). The choice of initial value $\phi_i$ is a matter of taste: string theorist would put $\phi_i \sim M_p$ while phenomenologically we can take it much larger.

Let us estimate the lower bound on $\phi_f$. In order to have effective mass $V_{\phi\phi}$ to be less than $M_p^2$ everywhere, we estimate $\phi_f^2 M_p^2 \gtrsim M^4$. On the other hand, we have $H^2 M_p^2 \sim M^4$ so that $\phi_f \gtrsim H$. If we take $\phi_i \sim M_p$, then we have $N \lesssim \ln\frac{M_p}{H} \sim 2\ln\frac{M_p}{M}$. On the other hand, the constraint on $N$ is related to the energy scale of inflation via the formula (33). Combining both formulas for $N$, we obtain a condition for $M$. The result is $M \lesssim 10^8 - 10^{10}$ GeV.

Now, if we increase $\phi_i$, the value of $M$ will increase. Increase of $\phi_f$, in contrast, decreases $M$.

Similar estimation can be made for the hybrid inflationary model with the potential $V(\phi,\sigma) = \frac{\lambda}{4}(\sigma^2 - v^2)^2 + \frac{g^2}{2}\phi^2\sigma^2$ plus conformal couplings. In this case $\phi_f$ is associated with the critical value $\phi_c = \frac{\sqrt{\lambda}}{g}v$. The result for $M$ will depends on $\phi_i$ and the ratio $g/\lambda^{1/4}$.

## 7 Cosmological Fluctuations in Conformal Inflation

At this point the model we are investigating is far from being complete. We have not elaborated the picture of cosmological fluctuations in the models of conformal inflation. In this section we just discuss some ideas about generation of cosmological fluctuations in the model. Conformally coupling inflaton cannot be responsible for generation of primordial fluctuations. Indeed, conformal inflaton with the constant potential is described by the equation $\left(\nabla_\mu\nabla^\mu + \frac{1}{6}R\right) = 0$, which can be mapped by conformal transformation $\phi = \varphi/a$ to the wave equation in the flat space-time. Thus, conformal inflaton fluctuations cannot be generated during inflation.

Cosmological fluctuations, however, can be generated as the modulated (e.g. inhomogeneous reheating) fluctuations [9, 10] or curvaton fluctuations [11, 12]. Obvious candidate for these are fluctuations of the angular degrees of freedom. Indeed, setting of the warped brane-antibrane inflation includes $D3$ brane and anti $D3$ brane in six dimensional Klebanov-Strassler throat geometry [16]. While radial distance of $D3$-$\bar{D}3$ pair acts as an inflaton, its five bulk angular coordinates are other moduli fields. It turns out that angular fluctuations are effectively conformal scalar fields and cannot be responsible for the modulated fluctuations. This is demonstrated in the Appendix B. If inflaton is not conformal, angular fluctuations are also non-conformal and generated during inflation. Similarly, modulated fluctuations in principle can be provided by the scalars of SM sector (the Higgs fields) or from the moduli field coupled with the SM sector. For this we need a light scalar field with the effective mass smaller than the Hubble parameter. If we deal with the fields with the mass term $m^2\phi^2$ which is not changing during the inflation and reheating, we encounter here new difficulties. Indeed, the Hubble parameter in this low-energy inflation is very small, $H \sim M^2/M_p$, $10^{-3}eV < H < 10eV$. Potential candidates for very light scalars would be the Kähler moduli or axions associated with the four-cycles of the Calabi-Yau manifold. Kähler moduli $\tau$ may have very flat potential at large values of $\tau$, while axion is very light. Another potential candidate of the light scalar is the scalar mode that is the super partner of the Goldstone mode associated with $U(1)_B$ symmetry breaking in KS geometry [18, 19]. It is massless in the limit of infinite throat but shall acquire mass when the throat is compactified. Other possibilities is to have the scalar field potential such that moduli or curvaton scalar is very light scalars during inflation but became massive at lower energies after inflation. The amplitude of modulated or curvaton fluctuations is proportional to $H/\phi_*$, where $\phi_*$ is the mean value amplitude of the moduli/curvaton during inflation. One has to adjust $\phi_*$ to be small to reach required level of fluctuations $10^{-5}$. Potential modulated or curvaton fluctuations require further investigations which are out of the scope of this presentation.

In other models of conformal inflation, where its energy scale is larger, the value $\phi_*$ can be much larger. For instance, for $M \sim 10^{10}$ GeV, $\phi_* \sim 10^7$ GeV.

In the following we shall estimate the scalar spectral index and its running for the modulated reheating in almost conformal inflation.

### 7.1 Basic equations

For a general value of the curvature coupling $\xi$, basic equations are

$$\begin{aligned} 3M_p^2\left(1-\xi\phi^2/M_p^2\right)H^2 &= \frac{1}{2}\dot{\phi}^2+6\xi H\phi\dot{\phi}+V(\phi), \\ \ddot{\phi}+3H\dot{\phi}+6\xi(\dot{H}+2H^2)\phi+V' &= 0. \end{aligned} \tag{41}$$

By introducing $\pi \equiv \dot{\phi}+6\xi H\phi$, they are simplified as

$$\begin{aligned} 3M_p^2\left[1-\xi(1-6\xi)\phi^2/M_p^2\right]H^2 &= \frac{1}{2}\pi^2+V(\phi), \\ \dot{\pi}+3(1-2\xi)H\pi+V'-6\xi(1-6\xi)H^2\phi &= 0. \end{aligned} \tag{42}$$

It is easy to show that

$$\begin{aligned} \frac{\dot{H}}{H^2} &= -\frac{3\left[\frac{1}{2}(1-2\xi)\pi^2-2\xi(1-6\xi)H\pi\phi+\xi V'\phi\right]}{\frac{1}{2}\pi^2+V}-\frac{6\xi^2(1-6\xi)\phi^2/M_p^2}{1-\xi(1-6\xi)\phi^2/M_p^2} \\ &= -\frac{3\left[\frac{1}{2}(1-2\xi)\Pi^2+2\xi\tilde{\epsilon}\right]}{\frac{1}{2}\Pi^2+1}+\frac{2\sqrt{3}\kappa\Pi}{\sqrt{\left(\frac{1}{2}\Pi^2+1\right)(1-\kappa\Phi)}}-\frac{6\xi\kappa\Phi}{1-\kappa\Phi}, \\ \frac{\ddot{H}}{H^3} &= \frac{4\xi(1-6\xi)(\pi/H-6\xi\phi)\phi}{M_p^2\left[1-\xi(1-6\xi)\phi^2/M_p^2\right]}\frac{\dot{H}}{H^2}+\frac{3\left[(3-10\xi)\pi^2-12\xi(1-6\xi)H\pi\phi\right]}{\frac{1}{2}\pi^2+V} \\ &\quad +\frac{12\xi^2(1-6\xi)\phi^2/M_p^2}{1-\xi(1-6\xi)\phi^2/M_p^2}+\frac{3\left[(1-3\xi)\pi/H-2\xi(1-9\xi)\phi\right]V'}{\frac{1}{2}\pi^2+V}-\frac{3\xi\left(\pi/H-6\xi\phi\right)V''\phi}{\frac{1}{2}\pi^2+V} \\ &= 4\left[\frac{\sqrt{3}\kappa\Pi}{\sqrt{\left(\frac{1}{2}\Pi^2+1\right)(1-\kappa\Phi)}}-\frac{6\xi\kappa\Phi}{1-\kappa\Phi}\right]\frac{\dot{H}}{H^2}+\frac{3\left[(3-10\xi)\Pi^2-4\xi(1-9\xi)\tilde{\epsilon}+6\xi^2\Phi^2\eta\right]}{\frac{1}{2}\Pi^2+1} \\ &\quad +\frac{12\xi\kappa\Phi}{1-\kappa\Phi}-\frac{12\sqrt{3}\kappa\Pi}{\sqrt{\left(\frac{1}{2}\Pi^2+1\right)(1-\kappa\Phi)}}+3\left[\sqrt{6}(1-3\xi)e-\sqrt{3}\xi\Phi\eta\right]\Pi\sqrt{\frac{1-\kappa\Phi}{\left(\frac{1}{2}\Pi^2+1\right)^3}}, \end{aligned} \tag{43}$$

where

$$\Phi \equiv \frac{\phi}{M_p}, \quad \Pi \equiv \frac{\pi}{\sqrt{V}}, \quad \epsilon \equiv \frac{M_p^2}{2}\left(\frac{V'}{V}\right)^2, \quad \eta \equiv \frac{M_p^2 V''}{V}, \quad \tilde{\epsilon} \equiv \frac{V'\phi}{2V}, \tag{44}$$

and

$$\kappa \equiv \frac{\xi(1-6\xi)\phi}{M_p}, \quad e \equiv \frac{M_p V'}{\sqrt{2}V}. \tag{45}$$

Note that

$$e^2 = \epsilon, \quad e\Phi = \sqrt{2}\tilde{\epsilon}. \tag{46}$$

### 7.2 Approximate equations

Let us define

$$V_0 \equiv \frac{V}{1-\xi(1-6\xi)\phi^2/M_p^2}, \quad V_1 \equiv V' - 2\xi(1-6\xi)V_0\phi/M_p^2, \tag{47}$$

$$\epsilon_\xi \equiv \frac{M_p^2}{2}\left(\frac{V_1}{V_0}\right)^2, \quad \eta_\xi \equiv \frac{M_p^2 V_1'}{V_0}, \quad \tilde{\epsilon}_\xi \equiv \frac{V_1\phi}{2V_0}, \quad \tilde{\eta}_{\xi,c} \equiv \eta_\xi + \frac{c+3(1-2\xi)}{3}\left(\frac{6\xi V_1'\phi}{V_1} + c\right), \tag{48}$$

and

$$e_\xi \equiv \frac{M_p}{\sqrt{2}}\frac{V_1}{V_0}. \tag{49}$$

Note that

$$\begin{aligned} e_\xi &= (1-\kappa\Phi)e - \sqrt{2}\kappa, \\ \epsilon_\xi &= \left[(1-\kappa\Phi)e - \sqrt{2}\kappa\right]^2, \\ \eta_\xi &= (1-\kappa\Phi)\eta - 4\xi(1-6\xi)\tilde{\epsilon} - \frac{2\xi(1-6\xi)(1+\kappa\Phi)}{1-\kappa\Phi}, \\ \tilde{\epsilon}_\xi &= (1-\kappa\Phi)\tilde{\epsilon} - \kappa\Phi. \end{aligned} \tag{50}$$

When

$$\epsilon_\xi \ll 1, \quad |\tilde{\epsilon}_\xi| \ll 1, \tag{51}$$

and the constant $c$ is chosen so that

$$|\tilde{\eta}_{\xi,c}| \ll 1, \tag{52}$$

eqs. (42) are apoproximated as

$$3M_p^2 H^2 \simeq V_0(\phi), \tag{53}$$

$$\tilde{c}H\pi \simeq -V_1(\phi), \tag{54}$$

where

$$\tilde{c} = c + 3(1-2\xi). \tag{55}$$

The error to the first approximate equation is given by

$$\frac{3M_p^2H^2 - V_0}{V_0} = \frac{1}{2}\Pi^2. \tag{56}$$

To estimate the error to the second equation, let us define

$$\Delta \equiv \frac{\tilde{c}H\pi + V_1}{V_1}. \tag{57}$$

It is straightforward to show that $\Delta$ satisfies the following differential equation.

$$H^{-1}\dot{\Delta} + f\Delta = s, \tag{58}$$

where

$$\begin{aligned} f &= \tilde{c} + \frac{3}{\tilde{c}}\left(\tilde{\eta}_{\xi,c} - \eta_\xi\right) + \frac{2}{\tilde{c}}\left[-3\tilde{\eta}_{\xi,c} + \frac{3\eta_\xi\Pi^2/2}{1+\Pi^2/2} + \frac{6\xi(1-\xi)}{1-\xi(1-6\xi)\Phi^2}\cdot\frac{\tilde{\epsilon}_\xi}{1+\Pi^2/2}\right] - \frac{\dot{H}}{H^2} \\ &\simeq \tilde{c} - \frac{3\eta_\xi}{\tilde{c}}, \\ s &= \frac{1+\Delta^2}{\tilde{c}}\left[-3\tilde{\eta}_{\xi,c} + \frac{3\eta_\xi\Pi^2/2}{1+\Pi^2/2} + \frac{6\xi(1-\xi)}{1-\xi(1-6\xi)\Phi^2}\cdot\frac{\tilde{\epsilon}_\xi}{1+\Pi^2/2}\right] - \frac{\dot{H}}{H^2}. \end{aligned} \tag{59}$$

Provided that $f > 0$ and that $|s|$ is small, $|\Delta|$ approarches a small value.

It is also easy to show that $\Pi$ satisfies

$$\Pi\sqrt{1+\frac{1}{2}\Pi^2} = \Pi_0(1-\Delta), \quad \Pi_0 = -\frac{\sqrt{6}e_\xi}{\tilde{c}\sqrt{1-\kappa\Phi}}. \tag{60}$$

Now, let us introduce a small number $\epsilon_1$ and assume that

$$e = O(\epsilon_1), \quad \tilde{\eta}_{\xi,c} = O(\epsilon_1), \quad \xi\Phi\eta = O(\epsilon_1), \quad \kappa = O(\epsilon_1), \quad \Phi = O(1). \tag{61}$$

Then

$$\epsilon = O(\epsilon_1^2), \quad \tilde{\epsilon} = O(\epsilon_1), \quad \tilde{\epsilon}_\xi = O(\epsilon_1), \quad e_\xi = O(\epsilon_1), \quad \Pi_0 = O(\epsilon_1). \tag{62}$$

If $\Delta = O(\epsilon_1)$ then we obtain

$$\Pi = \Pi_0 + O(\epsilon_1^2), \tag{63}$$

and

$$\frac{\dot{H}}{H^2} = O(\epsilon_1), \quad s = O(\epsilon_1). \tag{64}$$

Therefore, $\Delta = O(\epsilon_1)$ is indeed consistent with (58), provided that $f > 0$.

### 7.3 Spectral index and running

In general the spectral index and its running are written as

$$\begin{aligned} n_s - 1 &= \frac{d\ln H^2}{d\ln(aH)} = 2\left(1+\frac{\dot{H}}{H^2}\right)^{-1}\frac{\dot{H}}{H^2}, \\ \frac{dn_s}{d\ln(aH)} &= 2\left(1+\frac{\dot{H}}{H^2}\right)^{-3}\left[\frac{\ddot{H}}{H^3} - 2\left(\frac{\dot{H}}{H^2}\right)^2\right]. \end{aligned} \tag{65}$$

By substituting (63) into (43), we obtain

$$n_s - 1 \simeq -2\tilde{\epsilon} - \left(\frac{12}{\tilde{c}^2}+\Phi^2\right)\epsilon + 2\Phi^2\delta\xi + O(\epsilon_1^3), \tag{66}$$

and

$$\frac{dn_s}{d\ln(aH)} \simeq \eta\Phi^2 + \left(2+3\eta\Phi^2+\frac{6\eta}{\tilde{c}}\right)\tilde{\epsilon} + \left[\frac{6(8-3\tilde{c})}{\tilde{c}^2}+\Phi^2\right]\epsilon - 4\Phi^2\delta\xi + O(\epsilon_1^3), \tag{67}$$

Here, we have supposed that

$$e = O(\epsilon_1), \quad \tilde{\eta}_{\xi,c} = O(\epsilon_1), \quad \eta = O(\epsilon_1), \quad \delta\xi \equiv \xi - \frac{1}{6} = O(\epsilon_1^2), \quad \Phi = O(1). \tag{68}$$

This is a bit stronger than (61). By using the identity

$$\eta\Phi = \sqrt{2}e\left[\frac{3}{\tilde{c}}(\tilde{\eta}_c - \eta) - \tilde{c} + 2\right], \quad \tilde{\eta}_c \equiv \eta + \frac{\tilde{c}}{3}\left(\frac{V''\phi}{V'} + \tilde{c} - 2\right), \tag{69}$$

(67) can be rewritten as

$$\frac{dn_s}{d\ln(aH)} \simeq 2\left(3-\tilde{c}+\frac{3}{\tilde{c}}\tilde{\eta}_c\right)\tilde{\epsilon} + \left[\frac{6(8-3\tilde{c})}{\tilde{c}^2}+(7-3\tilde{c})\Phi^2\right]\epsilon - 4\Phi^2\delta\xi + O(\epsilon_1^3). \tag{70}$$

Based on the formulas (66) and (70), observational constraints are given in ref. [20].

# 8 Summary

Contrary to the folklore that conformal coupling should spoil inflationary behavior, we have shown that a conformally coupled scalar field can actually drive inflation. This surprising result potentially has significant impacts on development of the phenomenological inflationary models as well as inflationary models in string theory since an inflaton for warped brane inflation typically has conformal coupling. Indeed, the inflaton potential (without further fine-tuning) for warped brane inflation (1) satisfies all the conditions for conformal inflation (23), (24) and (25) in the regime (27). As shown in Figure 1, we have confirmed the onset of inflationary behavior (for a wide range of initial conditions) by using the phase portrait method.

We have shown the following three features of the conformal inflation:

i) The conformal inflation is a rapid roll inflation. The inflaton rolls as $\phi = \varphi_0/a$, where $\varphi_0$ is a constant and $a$ is the scale factor.

ii) For the conformal inflation in the context of warped extra dimensions, the number of inflationary efoldings and the scale of inflation are related to the warp factor of the throat geometry. As a result, the scale of inflation $M$ is bounded from above as $M \lesssim 10^6 GeV$. It has similar bound in the context of the hybrid inflation (unless $\phi$ is not bounded by $M_p$ from above).

iii) Cosmological fluctuations originated from modulated or curvaton fluctuations.

Since the conformal inflation arises rather naturally in the context of warped brane inflation in string theory, it is certainly worthwhile investigating more details of its properties.

Another interesting question is about how realization of inflation depends on the value of the curvature coupling $\xi$. The best way to study this issue apparently would be to construct the phase portrait of the dynamical system (6)-(7), and to seek for the inflationary separatrices. For instance, as it was done in [21]. Looking at the constraint equation (6), we notice that at least the cases $\xi = 0$ and $1/6$ have nice properties: the kinetic terms can be separated from other terms, as $\dot{\phi}^2$ for $\xi = 0$ and as $\pi^2$ for $\xi = 1/6$. We leave this issue for the future investigation.

We have given formulas of spectral index and its running, taking into account possible deviation from exact conformal coupling. Observational constraints on the brane-antibrane inflation model were given in ref. [20].

# Appendix A: Conditions for Inflation

This Appendix continues discussion of the conditions for conformal inflation. We suppose that the equations of motion (19) are approximated by

$$\begin{aligned} H^2 &\simeq \frac{1}{3M_p^2}V, \\ \tilde{c}H\pi &\simeq -V', \end{aligned} \tag{71}$$

where $\tilde{c}$ is a constant being determined in the following argument.

Validity of the first approximate equation requires that $\pi^2/V$ be sufficiently smaller than unity. Since

$$\frac{\pi^2}{V} = \frac{(\tilde{c}H\pi)^2}{\tilde{c}^2H^2V} \simeq \frac{6}{\tilde{c}^2}\epsilon, \ . \tag{72}$$

This demands that $\epsilon \ll 1$. Before seeking the consistency condition for the second approximate equation, let us consider the condition for the Hubble expansion rate $H$ to be almost constant. This condition is written as $|\dot{H}/H^2| \ll 1$, where $\dot{H}/H^2$ is estimated by using (21) and (72) as

$$-\frac{\dot{H}}{H^2} = \frac{\pi^2/V + \tilde{\epsilon}}{\pi^2/2V + 1} \simeq \frac{6\epsilon/\tilde{c}^2 + \tilde{\epsilon}}{1 + 3\epsilon/\tilde{c}^2} \simeq \tilde{\epsilon} \ . \tag{73}$$

and we have used the condition $\epsilon \ll 1$. Thus, in order for the Hubble expansion rate $H$ to be almost constant, it is required that $|\tilde{\epsilon}| \ll 1$.

Now let us seek the consistency condition for the second approximate equation. Since it ignores $\dot{\pi} - (\tilde{c}-2)H\pi$ relative to $\tilde{c}H\pi$, we have to estimate $|[\dot{\pi} - (\tilde{c}-2)H\pi]/(\tilde{c}H\pi)|$ and to demand that it be sufficiently smaller than unity. For this purpose let us take the time derivative of the second approximate equation as

$$H\dot{\pi} + \dot{H}\pi \simeq -\frac{1}{\tilde{c}}V''(\pi - H\phi). \tag{74}$$

Hence,

$$\dot{\pi} - (\tilde{c}-2)H\pi \simeq -\frac{V''\pi}{\tilde{c}H} - \frac{\dot{H}\pi}{H} + \frac{V''\phi}{\tilde{c}} - (\tilde{c}-2)H\pi. \tag{75}$$

Thus, by using the approximate equations and the condition $|\dot{H}/H^2| \ll 1$ demanded above, we obtain

$$\begin{aligned}\frac{\dot{\pi} - (\tilde{c}-2)H\pi}{\tilde{c}H\pi} &\simeq -\frac{V''}{\tilde{c}^2H^2} - \frac{\dot{H}}{\tilde{c}H^2} + \frac{V''\phi}{\tilde{c}^2H\pi} - \frac{\tilde{c}-2}{\tilde{c}} \\ &\simeq -\frac{3\eta}{\tilde{c}^2} - \frac{1}{\tilde{c}}\left[\frac{V''\phi}{V'} + (\tilde{c}-2)\right] = -\frac{3\eta_c}{(c+2)^2},\end{aligned} \tag{76}$$

where we have set $\tilde{c} = c+2$ in the last equality. Therefore, $|[\dot{\pi} - (\tilde{c}-2)H\pi]/(\tilde{c}H\pi)| \ll 1$ is equivalent to (25), and the second approximate equation is justified.

## Appendix B: Angular Fluctuations of Mobile Brane in the Warped Geometry

First we recall the KS geometry. The Klebanov-Strassler geometry has the simple ansatz:

$$ds^2 = h^{-1/2}(\tau)\eta_{\mu\nu}dx^\mu dx^\nu + h^{1/2}(\tau)ds_6^2, \tag{77}$$

where $x^\mu$ ($\mu = 0, \cdots, 3$) are 4-dimensional coordinates and $ds_6^2$ is the metric of the deformed conifold

$$ds_6^2 = \frac{\epsilon^{4/3}}{2}K(\tau)\left[\frac{1}{3K^3(\tau)}\left(d\tau^2 + (g^5)^2\right) + \cosh^2\left(\frac{\tau}{2}\right)\left((g^3)^2 + (g^4)^2\right) + \sinh^2\left(\frac{\tau}{2}\right)\left((g^1)^2 + (g^2)^2\right)\right]. \tag{78}$$

Here,

$$K(\tau) = \frac{(\sinh(2\tau) - 2\tau)^{1/3}}{2^{1/3}\sinh\tau}, \tag{79}$$

and $g^i$ ($i = 1, \cdots, 5$) are orthonormal basis defined by

$$\begin{aligned} g^1 &= \frac{e^1 - e^3}{\sqrt{2}}, \quad g^2 = \frac{e^2 - e^4}{\sqrt{2}}, \\ g^3 &= \frac{e^1 + e^3}{\sqrt{2}}, \quad g^4 = \frac{e^2 + e^4}{\sqrt{2}}, \quad g^5 = e^5, \end{aligned} \tag{80}$$

where

$$\begin{aligned} e^1 &\equiv -\sin\theta_1 d\phi_1, \quad e^2 \equiv d\theta_1, \\ e^3 &\equiv \cos\psi\sin\theta_2 d\phi_2 - \sin\psi d\theta_2, \\ e^4 &\equiv \sin\psi\sin\theta_2 d\phi_2 + \cos\psi d\theta_2, \\ e^5 &\equiv d\psi + \cos\theta_1 d\phi_1 + \cos\theta_2 d\phi_2. \end{aligned} \tag{81}$$

Because of the warp factor $h^{-1/2}(\tau)$, this geometry is often called the warped deformed conifold. The R-R 3-form field strength $F_3$ and the NS-NS 2-form potential $B_2$ also have the $Z_2$ symmetric $((\theta_1,\, \phi_1) \leftrightarrow (\theta_2,\, \phi_2))$ ansatz:

$$\begin{aligned} F_3 &= \frac{M\alpha'}{2}\left\{g^5 \wedge g^3 \wedge g^4 + d[F(\tau)(g^1 \wedge g^3 + g^2 \wedge g^4)]\right\}, \\ B_2 &= \frac{g_s M\alpha'}{2}[f(\tau)g^1 \wedge g^2 + k(\tau)g^3 \wedge g^4], \end{aligned} \tag{82}$$

where $F(0) = 0$ and $F(\infty) = 1/2$. For this ansatz with the additional condition

$$g_s^2 F_3^2 = H_3^2, \tag{83}$$

we can consistently set the dilaton $\phi$ and the R-R scalar $C_0$ to zero. The BPS saturated solution found by Klebanov and Strassler is

$$\begin{aligned} F(\tau) &= \frac{\sinh\tau - \tau}{2\sinh\tau}, \\ f(\tau) &= \frac{\tau\coth\tau - 1}{2\sinh\tau}(\cosh\tau - 1), \\ k(\tau) &= \frac{\tau\coth\tau - 1}{2\sinh\tau}(\cosh\tau + 1), \end{aligned} \tag{84}$$

and

$$h(\tau) = 2^{2/3} \cdot (g_s M\alpha')^2 \epsilon^{-8/3} I(\tau), \tag{85}$$

where

$$I(\tau) = \int_\tau^\infty dx \frac{x\coth x - 1}{\sinh^2 x}(\sinh(2x) - 2x)^{1/3}. \tag{86}$$

For this solution,

$$C_4 = h^{-1} dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \tag{87}$$

in a particular gauge. For large $g_s M$ the curvature is small everywhere and we can trust the supergravity description.

When $g_s M$ is sufficiently large, we can treat a $D3$-brane as a probe brane. The action for the probe $D3$-brane is

$$S_{D3} = -T_3 \int d^4\xi e^{-\phi}\sqrt{-\det(G_{\alpha\beta} - B_{\alpha\beta})} + T_3 \int d^4\xi C_4, \tag{88}$$

where $\xi^\alpha$ $(\alpha = 0, \cdots, 3)$ are intrinsic coordinates on the $D3$-brane, $T_3$ is the tension and

$$G_{\alpha\beta} = G_{MN}\frac{\partial x^M}{\partial\xi^\alpha}\frac{\partial x^N}{\partial\xi^\beta}, \quad B_{\alpha\beta} = (B_2)_{MN}\frac{\partial x^M}{\partial\xi^\alpha}\frac{\partial x^N}{\partial\xi^\beta}. \tag{89}$$

In the following we shall adopt a gauge in which brane coordinates $\xi^\alpha$ coincide with $x^\alpha$:

$$x^\alpha = \xi^\alpha, \quad \psi^m = \psi^m(\xi^\alpha), \tag{90}$$

where $\{\psi^m\}$ $(m = 5, \cdots, 10)$ represents $\{\tau, \psi, \theta_1, \phi_1, \theta_2, \phi_2\}$. In the non-relativistic limit,

$$S_{D3} = -\frac{T_3}{2}\int d^4\xi \gamma_{mn}\eta^{\alpha\beta}\frac{\partial\psi^m}{\partial\xi^\alpha}\frac{\partial\psi^n}{\partial\xi^\beta}. \tag{91}$$

where $\gamma_{mn} d\psi^m d\psi^n = ds_6^2$ is the metric of the deformed conifold given by (78). For a cosmological background, if the energy scale is sufficiently low, we can replace $\eta_{\mu\nu}$ by $g_{\mu\nu}$ to obtain

$$S_{D3} = -\frac{T_3}{2}\int d^4\xi \sqrt{-g}\gamma_{mn} g^{\mu\nu}\frac{\partial\psi^m}{\partial x^\mu}\frac{\partial\psi^n}{\partial x^\nu}. \tag{92}$$

For large $\tau$, $K(\tau)$ and $I(\tau)$ behaves as

$$K(\tau) \simeq 2^{1/3} e^{-\tau/3}, \quad I(\tau) \simeq \frac{3}{2^{1/3}} e^{-4\tau/3}. \tag{93}$$

Thus, we have

$$\gamma_{mn} \partial\psi^m \partial\psi^n = \frac{\epsilon^{4/3}}{2^{2/3}} e^{2\tau/3} \left[\frac{1}{6}\left(d\tau^2 + (g^5)^2\right) + (g^3)^2 + (g^4)^2 + (g^1)^2 + (g^2)^2\right]. \tag{94}$$

By introducing the canonically normalized variable $\phi$ as

$$\phi = \frac{\epsilon^{2/3}\sqrt{3T_3}}{2^{5/6}} e^{\tau/3}, \tag{95}$$

we obtain

$$T_3 \gamma_{mn} \partial\psi^m \partial\psi^n = \partial\phi\partial\phi + \frac{2}{3}\phi^2 \left[\frac{1}{6}(g^5)^2 + (g^3)^2 + (g^4)^2 + (g^1)^2 + (g^2)^2\right]. \tag{96}$$

Therefore,

$$\begin{aligned} S_{D3} \ = \ & -\frac{1}{2}\int d^4\xi \sqrt{-g} g^{\mu\nu} \left\{\partial_\mu\phi\partial_\nu\phi + \frac{2}{3}\phi^2 \left[\frac{1}{6} g^5(\partial_\mu) g^5(\partial_\nu)\right.\right. \\ & \left.\left. + g^3(\partial_\mu)g^3(\partial_\nu) + g^4(\partial_\mu)g^4(\partial_\nu) + g^1(\partial_\mu)g^1(\partial_\nu) + g^2(\partial_\mu)g^2(\partial_\nu)\right]\right\}. \end{aligned} \tag{97}$$

Note that the kinetic term of angular coordinates has the overall factor $\phi^2$. In more general situation where the KS geometry is modified, the overall factor will be the angular component of the 6-dimensional metric multiplied by the $D3$-brane tension $T_3$.

For simplicity, we continue our analysis with single angular coordinate. Result will be similar for the full geometry (97). Let us consider a scalar field $\theta$ coupled to the conformal inflaton $\phi$ described by

$$I = \int d^4x \sqrt{-g} \left[\frac{M_p^2 R}{2} - \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - \frac{1}{2}\phi^2 \partial^\mu\theta\partial_\mu\theta - V(\phi) - \frac{\xi}{2} R\phi^2\right], \tag{98}$$

where $\xi = 1/6$ and the potential $V(\phi)$ is assumed to satisfy the conditions for conformal inflation. During the conformal inflation,

$$\phi \simeq \frac{\varphi_0}{a}, \tag{99}$$

where $\varphi_0$ is a constant. Hence, the equation of motion for $\theta$ is

$$\ddot{\theta}_{\vec{k}} + H\dot{\theta}_{\vec{k}} + \frac{\vec{k}^2}{a^2}\theta_{\vec{k}} = 0 \ . \tag{100}$$

Switching to the conformal time brings this equation to the wave equation in the flat spacetime.

$$\theta''_{\vec{k}} + \vec{k}^2 \theta_{\vec{k}} = 0 \ . \tag{101}$$

Thus, angular fluctuations are effectively described by the equation for the conformal fields and, thus, cannot be responsible for the cosmological modulated fluctuations.

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